Mechanism for Charge Bunching of Bosons in High-Energy Collisions

C. S. Lam and S. Y. Lo[(a)]
Department of Physics, McGill University, Montreal, Quebec H3A 2T8, Canada
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Induced transitions give rise to enhanced probabilities of finding a large number of pions of the same charge. The exact charge distribution with induced emission fully taken into account is solved in a simple model. The enhancements are found to be large. These effects are observable in high-energy heavy-ion collisions, and possibly are responsible for the cosmic-ray Centauro-like events.

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Unusual charge bunching of shower particles has been observed in cosmic-ray experiments in the form of the so-called Centauro and mini-Centauro events.[1] These are events with large charge multiplicities but very few accompanying neutral pions. On the other hand, such events are not found in the CERN $\bar{p}p$ collider.[2] One possibility consistent with these observations is that such events occur only when complex nuclear targets and large numbers of collisions are involved. This motivates us to consider induced transition as a possible mechanism for producing abnormal charge bunchings. Similar to the laser mechanism, we will refer to this mechanism for amplifying same-charge pions, kaons, and in general bosons, as the PASER, KASER, and BASER mechanisms, respectively.

According to the BASER mechanism, the probability of having several final-state bosons in the same quantum state (charge, energy, momentum, etc.) is enhanced compared to those not in the same quantum state, on account of the Bose statistics. Take for example heavy-ion collisions at a suitable energy when N Δ^0 resonances are produced approximately at rest. Since each isolated Δ^0 has $\frac{1}{3}$ probability of decaying into $p+\pi^-$ (channel A), and $\frac{2}{3}$ probability decaying into $n+\pi^0$ (channel B), the probability of N isolated Δ^0 decaying into a π^-'s and $b=N-a$ π^0's is given by the binomial distribution $\binom{N}{a}(\frac{1}{3})^a(\frac{2}{3})^b$, which is very small for unusual charge bunchings (N large and either a or b small). But in reality these N Δ^0's are not isolated. If the first Δ^0 decays via channel A, the PASER mechanism gives rise to an enhanced probability for the subsequent Δ^0's to decay also via channel A, thus fortifying the probability for same-charge bunching to occur. We will calculate in the rest of this paper this enhancement and will show that it can be very large.

This PASER mechanism for Δ decay might explain unusual charge bunchings of Centauro-like events if the primary cosmic-ray particle is a heavy nucleus. If it is a proton, induced production of charged pions and neutral ρ in the beam fragmentation region (where all pions typically have comparable energies of several hundreds of megaelectronvolts in the rest frame of the beam particle) conceivably can still produce the unusual charge bunchings. In any case, the BASER mechanism for abnormal charge bunchings exists, is important, and should be looked for in high-energy heavy-ion collision experiments.

The exact calculation of the amplification factor of charge bunching is difficult on account of our relative ignorance of the particle production mechanism. Nevertheless, once we assume the existence of a given number of resonances, the amplification factor due to induced decay can be calculated, at least in the context of the simplified model discussed below. It is important to emphasize in this connection that our BASER "device" is a nucleus. It is highly microscopic; thus full-fledged quantum mechanics must be used. Moreover, because of the smallness of the BASER device, momentum spread becomes important. Thus the usual signature of laser light, that of momentum bunching, will be diluted and is probably not as good a signature for the BASER mechanism as charge bunching.

We will devote the rest of this paper to a solvable model describing the decay of N Δ^0's approximately at rest. We assume the N resonances to be confined to a rectangular volume V, with small, distinct, and discrete Fermi momenta[3] $\vec{q}_i$ $(1\leq i\leq N)$. Theoretically, the pion and the nucleon momenta should lie in the continuum, but we adopt a coarse-grain description of them in this model. We approximate the continuum by those discrete states that satisfy periodic boundary condition in the volume. In calculating decay matrix elements, the final-state wave functions are always multiplied by the Δ wave functions, which vanish outside the volume. So effectively we are staying inside the volume V where the different momentum states form an orthonormal complete set. The passage back to the continuum is given by the familiar rule $\sum_{\vec{k}}(2\pi)^3/V = \int d^3k$.

The effective interaction Hamiltonian governing the decay of Δ^0 is[3]

$$H = g[(2\pi)^3/V]^{1/2} \sum_{\vec{1}_1, \vec{1}_2, \vec{1}_3} [(\tfrac{1}{3})^{1/2} p^+(\vec{1}_3)\pi_-^{\pm}(\vec{1}_2) + (\tfrac{2}{3})^{1/2} N^+(\vec{1}_3)\pi_0^+(\vec{1}_2)]\Delta(\vec{1}_1)\delta_{\vec{1}_1, \vec{1}_2 + \vec{1}_3} + \text{H.c.}, \tag{1}$$

where the particle symbols represent the corresponding field operators normalized in such a way that they create normalized momentum states from the vacuum. An n-particle momentum state $|\vec{k}\rangle \equiv |\vec{k}_1, \ldots, \vec{k}_n\rangle$ is normalized to unity when all the momenta $\vec{k}_i$'s are different. The coupling constant g is related to the width Γ of Δ^0 by

$$\Gamma = g^2[(2\pi)^3/V]\sum_{\vec{k}}(2\pi)\delta[(k^2+\mu^2)^{1/2} + (k^2+m^2)^{1/2} - M] = 8\pi^2 g^2 k k_1^0 k_2^0/M, \tag{2}$$

where μ, m, and M are respectively the mass of pion, nucleon, and Δ; k, k_1^0, k_2^0 are the center-of-mass momentum and energies of the decay products.

Suppose a of the N resonances decay via channel A and $b = N - a$ decay via channel B. Let Δ_i^A (Δ_j^B) denote a resonance with Fermi momentum $\vec{q}_i$ ($\vec{q}_{a+j}$) decaying via channel A (B). The corresponding nucleon momentum[3] will be denoted by $\vec{p}_i$ ($\vec{p}_j'$). The initial state is then $|\Delta^A;\Delta^B\rangle \equiv |\Delta_1^A, \ldots, \Delta_a^A;\Delta_1^B, \ldots, \Delta_b^B\rangle$, and the final state is $\langle\vec{k}, \vec{k}';\vec{p}, \vec{p}'|$. Note that the a (b) π^-'s (π^0's) with momenta $\vec{k}_i$ ($\vec{k}_j'$) are identical particles so that the pion from the decay of Δ_i^A does not necessarily have momentum $\vec{k}_i$, although the corresponding nucleon momentum[3] is always labeled by $\vec{p}_i$. The lowest-order transition rate is given by Nth-order perturbation theory to be

$$R = \frac{N!}{a!b!}\sum_{\substack{\vec{k}, \vec{k}' \\ \vec{p}, \vec{p}'}} \frac{1}{a!b!} |\langle\vec{k}, \vec{k}';\vec{p}, \vec{p}'|H[(\Delta\hat{E} + i\tfrac{1}{2}\Gamma)^{-1}H]^{N-1}|\Delta^A;\Delta^B\rangle|^2 2\pi\delta(E_f - E_i), \tag{3}$$

where $\Delta\hat{E}$ is the energy denominator operator. The combinatorial factor in front of the sum accounts for the possibility that a Δ's with any Fermi momenta may decay via channel A, and not just those occupying the first a Fermi momenta. The $(a!b!)^{-1}$ factor after the sum comes about because the pions are treated as identical particles.

Inserting into (2) intermediate states, and after carrying out a somewhat involved combinatorial calculation,[4] we arrive at the result

$$R = R_s \sum_{\pi, \pi'} \prod_{i=1}^{a} \sum_{\vec{k}_i} \frac{\eta\delta_\Gamma(E_i - M)}{\delta_\Gamma(0)} \delta_{\vec{k}_i, \vec{k}_{\pi(i)}} \prod_{j=1}^{b} \sum_{\vec{k}_j'} \frac{\eta\delta_\Gamma(E_j' - M)}{\delta_\Gamma(0)} \delta_{\vec{k}_j', \vec{k}'_{\pi'(j)}}, \tag{4}$$

where

$$R_s = N\Gamma\frac{N!}{a!b!}(\tfrac{1}{3})^a(\tfrac{2}{3})^b. \tag{5}$$

Moreover, $E_i \equiv (k_i^2+\mu^2)^{1/2} + (k_i^2+m^2)^{1/2}$, $E_j' \equiv (k_j'^2+\mu^2)^{1/2} + (k_j'^2+m^2)^{1/2}$, and sums are taken over all permutations π and π' of a and b objects, respectively. The constant η and the spreadout δ function δ_Γ are given by

$$\eta = \frac{4g^2}{\Gamma^2}\frac{(2\pi)^3}{V} = \frac{3M}{\Gamma R^3}\frac{1}{k k_1^0 k_2^0}, \tag{6}$$

$$\delta_\Gamma(x) = \frac{\Gamma/2\pi}{x^2 + \Gamma^2/4}. \tag{7}$$

Equation (2) has been used in (6) and $R^3 \equiv (3/4\pi)V$ has been introduced. The Fermi momenta $\vec{q}_i$ have been assumed to be small compared to m. Moreover, a narrow width approximation $\delta(x) \simeq \delta_\Gamma(x)$ has been made in (4). In the same spirit, we will also approximate $[\delta_\Gamma(x)]^k$ below by $[\delta_\gamma(0)]^{k-1}\delta_\Gamma(x)$.

The rate R in (4) includes the sum of spontaneous and all induced transition rates. If we specialize to a configuration where induced emission is impossible, viz., when all $\vec{k}_i$ and separately all $\vec{k}_j'$ are different, then we can extract out of (4) the spontaneous transition rate. It is just given by R_s in (5), which is what one could expect from the independent decay of N Δ's. As to induced emissions, there are many, depending on how many and which of the $\vec{k}_i$ and $\vec{k}_j'$ are equal. For example, if all $\vec{k}_i$ and separately all k_j' are equal, then one can get from (4) in the limit of large a and b, this "completely induced" transition rate to be

$$R_c = R_s \eta^{N-2} a!b!. \tag{8}$$

The extra factor $a!b!$, which comes from the π and π' summation in (4), results from the Bose statistics of pions and is largest when $a = N$ or $b = N$. This PASER amplification factor therefore favors extreme charge bunchings. The factor η^{N-2} is the phase-space factor needed to force the remaining $N-2$ pions to have the same momenta as $\vec{k}_1$ or $\vec{k}_1'$.

To get the final result we must also sum over all intermediate induced transitions. It turns out that this can be done. Of the a (b) momenta $\vec{k}_i$ ($\vec{k}_j'$), suppose there are ν_l (ν_l') groups of l momenta ($l = 1, 2, 3, \ldots$) that are the same. Then it can be shown[4] that (3) becomes

$$R = R_s \sum a!\left(\prod_l \nu_l! \, l^{\nu_l}\right)^{-1} b!\left(\prod_l \nu_l'! \, l^{\nu_l'}\right)^{-1} \exp\{[N - \textstyle\sum_l(\nu_l + \nu_l')]\ln\eta\}, \tag{9}$$

where the first sum is taken over all ν_l and ν_l' subject to the restriction that $\sum_l l\nu_l = a$ and $\sum_l l\nu_l' = b$. The case of spontaneous and completely induced emissions correspond to $\nu_1 = a$, $\nu_1' = b$, $\nu_i = \nu_i' = 0$ ($i \neq 1$), and to $\nu_a = 1 = \nu_b'$, $\nu_i = \nu_j' = 0$ ($i \neq a$, $j \neq b$), respectively.

The sum in (9) can be carried out.[4] The final result is

$$R = R_s \prod_{i=1}^{a-1}(1+i\eta)\prod_{j=1}^{b-1}(1+j\eta) \equiv R_s A. \tag{10}$$

It is convenient to express the amplification factor A in terms of the parameter $\xi = N\eta$, which is expected to be approximately N independent if the radius $R_0 \equiv R/N^{1/3}$ of the region occupied by a single Δ is approximately N independent. In the limit $\xi/N >> 1$,

$$A \simeq (\xi/N)^{N-2}(a-1)!(b-1)!, \tag{11}$$

and in the limit $\xi << 1$,

$$A \simeq \exp\{(\xi/2N)[a(a-1)+b(b-1)]\}. \tag{12}$$

Defining the relative amplification ratio $\alpha(a,N)$ to be the ratio of A at any a and b to its value at $a = b = N/2$, and letting $\alpha_0 = \alpha(N,N)$, we get

$$\alpha = \exp[(\xi N/4)] \tag{13}$$

in the limit $\xi << 1$. Thus even for small ξ when induced emission is least important, the relative amplification α_0 still grows exponentially with N.

If we take $R_0 = 1$ fm and the other parameters from a free Δ decay, then $\xi = 4.2$. In reality, besides R_0, Γ and k are also not known that well because of the possibility of pressure broadening and mass shift when the Δ's are put in an environment

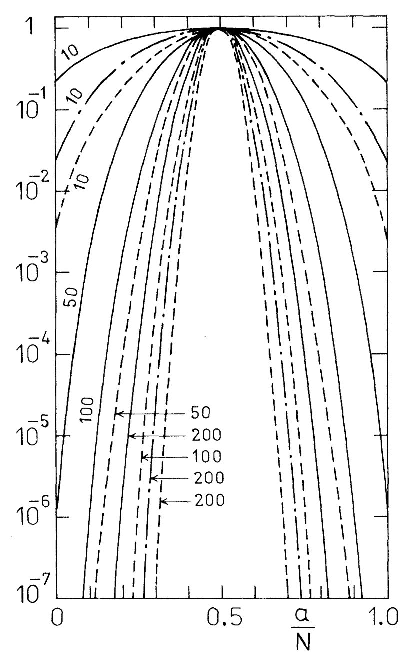

FIG. 1. Charge distribution $R/(\frac{1}{3})^a(\frac{2}{3})^b$ [see Eqs. (5) and (10)], normalized to unity at $a = b = N/2$, plotted as a function of a/N. Spontaneous emission distributions ($\xi = 0$) are shown as dashed lines; amplified emission distributions are shown as solid lines ($\xi = 4.2$) and dot-dashed lines ($\xi = 1.0$). Four values of N are shown: $N = 10$, 50, 100, and 500. These values of N are marked in the graph. The curves for $\xi = 1.0$, $N = 100$, and for $\xi = 1.0$, $N = 50$ are not shown because they lie almost on top of the curves $\xi = 4.2$, $N = 200$ and $\xi = 4.2$, $N = 100$, respectively.

of hadronic matter. We have also overestimated the rates by using the narrow width approximation and by ignoring the Pauli blocking effects of the nucleons. It is thus wise to treat ξ as a free parameter. The charge distributions with and without the amplification factor A are shown in Fig. 1. We see that α can be very large; charge distribution can differ substantially from the naive binomial distribution in (5). In short, the PASER mechanism is important and it greatly facilitates the occurrence of unusual charge-bunching events. Finally, similar treatments can be applied to calculate the induced decay of other resonances and the induced production of bosons.

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[a]On leave from School of Physics, University of Melbourne, Parkville, Victoria 3052, Australia.

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[3]Spins are unessential and will be ignored. The Δ's are regarded as fermions in the sense that they all occupy different Fermi levels. The nucleons are treated as if they were distinguishable particles. Thus the Pauli blockings for nucleons are ignored in this model and this leads to an overestimate of the transition rate. The pions of each charge are treated as identical bosons to preserve the essence of the PASER mechanism.

[4]Details of the calculations will be published elsewhere.