Excitation of Internal Kink Modes by Trapped Energetic Beam Ions

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(Received 18 October 1983)

Energetic trapped particles are shown to have a destabilizing effect on the internal kink mode in tokamaks. The growth rate is near the ideal magnetohydrodynamic value, but the frequency is comparable to the trapped-particle precession frequency. A model for the instability cycle gives stability properties, associated particle losses, and neutron emissivity consistent with the "fishbone" events observed in poloidal divertor experiments.

PACS numbers: 52.35.Py

In recent poloidal divertor experiments (PDX) with high-power, nearly perpendicular beam injection, bursts of large-amplitude magnetohydrodynamic (MHD) fluctuations, dubbed "fishbones," have been observed.[1,2] These fishbone bursts are found to be correlated with significant losses of energetic beam ions and thus have serious implications for the achievable β $(=8\pi nT/B^2)$ values in tokamaks. Detailed experimental measurements have identified the mode structure of the fishbone as an $m=1$, $n=1$ mode with additional $m\geqslant 2$ components. (Here m and n are, respectively, poloidal and toroidal mode numbers.)

We consider a large-aspect-ratio tokamak plasma consisting of core (c) and hot (h) components. For the purpose of formal orderings, we use $\epsilon=a/R \ll 1$ as the small parameter. Since we are interested in the parameter range of the first stability boundary of the internal kink mode,[3] we order $\beta_{pc}\sim O(1)$ and, for simplicity, $\beta_{ph}\sim O(\epsilon)$. ($\beta_p$ is the poloidal beta.) Temperatures are ordered as $T_c/T_h\sim O(\epsilon^2)$, which implies $n_h/n_c\sim O(\epsilon^3)$ and, hence, overall charge neutrality may be assumed. We also have, for PDX parameters, $|\omega/\omega_A|\sim|\overline{\omega}_{dh}/\omega_A|\sim O(\epsilon^2)$, similar to the usual internal kink ordering.[3] Here $\overline{\omega}_{dh}$ is the toroidal precession frequency of the trapped hot particles, and ω_d denotes the magnetic drift frequency.

Consistent with the above orderings, we adopt the ideal MHD description for the core plasma. For the hot component, however, we employ the gyrokinetic description,[4,5] neglecting the finite-Larmor-radius correction. Summing the collisionless equations of motion for each species, we obtain

$$-\omega^2\rho_m\vec{\xi}=c^{-1}(\delta\vec{j}\times\vec{B}+\vec{j}\times\delta\vec{B})-\nabla\delta P_c-\nabla\cdot\delta\vec{P}_h,\quad(1)$$

where $\vec{\xi}$ is the usual fluid displacement vector. In Eq. (1), noting that $n_h/n_c\sim O(\epsilon^3)$, we have $\rho_m=n_{0i}m_i$. The following ideal MHD relations hold: $\delta P_c=-[\vec{\xi}\cdot\nabla P_c+\gamma P_c(\nabla\cdot\vec{\xi})]$, $\delta\vec{E}_\perp=i\omega\vec{\xi}\times\vec{B}/c$, $\delta E_\parallel=0$, $\delta\vec{B}=\nabla\times(\vec{\xi}\times\vec{B})$, and $\delta\vec{J}=c\nabla\times\delta\vec{B}/4\pi$. The perturbed distribution of the hot component, δF_h, is given by[4,5]

$$\delta F_h=\frac{e}{m}\left(\delta\phi\frac{\partial}{\partial E}-\frac{\mu}{\omega_c}\frac{\delta B_\parallel}{c}\frac{\partial}{\partial\mu}\right)F_{0h}+\delta H_h,\quad(2)$$

$$[v_\parallel\partial/\partial l-i(\omega-\omega_{dh})]\delta H_h=i(e/m)Q\delta\psi,\quad(3)$$

where $E=v^2/2$, $\mu=v_\perp^2/2B$, ω_c is the cyclotron frequency, $\partial/\partial l\equiv\vec{e}_\parallel\cdot\nabla$, $\delta\psi=\delta\phi-v_\parallel\delta A_\parallel/c+v_\perp^2\times\delta B_\parallel/2\omega_c c$, $Q=(\omega\,\partial/\partial E+\hat{\omega}_{*h})F_{0h}$; $\hat{\omega}_{*h}=-(i/\omega_c)(\vec{e}_\parallel\times\nabla\ln F_{0h})\cdot\nabla$, $\omega_{dh}=-i\vec{v}_{dh}\cdot\nabla$, $\vec{v}_{dh}$ is the magnetic drift velocity, and $\delta\phi$ and $\delta A_\parallel$ are related to $\vec{\xi}$ by $c\nabla\delta\phi=-i\omega\vec{\xi}\times\vec{B}$ and $\omega\delta A_\parallel/c=-i\,\partial\delta\phi/\partial l$. When one notes that the frequencies are much smaller than the hot-particle transit and bounce frequencies, Eq. (3) can be solved readily for both trapped (t) and untrapped (u) particles. We find that $\delta H_{h,u}=-eQ\delta\phi/m\omega$ and $\delta H_{h,t}=-eQ\times\delta\phi/m\omega+\delta G_{h,t}$, where $\delta G_{h,t}=2QE\overline{J}/(\omega-\overline{\omega}_{dh})$. $\overline{A}\equiv(\oint A\,dl/v_\parallel|)/(\oint dl/|v_\parallel|)$ denotes bounce averaging, and $J=(\alpha B/2)\nabla\cdot\vec{\xi}_\perp-(1-3\alpha B/2)\times\vec{\xi}_\perp\cdot\vec{\kappa}$, with $\alpha=\mu/E$ and $\vec{\kappa}=\partial\vec{e}/\partial l$. Substituting δH into Eq. (2), we have $\delta\vec{P}_h$ given by

$$\delta\vec{P}_h=-\vec{\xi}_\perp\cdot\nabla[P_\perp\vec{I}+(P_\parallel-P_\perp)\vec{e}_\parallel\vec{e}_\parallel]_h+\delta\hat{P}_\perp\vec{I}+(\delta\hat{P}_\parallel-\delta\hat{P}_\perp)\vec{e}_\parallel\vec{e}_\parallel,$$

where

$$\begin{Bmatrix}\delta\hat{P}_\perp\\ \delta\hat{P}_\parallel\end{Bmatrix}=2^{7/2}\pi m_h B\int_{B_{\max}^{-1}}^{B^{-1}}d\alpha(1-\alpha B)^{1/2}\int_0^\infty dE\,\frac{E^{5/2}Q}{\omega-\overline{\omega}_{dh}}\,\overline{J}\begin{Bmatrix}\alpha B/2(1-\alpha B)\\ 1\end{Bmatrix}\quad(4)$$

correspond to kinetic contributions due to the trapped energetic particles. Substituting $\delta\vec{P}_h$ into Eq. (1), we have a complete normal-mode equation in terms of $\vec{\xi}$.

We now derive a dispersion relation variationally. Perform $\int d^3x\, \vec{\xi}^*\cdot$ on Eq. (1) and assume a fixed conducting boundary. We have $D[\vec{\xi}] = \delta W_{\rm MHD} + \delta W_k + \delta I$, where, with $P \equiv P_c + (P_\perp + P_\parallel)_h/2$,

$$\delta W_{\rm MHD} = \frac{1}{2}\int d^3x \left\{ \frac{|\delta\vec{B}_\perp|^2}{4\pi} - \frac{j_\parallel}{c}(\vec{\xi}_\perp^* \times \vec{e}_\parallel)\cdot\delta\vec{B}_\perp - 2(\vec{\xi}_\perp\cdot\nabla P)(\vec{\xi}_\perp^*\cdot\vec{\kappa}) + B^2|\nabla\cdot\vec{\xi}_\perp\cdot\vec{\kappa}|^2 + \gamma P_c|\nabla\cdot\vec{\xi}|^2 \right\}, \tag{5}$$

$$\delta W_k = -2^{9/2}\pi^3 m_h \int RBr\, dr \int_{B_{\max}^{-1}}^{B_{\min}^{-1}} d\alpha \int_\theta^\infty dE\, E^{5/2} K_b \bar{J}^* \frac{Q}{\omega - \bar{\omega}_{dh}} \bar{J}; \tag{6}$$

$K_b = \oint (d\theta/2\pi)(1-\alpha B)^{-1/2}$; and $\delta I = -\frac{1}{2}\omega^2 \int d^3x\, \rho_m |\vec{\xi}|^2$ is the inertial term. Note that in the high- and low-frequency limits δW_k reduces, respectively, to that of the collisionless[6,7] and the low-frequency kinetic energy principles.[8-10] To apply the variational method, we have, for the present orderings,

$$\delta W_k^{(2)} \sim (\beta_{h,t}/\epsilon)(B^2|\vec{\xi}/R|^2 V) \sim \epsilon^2(B^2|\vec{\xi}/R|^2 V) \sim \delta I^{(2)}.$$

Here, V is the volume, superscripts denote the orderings, and we have noted in ordering δI the existence of an inertial singular layer with a width $\Delta_A \sim (\omega/\omega_A)a \sim \epsilon^2 a$ at $q(r_s) \equiv rB_t/RB_p = 1$. The variational scheme then is to find a trial function, $\vec{\xi}_t$, which minimizes D to $O(\epsilon^3)$ or smaller. Since both $\delta W_{\rm MHD}$ and δI [with the assumption that $|{\rm Im}\omega^2|/|{\rm Re}\omega^2| \sim O(\epsilon)$, i.e., near marginal stability in the present case] are variational, this minimizing procedure is identical to that of ideal MHD.[5] Let $D(\vec{\xi}_t)$ be $D_e + D_s$ where D_e and D_s are the contributions from outside and inside the singular layer, respectively. For the case of circular cross sections, for $|r - r_s| >> \Delta_A$ we have $\vec{\xi}_t^e$ as given by Bussac *et al.*[3] We then obtain $D_e \equiv D[\vec{\xi}_t^e]$ as $D_e = \delta W_{\rm MHD}^{(2)}[\vec{\xi}_t^e] + \delta W_k^{(2)}[\vec{\xi}_t^e] + O(\epsilon^4)$, where, for $n = 1$,

$$\frac{\delta W_{\rm MHD}^{(2)}}{2\pi R_0} = \frac{\pi}{4}\left(\frac{r_s^2}{R_0^2}B_0\right)^2 |\xi_{r0}|^2 \delta\tilde{W}_T \equiv |\xi_{r0}|^2 \left(\frac{r_s B_0}{2R_0}\right)^2 \delta\hat{W}_f \tag{7}$$

with $\delta\tilde{W}_T$ given in Ref. 3 and, to $O(\epsilon^3)$,

$$\frac{\delta W_k^{(2)}}{2\pi R_0} \cong \pi^2 m_h 2^{3/2} \frac{|\xi_{r0}|^2}{R_0^2} \int_0^{r_s} r\, dr \int_{1-r/R}^{1+r/R} d(\alpha B) \int_0^\infty dE\, E^{5/2} \frac{K_2^2}{K_b} \left[\frac{Q}{\bar{\omega}_{dh} - \omega}\right]_{1,1} \equiv |\xi_{r0}|^2 \left(\frac{r_s B_0}{2R_0}\right)^2 \delta\hat{W}_k; \tag{8}$$

$K_2 = \oint (d\theta/2\pi)\cos\theta(1-\alpha B)^{-1/2}$; (1,1) refers to $m = 1$, $n = 1$; $B \simeq B_0(1 - r\cos\theta/R)$, and $\Delta q = 1 - q(0) \sim O(\epsilon)$ is assumed. Note that, assuming a parabolic q profile,[3] we have $\delta\hat{W}_f \simeq 3\pi\Delta q r_s^2(\frac{13}{144} - \beta_{ps}^2)/R_0^2$ with $\beta_{ps} = -(R_0/r_s^2)^2 \int_0^{r_s} r^2\beta'\, dr$.

Near the singular $q = 1$ surface, we have $|x| \equiv |r - r_s| \sim |\Delta_A|$ and the Euler equation for $\vec{\xi}_t^s$ is $d[(3\omega^2 - |k_\parallel'|^2 V_A^2 x^2)(d\xi_{rt}^s/dx)]/dx = 0$, with $|k_\parallel'| = q_s'/R_0$. This equation can be solved readily and ξ_{rt}^s matched to ξ_{rt}^e by use of the causality condition. It is then straightforward to show that

$$D_s = \delta W_{\rm MHD}^{(2)}[\vec{\xi}_t^s] + \delta I^{(2)}[\vec{\xi}_t^s] + O(\epsilon^4) = 2\pi R_0(B_0 r_s/2R_0)^2|\xi_{r0}|^2(-i\omega/\tilde{\omega}_A) + O(\epsilon^4), \tag{9}$$

with $\tilde{\omega}_A \equiv V_A/(3^{1/2}R_0\hat{s})$ and $\hat{s} = r_s q_s'$. Combining D_e and D_s then yields the dispersion relation:

$$-i\omega/\tilde{\omega}_A + \delta\hat{W}_f + \delta\hat{W}_k = 0. \tag{10}$$

The terms in Eq. (10) are all formally of the same order.

Without the trapped-particle term, $\delta\hat{W}_k$, we recover the ideal MHD results. Within the present orderings, the inclusion of $\delta\hat{W}_k$ has the most interesting effect of introducing a trapped-particle–induced branch which can become unstable at $\omega = \omega_r$ and $\delta\hat{W}_f > 0$. By substituting into $\delta\hat{W}_k$ a monoenergetic, single magnetic moment distribution F_{0h} we find a thresholdless unstable solution with $\omega_r \simeq \bar{\omega}_{dh}$ and ω_i increasing with $\langle\beta_{h,t}\rangle$ (the average trapped-particle β within the $q = 1$ surface) and $\omega_{*h}/\bar{\omega}_{dh} > 0$. This new instability mechanism thus has the character of coupling between a negative-energy/dissipation trapped-particle precession mode and a core-plasma MHD mode, which is positively dissipated because of the $\omega_r \sim k_\parallel V_A$ Alfvén resonance. Many interesting features of Eq. (10) can be derived by assuming a model distribution function for the slowing-down beam ions; $F_{0h} = c_0 E^{-3/2}\delta(\alpha - \alpha_0)$ for $0 \leqslant E \leqslant E_m$, where $c_0(r) = P_h(r)/(\pi K_{b0} m_h B_0 2^{3/2} E_m)$, $K_{b0} = K_b(\alpha = \alpha_0)$. The corresponding dispersion relation is then given by

$$-i\,\Omega\,(\overline{\omega}_{dm}/\tilde{\omega}_A) + \delta\hat{W}_{fc} + \langle \beta_{h,t}\hat{I}_0\rangle\,\Omega\,\ln(1 - 1/\Omega) = 0, \tag{11}$$

where

$$\overline{\omega}_{dm} \equiv \overline{\omega}_{dh}(E = E_m),\quad \Omega = \omega/\overline{\omega}_{dm},\quad \langle y\rangle \equiv (2/r_s^2)\int_0^{r_s} yr\,dr,\quad \hat{I}_0 = (1/2K_{b0})[\alpha_0 I'(\alpha_0) + I(\alpha_0)\hat{\omega}_{*h}/\hat{\omega}_{dh}],$$

$$I(\alpha_0) = (2R_0/r)^{1/2}[2E(k_0^2) - K(k_0^2)]^2/\pi K(k_0^2),\quad K_{b0} = (2R_0/r)^{1/2}K(k_0^2)/\pi,$$

$$k_0^2 = (1 + r/R_0 - \alpha_0 B_0)R_0/2r,$$

$E(k_0^2)$ and $K(k_0^2)$ being the complete elliptic integrals, $\hat{\omega}_{*h} = (d\ln P_{h,t}/dr)/r\omega_c$, $\hat{\omega}_{dh} = -[2E(k_0^2)/K(k_0^2) - 1]/rR\omega_c$, and $\delta\hat{W}_{fc}$ corresponds to $\delta\hat{W}_f$ with only the core-plasma pressure contribution. Simple analysis of Eq. (11) then reveals that, even for $\delta\hat{W}_{fc} > 0$, the internal kink mode is destabilized if $\beta_{h,t}$ exceeds a critical value, $\langle\beta_{h,t}\hat{I}_0\rangle_{\rm crit} = \overline{\omega}_{dm}/\pi\tilde{\omega}_A$, which is typically $\lesssim O(10^{-2})$ and is consistent with the observations.[1,2] Meanwhile, the growth rate is peaked near $\delta\hat{W}_{fc} \simeq 0$ and drops sharply as $\delta\hat{W}_{fc}$ increases. In fact, for a more realistic F_{0h}, stabilization can be expected for $\delta\hat{W}_{fc} > \overline{\omega}_{dm}/\pi\tilde{\omega}_A$. This may account for the predominant occurrences of "fishbones" near ideal MHD marginal stability. Taking $1 - q(0) = O(10^{-1})$, we find $\pi\delta\hat{W}_{fc}\tilde{\omega}_A/\overline{\omega}_{dm} \lesssim O(1)$, $\overline{\omega}_{dm} > \omega_r \gtrsim \overline{\omega}_{dm}/2$, and the growth rate

$$\omega_i \simeq \tilde{\omega}_A(\pi^2/4)(\langle\beta_{h,t}\hat{I}_0\rangle - \langle\beta_{h,t}\hat{I}_0\rangle_{\rm crit}) \tag{12}$$

tends to be of the same order as the usual ideal MHD growth rate.

The beam loss process due to the beam-ion–induced internal kink mode has already been considered,[11] and allows us to model the full fishbone cycle. If we neglect variations of the core plasma component, the internal kink mode is destabilized by the trapped particles within the $q = 1$ surface. Assuming the trapped particles to be uniformly distributed within the $q = 1$ surface, we then have, from Eq. (12), for the amplitude of the kink mode ($A = \delta B_r/B$),

$$dA/dt = A\,\Gamma(\beta_h - \beta_{\rm crit}) \tag{13}$$

with $\Gamma = \tilde{\omega}_A(\pi^2/4)\langle\hat{I}_0\rangle$. This equation for the mode has been used in Monte Carlo simulations using the formalism of Ref. 11, which will be reported in a future publication, but the essential results can be reproduced by replacing the particle loss mechanism with a simple model equation. Beam loss takes the form of secular outward drift of those trapped particles in resonance with the mode. Since the loss occurs on a time scale much shorter than the beam deposition time, the rate of particle loss through the $q = 1$ surface induced by the $m = 1$ perturbation is approximately constant until a significant fraction of the particles are lost. Thus

$$d\beta_h/dt = D - AZ\beta_{\max}\theta(\beta_h - \beta_{\min}), \tag{14}$$

where D is the net deposition rate of trapped particles within the $q = 1$ surface, and Z is a measure of the particle loss rate. The Heaviside θ function reflects the fact that only a certain fraction f of the trapped particles can be ejected. Note that the presence of $m \geqslant 2$ components, which extend to the plasma boundary, is necessary for the complete loss of trapped particles. An examination of Eqs. (13) and (14) in the (β_h, A) plane, and in particular the symmetry of these equations about $\beta_{\rm crit}$, leads to the result that the motion is periodic with $\beta_{\max} = \beta_{\rm crit}/(1 - \frac{1}{2}f)$, and $\beta_{\min} = (1 - f)\beta_{\max}$.

We illustrate the solution of Eqs. (13) and (14) for a PDX case with $B = 10$ kG, $r_s = a/2$, and 4 MW of near-perpendicular 50-keV neutral beam injection. This gives $D \sim 0.5\ \sec^{-1}$. The beam ejection efficiency has been obtained for this case with Monte Carlo simulations,[11] giving $Z = 2.5\times10^6\ \sec^{-1}$ and $f = 0.4$. Using the expression for $\hat{I}_0$ following Eq. (11), $n = 5\times10^{13}$, the bounce angle $\theta_b \sim \pi/4$, and $k_0^2 = \sin^2(\theta_b/2) \ll 1$, we find $\hat{I}_0 \sim 4$, and thus $\Gamma \sim 1.1\times10^7\ \sec^{-1}$, and $\beta_{\rm crit} \sim 0.0025$. The solution to Eqs. (13) and (14) for these parameters is shown in Fig. 1, to be compared with Fig. 1 of Ref. 1. The fishbone period $\tau_{\rm fb} \sim \Delta\beta_h/D = f\beta_{\rm crit}/D[1 - \frac{1}{2}f]$, which is about 2.5 msec in this case. The $\sim 30\%$ variation of β_h, determined by the beam loss, and its time dependence are in good agreement with the observed variation of neutron emissivity. The maximum value of A is consistent with the typical observed values of the Mirnov loop signals. The width of the fishbone burst is $\Delta t = 4/\Gamma(\beta_{\max} - \beta_{\rm crit})$, about half a millisecond in this case, also in agreement with the

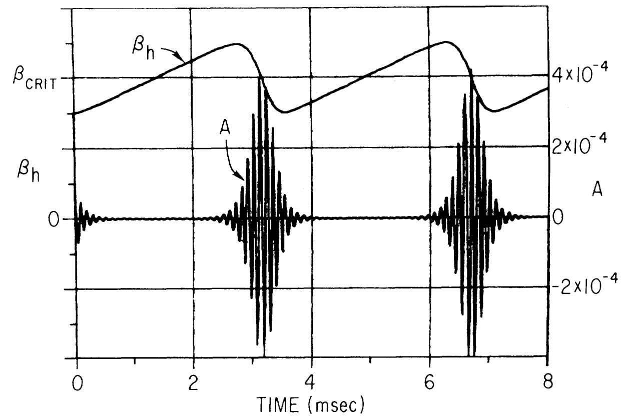

FIG. 1. The kink-mode amplitude, $A(t)\cos(\bar{\omega}_{dh}t)$, and the beam-particle beta, $\beta_h(t)$, vs time as obtained from Eqs. (13) and (14), for PDX parameters.

experimental results.

In summary, we have shown that energetic trapped particles can be destabilize the internal kink mode at a plasma pressure threshold lower than that predicted by the ideal MHD theory. This trapped-particle–induced instability has a real frequency comparable to the trapped-particle toroidal precession frequency and a growth rate of the order of the ideal MHD value. A simple model for the coupled kink-mode and trapped-particle system produces a time dependence for these quantities in good agreement with experimental results. Finally, we remark that since the instability mechanism is of sufficiently general nature, it may be desirable to extend our theoretical calculations to other regimes, such as $\beta_p \sim O(\epsilon^{-1})$, radio-frequency heated plasmas, alpha-particle effects, and so on. In this respect, we note that the ballooning-mode analog has been discussed by Rosenbluth *et al.*[12]

The authors would like to acknowledge useful discussions with H. L. Berk, J. W. Van Dam, and many members of the Plasma Physics Laboratory, in particular R. Goldston, K. McGuire, R. M. Kulsrud, J. Manickam, D. A. Monticello, C. R. Oberman, F. W. Perkins, Jr., and P. H. Rutherford.

This work was supported by U. S. Department of Energy Contracts No. DE-AC02-76-CHO-3073 and No. DE-FG-05-80ET-53088.

[1]K. McGuire *et al.*, Phys. Rev. Lett. 50, 891 (1983).

[2]D. Johnson *et al.*, in *Proceedings of the Ninth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Baltimore, 1982* (International Atomic Energy Agency, Vienna, 1983).

[3]M. N. Bussac, R. Pellat, D. Edery, and J. L. Soule, Phys. Rev. Lett. 35, 1638 (1975).

[4]T. M. Antonsen, Jr., and B. Lane, Phys. Fluids 23, 1205 (1980).

[5]P. J. Catto, W. M. Tang, and D. E. Baldwin, Plasma Phys. 23, 639 (1981).

[6]M. D. Kruskal and C. R. Oberman, Phys. Fluids 1, 275 (1958).

[7]M. N. Rosenbluth and N. Rostoker, Phys. Fluids 2, 23 (1959).

[8]J. W. Van Dam, M. N. Rosenbluth, and Y. C. Lee, Phys. Fluids 25, 1349 (1982).

[9]T. M. Antonsen, Jr., B. Lane, and J. J. Ramos, Phys. Fluids 24, 1465 (1981).

[10]T. M. Antonsen, Jr., and Y. C. Lee, Phys. Fluids 25, 132 (1982).

[11]R. B. White *et al.*, Phys. Fluids 26, 2958 (1983).

[12]M. N. Rosenbluth, S. T. Tsai, J. W. Van Dam, and M. G. Engquist, Phys. Rev. Lett. 51, 1967 (1983).