Asymptotic Q^2 for Exclusive Processes in Quantum Chromodynamics

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It is found that at available Q^2 the calculable perturbative contributions to the pion electric form factor $F_\pi(Q^2)$ and the nucleon magnetic form factors $G_M^N(Q^2)$ are much smaller than the data, which can probably be explained by soft contributions. Both hard and soft effects are estimated from light-cone/infinite-momentum-frame wave functions suggested by quark models, but the main conclusions have a more general validity.

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It has been convincingly argued[1-3] that the asymptotic behavior of many exclusive processes is calculable in perturbative QCD. We show here that in the case of elastic form factors these calculable contributions are unlikely to dominate at available momentum transfers. We therefore wish to sound a note of caution about attempts[2,3] to explain existing exclusive data by perturbative QCD.

Our main conclusion can be anticipated by noting that, up to logarithms, the calculable "hard scattering" contribution to $Q^4G_M^N(Q^2)$ behaves as $[\alpha_s(Q^2)/\pi]^2M^4$, where M has dimensions of mass and is determined by the "soft" nonperturbative low-p_T part of the hadronic wave function $\psi(x,p_T)$, in which x is the quark momentum fraction. In a model, discussed below, we find that $M\simeq 3\langle p_T^2\rangle^{1/2}$ so that a first guess that $\langle p_T^2\rangle^{1/2}=300$ MeV would give a value for $Q^4G_M^N$ which is two orders of magnitude below the measured value of about 1 GeV^4 between 4 and 30 GeV^2. Little is known about ψ and it could be imagined that $\langle p_T^2\rangle$ is much larger. Large $\langle p_T^2\rangle$ does not help, however, because perturbative QCD is only applicable[2,3] for $Q^2 \gg \langle p_T^2\rangle/\langle x^2\rangle$. We can make $\langle p_T^2\rangle$ sufficiently large that the perturbative formula fits data, but it then ceases to be valid in the region where data are available. On the other hand, we show that wave functions with $\langle p_T^2\rangle^{1/2}\simeq 300$ MeV can naturally generate "soft" nonleading terms which are as large as the data. Similar conclusions hold for the pion.[4] Calculations based on QCD sum rules[5] also generate soft contributions which fit the data for F_π and G_M.

Our calculations were based on the use of the light-cone quantization formalism[6] adopted by Brodsky, Lepage, and collaborators[2,3] or the equivalent infinite-momentum-frame formalism.[7] We cut off QCD at a scale μ, in order that ψ exists, and begin by taking μ small ($\lesssim 1$ GeV) so that it is presumably true that hadrons are dominated by $q\bar{q}$ and qqq configurations which can be approximated by those of the quark model. Accordingly, we use quark-model wave functions as a basis for making first guesses for infinite-momentum-frame/light-cone wave functions.[8] These wave functions can be used to estimate the "leading twist" contributions to G_M and F_π for $x_i^2Q^2\lesssim\mu^2\lesssim 1$ GeV^2, which can then be evolved to higher Q^2, where they are expected to dominate, by means of QCD perturbation theory. A similar program works well for the twist-two part of inelastic structure functions which have been calculated successfully using bag-model wave functions.[9]

The quark-model wave functions we use are, for mesons and baryons, respectively,

$$\phi_M(\vec{p})=\pi^{-3/4}\beta^{-3/2}\exp\{-(p_z^2+p_T^2)/2\beta^2\}, \quad (1)$$

$$\phi_B(\vec{p}_\rho,\vec{p}_\lambda)=\pi^{-3/2}\alpha^{-3}\exp\{-(p_{\rho_z}^2+p_{\lambda_z}^2+p_{\rho_T}^2+p_{\lambda_T}^2)/2\alpha^2\}, \quad (2)$$

where $\vec{p}=\vec{p}_q-\vec{p}_{\bar{q}}$, $\vec{p}_\rho=2^{-1/2}(\vec{p}_1-\vec{p}_2)$, and $\vec{p}_\lambda=6^{-1/2}(\vec{p}_1+\vec{p}_2-2\vec{p}_3)$; with $\beta=0.22$ GeV and $\alpha=0.32$ GeV these wave functions are known to give reasonable descriptions of the low-energy properties of mesons and baryons.[10] We then take two prescriptions for the x dependence of the infinite-momentum-frame/light-cone wave functions $\psi(x,p_T)$ which control the asymptotic behaviors of F_π and G_M, keeping the same factorized p_T dependence as in (1) and (2):

I: We replace p_{i_z} in (1) and (2) by the weak-binding form $m(x_i\langle x_i\rangle^{-1}-1)$ where m is the constituent quark mass[11] and $\langle x_i\rangle=\frac{1}{2}$ $(\frac{1}{3})$ for mesons (baryons).

II: We use $(x_1x_2)^{\eta_M}$ for mesons and $(x_1x_2x_3)^{\eta_B}$ for baryons.

Assuming that these ψ's dominate the normalization condition (see, however, Ref. 8), we find reasonable values for f_π, $\langle r^2\rangle_\pi$, and $\langle r^2\rangle_N$: for example, $f_\pi/f_\pi^{\exp} = 1.4$ (1.2) for case I (II).

The asymptotic forms of F_π and G_M given by these wave functions are shown in Figs. 1 and 2. As explained above, our results should be evolved to higher Q^2; we have not in fact performed this evolution as its effect is very small in the Q^2 range considered. Asymptotically, any wave function evolves to the case II with $\eta_M = \eta_B = 1$. We therefore chose $\eta_M = 1$; $\eta_M > 1$ gives a smaller result while for $\eta_M < 1$ we find the difficulties discussed in Ref. 4. For $\eta_B = 1$, G_M^p is identically zero and for $\eta_B < 1$ it is negative. Our case-II curve in Fig. 2 is for $\eta_B = 1.4$ since this value maximizes the result. Our hard-scattering results are[11]

$$\begin{aligned} Q^2F_\pi^{\mathrm{I}} &= [\alpha_s(Q^2)/\pi]64\pi^{1/2}\beta^3/3m, \\ Q^4G_M^{p\mathrm{I}} &= [\alpha_s(Q^2)/\pi]^2 16\sqrt{3}\pi\alpha^6/m^2, \end{aligned} \tag{3}$$

$$\begin{aligned} Q^2F_\pi^{\mathrm{II}} &= [\alpha_s(Q^2)/\pi]40\beta^2, \\ Q^4G_M^{p\mathrm{II}} &\simeq [\alpha_s(Q^2)/\pi]^2 80\alpha^4. \end{aligned} \tag{4}$$

In the case of the pion it is reassuring that our models both give curves near the rigorous result.[1]

We now consider the soft contributions, defined as amplitudes to scatter without explicit gluon exchange calculated with ψ's (such as I and II) which also do not contain the effects of hard gluon exchange. We cannot calculate these contributions reliably as they depend on all Fock components of the wave function. At small Q^2 we can use a small cutoff μ and argue that the $q\bar{q}$ and qqq components dominate but, because of the problem discussed in Ref. 8, the answer we obtain depends on the frame we use and the component of the currents whose matrix elements we calculate. Calculating $j^0 + j^3$ in a frame in which $q^0 = q^3$, we find the results in Figs. 1 and 2 for which the corresponding formulae are

$$\begin{aligned} Q^2F_\pi(Q^2)^{\mathrm{II}} &\xrightarrow[Q^2 \gg 10\ \mathrm{GeV}^2]{} \frac{60\pi^{1/2}\beta^3}{Q}, \\ Q^4\frac{G_M^p(Q^2)^{\mathrm{II}}}{G_M^p(0)^{\mathrm{II}}} &\xrightarrow[Q^2 \gg 50\ \mathrm{GeV}^2]{} \frac{2700\alpha^6}{Q^2}. \end{aligned} \tag{5}$$

(We do not give results for case I in which the $x \to 0, 1$ behavior, which controls the soft contribution, is unphysical.) Although these results are subject to the problems discussed above,[14] this calculation demonstrates that ψ's with small $\langle p_T^2\rangle$ are capable of generating soft contributions of the same magnitude as the data. This alone is sufficient to undermine confidence in the utility of these form factors as tests of perturbative QCD at available Q^2.

We have searched for ways of avoiding the unfor-

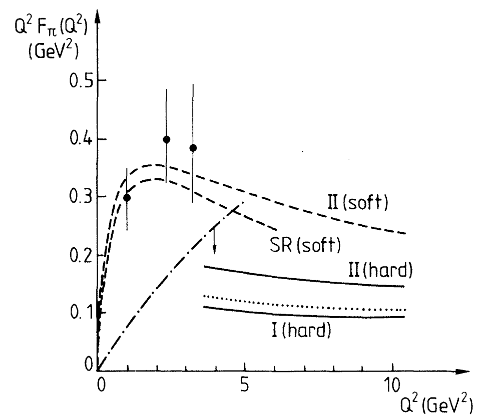

FIG. 1. Some contributions to $F_\pi(Q^2)$ compared to each other and to experiment (Ref. 12): The solid "hard" curves show the leading asymptotic contributions calculated with $\Lambda_{\mathrm{QCD}} = 150$ MeV; the dotted curve between them is the rigorous asymptotic result of Ref. 1. The dash-dotted curve is the bound on such contributions discussed in the text. The dashed "soft" contribution II (soft) is given asymptically by Eq. (5); SR (soft) is the result of the second paper of Ref. 5.

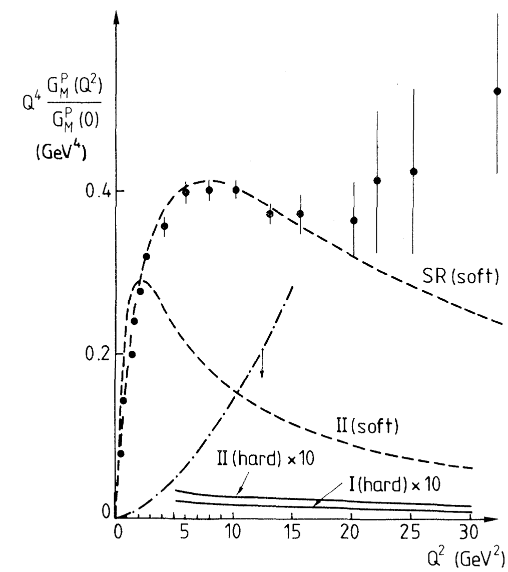

FIG. 2. Some contributions to $G_M^p(Q^2)$ compared to each other and to experiment (Ref. 13): The solid "hard" curves show the leading asymptotic contributions calculated with $\Lambda_{\mathrm{QCD}} = 150$ MeV; note that these two curves have been multiplied by a factor of 10 so that they can be seen. The dash-dotted curve is the bound on such contributions discussed in the text. The dashed "soft" contribution II(soft) is given asymptotically by Eq. (5); SR(soft) is the result of the third paper of Ref. 5. Note that contributions of order α_s/π, which might be important at intermediate Q^2, are not included.

tunate implications of these results. For example, with larger $\langle p_T^2 \rangle$ hard scattering could fit the data for G_M^2, but this possibility runs into two difficulties: (1) Since it would require[15] $\langle p_T^2 \rangle \gtrsim 1$ GeV^2, the perturbative calculations ought not to be compared to existing data: They are only valid[2,3] for $Q^2 \gg 10\langle p_T^2 \rangle \simeq \langle p_T^2 \rangle / \langle x^2 \rangle$ since it is only under this condition that the form factor is controlled by the calculable perturbative tail of the wave function. (2) The soft contribution would become even more important, as it increases more rapidly with $\langle p_T^2 \rangle$ than the hard contribution [see Eqs. (4) and (5)].

Since $Q^2 F_\pi$ is constrained by f_π (Ref. 1), it is impossible to vary the asymptotic prediction for it by changing $\langle p_T^2 \rangle$, but it might be hoped that subasymptotic x distributions with $\eta_M < 1$ could improve matters. However, a fit of the data with such an η_M would be neither a test of QCD nor consistent since smaller values of η_M would lead to even larger soft contributions to $Q^2 F_\pi$.[4]

While these remarks are all made in the context of generalizing our guesses for the ψ's, they presumably have a wider validity.[16] For factorizing ψ's with $\eta \geq 1$ we can use the observation[2,3] that the form factors depend on p_T integrals over ψ for $p_T^2 \leq x_i^2 Q^2$ to obtain the bounds

$$Q^2 F_\pi \ll [\alpha_s(Q^2)/\pi] 2Q^2/3, \tag{6}$$

$$Q^4 G_M^p \ll [\alpha_s(Q^2)/\pi]^2 35 Q^4/54, \tag{7}$$

for the calculable hard contributions shown in the figures. These bounds can only be saturated by allowing a behavior[17] for ψ which would render perturbative QCD inapplicable until much larger Q^2. Although factorization is implausible and the bounds could be weakened somewhat by changing the argument of α_s, they clearly indicate further difficulties for interpretations of the data in terms of perturbative QCD.

Brodsky, Lepage, and collaborators[3] fit the data with only hard scattering by taking a large $\langle p_T^2 \rangle > 1$ GeV^4, ignoring the various difficulties we have discussed here. With their $\langle p_T^2 \rangle$ the perturbative calculation is invalid for Q^2 in the measured range, as indicated by the violation of the bound (7). Moreover, their quoted qqq input wave function, with a probability of $\frac{1}{4}$ and a radius of 0.23 fm, would be expected to give a soft contribution to $Q^4 G_M^p$ of order $\frac{1}{4} \times 2.79(1 - \frac{1}{6}Q^2 R^2)$ for $\frac{1}{6}Q^2 R^2 \ll 1$, which is $\simeq 2$ GeV^4 at $Q^2 = 2$ GeV^2!

Our results constitute a *prima facie* case that in other exclusive process, such as πp or pp elastic scattering,[18] the calculable contributions due to n-gluon exchange will not dominate unless $Q^2 \gg (\alpha_s/\pi)^{-n}$ GeV^2. If, as we suspect, this is so, then it will be very difficult to test the many beautiful predictions of QCD for exclusive processes.

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[1]G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. 43, 246 (1979); A. V. Efremov and A. V. Radyushkin, Phys. Lett. 94B, 245 (1980); A. Duncan and A. H. Mueller, Phys. Lett. 90B, 159 (1980), and Phys. Rev. D 21, 1626 (1980).

[2]G. Peter Lepage and Stanley J. Brodsky, Phys. Rev. D 22, 2157 (1980).

[3]G. P. Lepage, S. J. Brodsky, T. Huang, and P. B. MacKenzie, in *Particles and Fields 2,* edited by A. Z. Capri and A. N. Kamal (Plenum, New York, 1983), p. 83; S. J. Brodsky, T. Huang, and G. P. Lepage, *ibid.,* p. 143.

[4]A perturbative fit to F_π would require either that $\Lambda_{QCD} \simeq 1$ GeV, which would invalidate the calculation in the region of the data, or that ψ is pathological [for the case $\eta = \frac{1}{4}$ of Fig. 12(c) of Ref. 2 more than 50% of the hard scattering is due to the end-point regions $x < 0.05$ and $x > 0.95$ (where the perturbative calculation is highly dubious) and there would be a large soft contribution which would grow with Q^2!].

[5]B. L. Ioffe and A. V. Smilga, Phys. Lett. 114B, 353 (1982); V. A. Nesterenko and A. V. Radyushkin, Phys. Lett. 115B, 410 (1982), and 123B, 439 (1983).

[6]J. B. Kogut and D. E. Soper, Phys. Rev. D 1, 2901 (1970); J. D. Bjorken, J. B. Kogut, and D. E. Soper, Phys. Rev. D 3, 1382 (1971).

[7]For the discussion of form factors it is convenient to use the infinite-momentum frame obtained by boosting a Breit frame in which a hadron scatters from momentum $-\frac{1}{2}\vec{Q}$ to $+\frac{1}{2}\vec{Q}$ to infinite momentum along the spin quantization axis.

[8]It is impossible to ensure that such wave functions have the correct angular momentum and parity without solving the theory. For example, in Refs. 2 and 3 only the $q\bar{q}$ helicity state which dominates F_π asymptotically is kept in ψ, which is normalized to f_π calculated using the A_+ component of the axial current; this is consistent as

only this piece of ψ contributes to $\langle 0|A_+|\pi\rangle$. However, calculation of $\langle 0|A_-|\pi\rangle$ then gives an inconsistent result for f_π unless there is an additional piece of the wave function (except in the weak-binding limit). Similar phenomena must occur for the nucleon. Thus even if μ is small and higher Fock states are negligible, the norms of the $q\bar{q}$ and qqq pieces which dominate the perturbative contributions to F_π and G_M^N are less than 1.

[9]R. L. Jaffe and G. G. Ross, Phys. Lett. 93B, 313 (1980).

[10]See, for example, Nathan Isgur, in *The New Aspects of Subnuclear Physics,* edited by A. Zichichi (Plenum, New York, 1980), p. 107.

[11]We take m to be 330 MeV. As explained in Ref. 10, ϕ_M and ϕ_B [Eqs. (1) and (2)] describe a world without short-range hyperfine interactions in which $M_\pi = M_\rho \simeq (\frac{1}{4}M_\pi + \frac{3}{4}M_\rho)_{\rm expt}$ and $M_N = M_\Delta \simeq (\frac{1}{2}M_N + \frac{1}{2}M_\Delta)_{\rm expt}$, so that $m = 330$ MeV is reasonably consistent with the weak-binding formulas $m_\pi = 2m$ and $M_N = 3m$. As ϕ_M and ϕ_B do not include hard-gluon hyperfine interactions, there is no double-counting problem in calculating hard and soft contributions to F_π and G_M.

[12]C Bebek *et al.*, Phys. Rev. D 17, 1693 (1978).

[13]M. D. Mestayer, SLAC Report No. 214, 1978 (unpublished), as quoted in Ref. 2.

[14]Furthermore, these soft contributions may be suppressed at high Q^2 by Sudakov form factors. Without such a suppression perturbative QCD is in any case inapplicable to G_M^N, but this suppression seems unlikely to be a very large effect at these Q^2 values. Note that the qualitatively very similar results of Ref. 5 for these soft contributions do not suffer from either this uncertainty or the flaw discussed in Ref. 8.

[15]For consistency with the known size of the proton, the qqq amplitude would have to be small if $\langle p_T^2\rangle$ is so large. With an amplitude less than 1, an even larger $\langle p_T^2\rangle$ would be needed to fit G_M^p.

[16]Since we submitted this paper, B. L. Ioffe has pointed out to us that QCD sum rules give a perturbative contribution to G_M^N very similar to that given by our models [V. M. Belyaev and B. L. Ioffe, Sov. Phys. JETP 56, 493 (1982)].

[17]The bound on G_M^p is mathematically very inefficient and cannot be saturated at all.

[18]Luckily, processes such as meson-baryon scattering are sufficiently rich that they allow elegant tests of the applicability of perturbative QCD which are free from *ad hoc* assumptions about hadronic wave functions. See G. R. Farrar, Rutgers University Report No. RU-83-32, 1983 (to be published).