Instability of Fusing Plasmas and Spin-Depolarization Processes

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A plasma mode driven by the anisotropy of the velocity distribution of charged fusion-reaction products can resonate with the spin precession frequency of one of the plasma components. When these spins are initially polarized, resonant depolarization can occur at a rate considerably faster than the fusion reaction rate, if spatial inhomogeneity effects do not depress excessively the mode amplitude.

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A series of advances in understanding the physics of thermonuclear plasmas that have occurred during the last few years have made it possible to suggest that so-called "neutronless" fusion reactors are feasible.[1] These are based on nuclear reactions that do not produce neutrons. In particular, considerable theoretical effort has been devoted to bringing the plasma parameters for which substantial thermonuclear burn can occur relatively close to a reasonable extrapolation of present day experiments, and to decreasing the rate of secondary reactions that produce neutrons. In this context it is desirable to have plasmas with properly spin-polarized nuclei as the relevant reactor fuel.[2] In fact, when this possibility was considered in the past it was, probably too hastily, dismissed by qualitative arguments based on the many orders of magnitude by which the reacting-plasma thermal energy is higher than the spin-flip energy, and upon consideration of the multitude of collective modes that, in principle, can be excited in a magnetically confined plasma. On the other hand, a recent serious analysis of the types of depolarization processes that can be envisioned has indicated that the resulting rate of depolarization may not be hopelessly high and may even be lower than the rate of fusion reaction.[2]

Here we point out that a multispecies plasma with nuclei having different values of the cyclotron frequency can sustain electromagnetic modes with frequencies resulting from combinations of the different nuclei cyclotron frequencies; in the case of a D-T plasma, these mode frequencies are close to the precession frequency of the deuteron spin. For a first approach to this problem we neglect the important effects of spatial inhomogeneities. Then we show that these modes are driven unstable by the anisotropy, in velocity space, of the distribution of fusion-reaction products that characterizes plasmas in which a considerable fraction of the fusing nuclei are spin polarized. We estimate that, for reasonable values of the mode amplitude, the rate of spin depolarization can be significantly faster than that of fusion reaction. This would lead us to conclude that after thermonuclear burn begins the deuterium spins would be rapidly scrambled. We think that the excitation of the same kind of mode should be considered also in the case where the magnetic confinement configuration is of the mirror type and the fusing nuclei have a "loss cone" type of distribution in velocity space.

In order to illustrate our point we consider the magnetic field $\vec{B}$ to be in the z direction and ignore at first its spatial variation. We look for electric field fluctuations of the form

$$\hat{\vec{E}} = \tilde{\vec{E}} \exp(-i\omega t + ik_\perp x + ik_\parallel z).$$

We refer to a D-T plasma and search for mode frequencies that can resonate both with the deuteron spin precession frequency $\Omega_D{}^p = 0.8574\Omega_D$ (Ω_j is the cyclotron frequency of species j) and with the distribution of the α particles that are produced by the D-T fusion reaction. Since $\Omega_\alpha = \Omega_D$, the relevant condition can be written as

$$\omega - \Omega_\alpha \simeq k_\parallel \langle v_\alpha \rangle ,$$

where $\langle v_\alpha \rangle^2 \equiv 2\langle \epsilon_\alpha \rangle / m_\alpha$ and $\langle \epsilon_\alpha \rangle$ is a representative energy of the α particle slowing-down distribution that is considered to be in the range of 3.5 MeV to about 0.6 MeV (see Ref. 3). In order to produce a significant growth rate we shall require that $k_\perp \langle v_\alpha \rangle / \Omega_\alpha \sim 1$. In addition, in order to avoid an appreciable Landau damping by the (Maxwellian) electron distribution we shall consider

$$\Omega_e{}^2 \gg \omega^2 \gg v_{the} |\tilde{E}_\parallel / \tilde{\varphi}|^2 ,$$

where $\hat{\vec{E}} = -\nabla\hat{\varphi} - (\partial\hat{\vec{A}}/\partial t)/c$ and $\nabla \cdot \hat{\vec{A}} = 0$.

Since the α-particle concentration n_α / n_e (n_e is the electron density) is small, and given the requirements on the mode wavelengths and frequencies that we have indicated earlier, we can derive the dispersion relation for the real part of the frequency by the *cold*-plasma approximation for both the electrons and the fuel nuclei. The relevant equations are $\tilde{n}_e=\tilde{n}_D+\tilde{n}_T$, $\tilde{\vec{J}}=(c/4\pi)k^2\tilde{\vec{A}}$, $\tilde{\vec{J}}=e(\tilde{\vec{u}}_D n_D+\tilde{\vec{u}}_T n_T-n_e\tilde{\vec{u}}_e)$, $\omega\tilde{n}=\vec{k}\cdot\tilde{\vec{u}}$ for each species, $-i\omega m_i\tilde{\vec{u}}_i=e(\tilde{\vec{E}}+\tilde{\vec{u}}_i\times\vec{B}/c)$ where $i=D,T$, $\tilde{u}_{i\parallel}=0$, $\tilde{\vec{E}}+\tilde{\vec{u}}_e\times\vec{B}/c=0$, and $-i\omega m_e\tilde{u}_{e\parallel}=-e\tilde{E}_\parallel$. In addition we consider $k^2 d_e^2\ll 1$, where $d_e=c/\omega_{pe}$, and notice that

$$|k_\perp|/k_\parallel|^2\simeq|\Omega_D/(\omega-\Omega_D)|^2\gg 1.$$

Here $\tilde{\vec{J}}$ is the perturbed current density, $\tilde{\vec{u}}$ is the perturbed average velocity of the different species, and the remaining symbols have a standard identification.

Then, after a systematic expansion in all the small parameters that we have indicated, we arrive at the following dispersion relation:

$$\omega^2(\omega^2-\bar{\bar{\Omega}}^2)/(\omega^2-\Omega_h^2)=k_\perp^2\bar{v}_A^2, \qquad (1)$$

where

$$\Omega_h\equiv(\Omega_D\Omega_T\bar{\bar{\Omega}}/\bar{\Omega})^{1/2} \qquad (2)$$

is the relevant ion hybrid frequency,

$$\bar{\Omega}\equiv\alpha_D\Omega_D+\alpha_T\Omega_T \text{ and } \bar{\bar{\Omega}}\equiv\alpha_T\Omega_D+\alpha_D\Omega_T, \qquad (3)$$

are weighted averages of the two ion-cyclotron frequencies, and $\alpha_D\equiv n_D/n_e$ and $\alpha_T\equiv n_T/n_e$ are the concentrations of deuterium and tritium ($\alpha_D+\alpha_T=1$). Two relevant Alfvên velocities are (see Fig. 1)

$$\bar{v}_A\equiv d_e(\Omega_e\bar{\Omega})^{1/2} \text{ and } \bar{\bar{v}}_A\equiv\bar{v}_A\Omega_h/\bar{\bar{\Omega}}. \qquad (4)$$

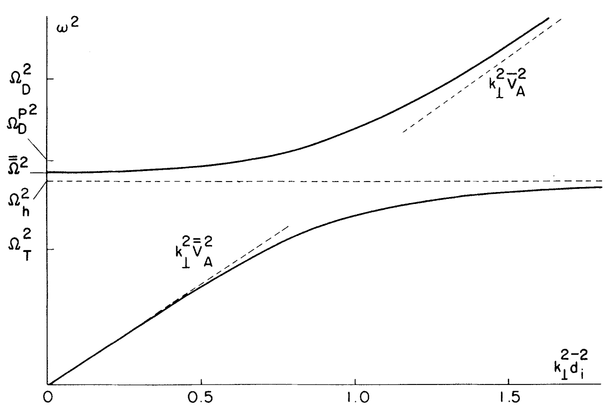

FIG. 1. Graphical representation of the dispersion relation (1), with the relevant symbols indicated in the main text. Notice that the two branches correspond to two magnetohydrodynamic waves: $\omega^2\simeq k_\perp^2\bar{v}_A^2$ for $\omega^2\gg\Omega_{D,T}^2$, and $\omega^2\simeq k_\perp^2\bar{\bar{v}}_A^2$ for $\omega^2\ll\Omega_{D,T}^2$. The scale of frequencies involved is represented on the vertical axis, where the close proximity of Ω_h, $\bar{\bar{\Omega}}$, and $\Omega_D{}^p$ is shown.

In the special interesting case where $\alpha_D=\alpha_T=\frac{1}{2}$, we have $\bar{\Omega}=\bar{\bar{\Omega}}=\frac{5}{6}\Omega_D$ and $\Omega_h=(\Omega_D\Omega_T)^{1/2}=(\frac{2}{3})^{1/2}\Omega_D$. The graphic representation of the dispersion relation (1) for this case, given in Fig. 1, indicates that a frequency exactly equal to $\Omega_D{}^p$ can be obtained from the upper branch for finite values of $k_\perp\bar{d}_i\equiv k_\perp\bar{v}_A/\bar{\Omega}$. The scale of frequencies involved in the present problem is shown on the vertical axis and $\Omega_D{}^p/\bar{\bar{\Omega}}=1.029$ while $\bar{\bar{\Omega}}/\Omega_h=1.021$. The close proximity of the frequencies Ω_h, $\bar{\bar{\Omega}}$, and $\Omega_D{}^p$ is evident. If we think of a realistic magnetic confinement configuration in which a mode with frequency ω around $\bar{\bar{\Omega}}$ is excited, it is clear that the magnetic field inhomogeneity will make the mode frequency equal to $\Omega_D{}^p$ and Ω_h in relatively close regions of space.

In particular we notice that if a low-β plasma regime and a large-aspect-ratio toroidal configuration are considered, the magnetic field spatial variation is represented by $B\simeq B_0{}^0(1-x/R_0{}^0)$, where $R_0{}^0$ is the distance of the inner edge of the plasma column from the axis of symmetry and x is the distance from the inner edge, on the equatorial plane. The distance between the (cylindrical) surfaces where $\omega=\bar{\bar{\Omega}}$ and $\omega=\Omega_D$ is given by

$$\tfrac{5}{6}(1-\bar{\bar{x}}/R_0{}^0)=1-x_D/R_0{}^0, \qquad (5)$$

and is about $x_D-\bar{\bar{x}}\simeq(R_0{}^0-\bar{\bar{x}})/6$. Since the ratio of the growth rate to the frequency of the modes we consider is smaller than the energetic α-particle concentration, an important question that remains to be resolved is whether, when the effects of spatial inhomogeneities are introduced in the theory, unstable normal modes or only convective modes can be found.[4] In the latter case the mode amplitude may not reach amplitudes that are significant.

A relevant factor for the problem under consideration is the polarization of the fluctuating fields in the plane perpendicular to the equilibrium magnetic field. In the cold-plasma limit considered thus far and for nearly perpendicular propagation ($k_\perp^2/k_\parallel^2\gg 1$) along the x direction, we obtain

$$\frac{\tilde{B}_\parallel}{B}=\frac{\tilde{n}_e}{n_e},\quad \frac{\tilde{B}_x}{B}=-\frac{k_\parallel}{k_\perp}\frac{\tilde{B}_\parallel}{B},\quad \frac{\tilde{B}_y}{B}=-i\lambda(\omega)\frac{k_\parallel\tilde{B}_\parallel}{k_\perp B}, \qquad (6)$$

where

$$\lambda(\omega)=\frac{\tilde{B}_y}{i\tilde{B}_x}=\frac{i\tilde{E}_x}{\tilde{E}_y}$$

$$=\frac{\omega[\alpha_D(\Omega_T{}^2-\omega^2)+\alpha_T(\Omega_D{}^2-\omega^2)]}{\alpha_D\Omega_D(\Omega_T{}^2-\omega^2)+\alpha_T\Omega_T(\Omega_D{}^2-\omega^2)}. \qquad (7)$$

Notice that $\lambda(\Omega_D)=\lambda(\Omega_T)=1$, $\lambda(\Omega_h)=\infty$, and $\lambda(\bar{\bar{\Omega}})=-1$. Hence the fluctuating perpendicular fields have right-circular, linear, and left-circular polarization, at the deuterium or tritium cyclotron frequencies, the resonance frequency Ω_h, and the cutoff frequency $\bar{\bar{\Omega}}$, respectively. At the deuteron spin precession frequency and for $\alpha_D=\alpha_T=\frac{1}{2}$, $\lambda(\Omega_D{}^p)\simeq 0.2$. Near this frequency λ is a rapidly varying function of ω as a result of the proximity of the pole of $\lambda(\omega)$ at Ω_h.

We now turn to the question of estimating the deuteron depolarization rate as a result of its interaction with the fluctuating magnetic field, maintaining the assumption that the plasma is infinite and homogeneous so that the interaction takes place over a long time. Then the resonance condition[4,5] between the deuteron spins and the excited mode is

$$\omega=\Omega_D{}^p+l\,\Omega_D+k_{\parallel}v_{\parallel},$$

where l is an integer and $l\,\Omega_D$ and $k_{\parallel}v_{\parallel}$ represent, respectively, the transverse and the longitudinal Doppler shifts of the mode frequency seen in the particle frame. In the cold-plasma limit that we have considered the transverse wavelength of the perturbation is much larger than the deuteron gyroradius and $\omega\gg k_{\parallel}v_{\mathrm{th\,D}}$. Therefore both Doppler shifts are negligible. Then the transition probability per precession period between two spin states $|m\rangle$ and $|m\pm1\rangle$ of the deuteron is

$$w=\left(\frac{2\pi}{\hbar}\right)|\langle m\pm1|\vec{\mu}\cdot\tilde{\vec{B}}|m\rangle|^2\frac{\delta(\Omega_D{}^p-\omega)}{\Omega_D{}^p},\qquad(8)$$

where $\vec{\mu}$ is the magnetic moment of the deuteron and the interaction Hamiltonian is $H=-\vec{\mu}\cdot\hat{\tilde{\vec{B}}}=-\vec{\mu}\cdot\tilde{\vec{B}}\exp(-i\omega t)$. Substituting the expressions (6) for the components of the fluctuating magnetic field into Eq. (8) we obtain

$$w=2\pi^2\Omega_D{}^p\left|\frac{\tilde{B}_\perp}{B}\right|^2\frac{[1-\lambda(\Omega_D{}^p)]^2}{1+\lambda(\Omega_D{}^p)^2}\,\delta(\Omega_D{}^p-\omega).\qquad(9)$$

In order to give a numerical estimate for the relevant rate of depolarization a further assumption has to be made on the nature of the spectrum of the excited fluctuations. First we consider a narrow spectrum of frequencies of width $\Delta\omega$ around the spin resonance, so that we write the density of states as $\rho(\Omega_D{}^p)\simeq 1/\Delta\omega$. The total transition probability per precession period is then

$$w=2\pi^2a_1(\Omega_D{}^p/\Delta\omega)|\tilde{B}_\perp/B|^2,\qquad(10)$$

where a_1 is an uncertainty constant of order unity ($a_1\simeq0.6$). On the other hand, if we estimate the spectral density of the fluctuations by counting the number of standing waves in a two-dimensional cavity perpendicular to B with cross-sectional area $\pi\bar{d}_i^{\,2}$, we obtain $\rho(\Omega_D{}^p)\simeq\frac{1}{2}\bar{d}_i^{\,2}\,\Omega_D{}^p\,dk_\perp{}^2(\Omega_D{}^p)/d\omega^2$. Thus, after using the dispersion relation (1), we find

$$w=2\pi^2a_2|\tilde{B}_\perp/B|^2,\qquad(11)$$

where a_2 is also of order unity ($a_2\simeq1.6$). The relative magnitude of the perpendicular fluctuating magnetic field is $|\tilde{B}_\perp/B|\simeq|k_{\parallel}/k_\perp|\,|\tilde{B}/B|$, where $|\tilde{B}/B|\simeq|\tilde{B}_{\parallel}/B|\simeq|\tilde{n}_e/n_e|$. The ratio of wave numbers $|k_{\parallel}/k_\perp|$ is determined by the condition that an unstable mode be excited through interaction with the anisotropic α-particle distribution function, and will next be shown to be of order 10^{-1}. Therefore normalizing the fluctuation level to a value of the order $|\tilde{B}/B|\simeq|\tilde{n}_e/n_e|\simeq10^{-2}$, we find the transition probability per unit time $\nu_{\mathrm{dep}}=w\Omega_D{}^p/2\pi$ to be

$$\nu_{\mathrm{dep}}=10^3\,a\left|\frac{10^3\tilde{B}_\perp}{B}\right|^2\left(\frac{B}{5\times10^4\ \mathrm{G}}\right)\mathrm{s}^{-1},\qquad(12)$$

where $a\simeq\Omega_D{}^p/3\Delta\omega$ under the narrow-spectrum hypothesis (10), and $a\simeq1$ if Eq. (11) holds.

This estimate is also valid when the main effects of the plasma spatial inhomogeneity on the mode-spin resonance are considered. These are the localization in space of the mode amplitude (of its left-circularly polarized component in particular) and the change along the particle orbit of the deuteron spin precession frequency. The first effect can be approximately accounted for by using an average along the deuteron orbits of the rational function of λ in Eq. (9). The second effect can be described, in the particle frame, as due to a time-dependent equilibrium magnetic field which we can model as $B=B_0(1+\epsilon\cos\omega_t t)$, where B_0 is the average value of B along the particle orbit, ϵ its dimensionless modulation, and ω_t the characteristic frequency of the particle orbit. By using the "adiabatic state" approximation[6] to describe the spin state of the deuterons one can show that this second effect splits the homogeneous plasma resonance condition $\omega=\Omega_D{}^p$ into a sum of resonances $\omega=\Omega_D{}^p(B_0)+l\omega_t$, where l is an integer such that $|l|\lesssim\epsilon\Omega_D{}^p(B_0)/\omega_t\gg1$. The strength of each of these resonances is reduced by a factor of order $\omega_t/\epsilon\Omega_D{}^p$ and, if the spectrum of the perturbations is such that $\Delta\omega\gtrsim\epsilon\Omega_D{}^p$, the sum over all these resonances yields again the same estimate.

In order to evaluate the mode growth rate, we recall that the angular distribution of the emitted α particles, when both the deuteron and the triton spins are oriented in the same direction along the magnetic field lines, is proportional to $\sin^2\theta = v_\perp^2/v^2$. Here $\epsilon = mv^2/2$ is the particle kinetic energy, $v^2 = v_\perp^2 + v_\parallel^2$, and $v_\perp$, $v_\parallel$, and ψ (the gyration angle) are used as coordinates in velocity space. Since the collisional slowing down does not change the particle pitch angle we consider the α-particle distribution to be of the form

$$f_\alpha = (v_\perp^2/v^2) f_\alpha^0(v), \tag{13}$$

where $f_\alpha^0(v)$ is isotropic and has the characteristic dependence $f_\alpha^0 \propto 1/(v^3 + v_c^3)$ in the range $m_\alpha v_c^2/2 \simeq 0.6$ MeV $\lesssim \epsilon_\alpha \leqslant 3.5$ MeV following the arguments given in Ref. 3. The evaluation of γ is obtained by adding the contribution of the perturbed α-particle distribution to both the positive charge density and the electrical current density. Given our choice of mode frequency and wave-number range, the α-particle contribution to the growth rate is dominated by the $\omega = k_\parallel v_\parallel + \Omega_\alpha$ resonance. In particular, after extensive algebraic manipulations we find the growth rate to be

$$\gamma = 2\pi^2 \left\{ \sum_{i=\mathrm{D,T}} \frac{\alpha_i \Omega_i [\Omega_i - \omega\lambda(\omega)]^2}{(\Omega_i^2 - \omega^2)^2} \right\}^{-1} \frac{\Omega_\alpha^3}{k_\perp^2 |k_\parallel| n_e} \int_0^\infty dv_\perp^2 \left[\lambda(\omega) J_1 - \frac{v_\perp k_\perp}{\Omega_\alpha} J_1'\right]^2 \times \left[\frac{\partial f_\alpha}{\partial v_\perp^2} + \left(\frac{\Omega_\alpha}{\omega} - 1\right)\left(\frac{\partial f_\alpha}{\partial v_\perp^2} - \frac{\partial f_\alpha}{\partial v_\parallel^2}\right)\right]_{v_\parallel = v_{\parallel 1}}, \tag{14}$$

where J_1 is the Bessel function of argument $v_\perp k_\perp/\Omega_\alpha$ and $v_{\parallel 1} = (\omega - \Omega_\alpha)/k_\parallel$. It is clear that, for $\omega \simeq \Omega_D{}^p$, the anisotropic features of the distribution function (14) result in a positive contribution to γ. In order to establish an instability condition we choose $k_\parallel$ in such a way that $m_\alpha v_{\parallel 1}^2 \geqslant (2 - 0.8\,\Omega_\alpha/\omega) \times 3.5$ MeV. A direct differentiation of the α-particle distribution function shows now that the integrand in Eq. (14) is positive definite except for the contribution of the sharp drop of f_α when ϵ_α reaches 3.5 MeV. This negative contribution is localized near $\epsilon_\alpha = 3.5$ MeV and may be overcome by bringing this point close to a zero of the multiplying function $[\lambda(\omega)J_1 - (k_\perp v_\perp/\Omega_\alpha)J_1']^2$. This can be realized for a certain domain of values of the magnetic energy per particle, $B^2/8\pi n_e$, and the mode frequency, ω, by selecting the proper value of $k_\parallel$. We recall that once $B^2/8\pi n_e$ and ω are given, $k_\perp$ is fixed by the dispersion relation (1). In particular, for $B \simeq 5\times10^4$ G, $n_e \simeq 10^{14}$ cm^{-3}, $\omega \simeq \Omega_D{}^p$, and $\alpha_D = \alpha_T = \frac{1}{2}$, a numerical evaluation of Eq. (14) yields an unstable mode with growth rate $\gamma/\Omega_\alpha \simeq 10^{-2} n_\alpha/n_e$ if the ratio of wave numbers is taken to be $|k_\perp/k_\parallel| \simeq 7$.

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