Rotational Structure and Nilsson Orbitals for Highly Deformed Odd-A Nuclei in the $A\sim100$ Region

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Six rotational bands have been found in ${}^{99}_{39}\mathrm{Y}_{60}$, ${}^{101}_{39}\mathrm{Y}_{62}$, ${}^{99}_{38}\mathrm{Sr}_{61}$, and ${}^{101}_{40}\mathrm{Zr}_{61}$. All bands have small and very similar values of $\hbar^2/2\mathcal{I}$ that indicate deformations of ~0.3–0.4 and imply that these nuclei are among the most deformed known, with $\mathcal{I}\sim0.8\mathcal{I}_{\mathrm{rigid}}$. Relative strengths of intraband γ transitions suggest Nilsson orbital assignments of $\pi\frac{5}{2}[422]$ for the Y ground bands.

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The region of deformed neutron-rich nuclei at mass $A\sim100$ was predicted theoretically in 1969 by Arseniev, Sobieczewski, and Soloviev.[1] The first experimental evidence was obtained by observing rotational bands in ^{252}Cf fission fragments with $B(E2)$ values enhanced by factors of 40–120,[2] indicating deformations β of 0.3–0.4,[3] which are reasonably well reproduced by calculations of Faessler *et al.*[4] Two-neutron separation energies S_{2n} and rms radii for Rb nuclei[5] also indicate prolate deformations of 0.2–0.4 with the maximum for ^{100}Rb.

Detailed spectroscopic studies of even-A Sr, Zr, Mo, and Ru nuclei have been pursued for years at on-line mass separator facilities and are summarized by Sistemich[6] and Pinston.[7] Characteristics of the shape transition for even-even nuclei at $A\sim100$ are shown in Fig. 1. The 2_1^+ energy decreases by a factor of 5.7 as N increases from 58 to 60 for Sr and Zr nuclei, but the decrease becomes more gradual as Z increases. The onset of deformation for Sr and Zr is thus the most abrupt known (for comparison, the 2_1^+ energy drops by a factor of 2.8 as N increases from 88 to 90 for Sm and Gd nuclei, which have the most abrupt onset for the rare-earth elements). Even-A Sr potential-energy calculations[8] indicate spherical minima for $N\leqslant58$ and prolate minima with $\beta\approx0.3$ for $N\geqslant60$. For $N>60$ the $4_1^+/2_1^+$ energy ratios have a common trend toward the rigid-rotor value of $\frac{10}{3}$. For $N<60$ the trend is reversed, with the vibrational ratios of ~2.2 decreasing as the subshell at $Z=38$ is approached. These ratios show that the $N=60$ isotones are transitional. This is strongly supported by the systematics of the 0_2^+ level energy which is exceptionally low for Sr and Zr at $N=60$ but is much higher for other N values.[6,7]

A microscopic explanation for the abrupt onset of deformation at $N=60$ and the existence of low-lying excited 0^+ states at the onset has been given by Federman and Pittel.[9] Crucial to this explanation is the strong attractive interaction between neutron-proton orbitals with large spatial overlap, such as the spin-orbit partner orbitals $\pi g_{9/2}$ and $\nu g_{7/2}$. The ability of the n-p interaction to cause strong deformation requires not only such orbitals near the Fermi surface prior to deformation but that orbitals with large spatial overlap also occur at the Fermi surface once the nucleus becomes deformed. Casten *et al.*[10] pointed out that it is precisely this requirement that causes the largest deformation to occur for Sm and Gd in the rare-earth region.

Prior to this Letter, only a single rotational band for an odd-A nucleus in the $A\sim100$ region has been reported.[11] This was for the decay of an 8.6-μs isomer of ^{99}Y that populates eight levels of a $K=\frac{5}{2}$ ground band. The level energies closely follow the general form[12] for an odd-A rotor with negligibly small higher-order terms, indicating a nearly rigid rotor. (The inertial parameter $\hbar^2/2\mathcal{I}$ of 17.8 keV changes by less than 0.1 keV if the higher-order terms are included.)

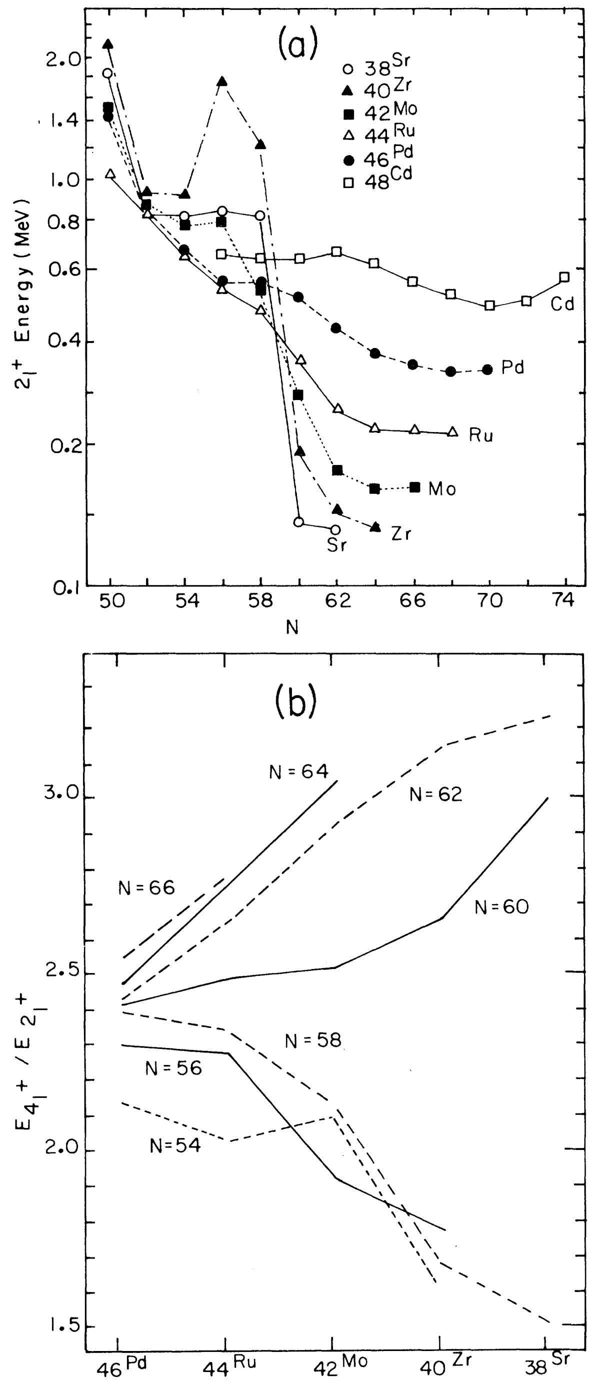

FIG. 1. Systematics of even-even nuclei in the $A \sim 100$ region.

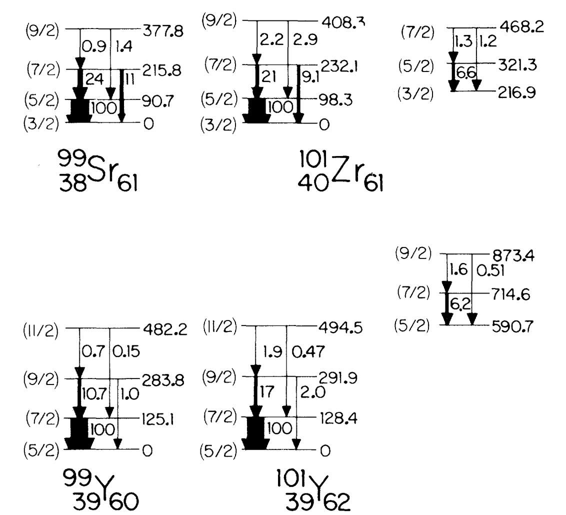

FIG. 2. Rotational bands in deformed odd-A Y isotopes and $N=61$ isotones. Intraband γ transitions and their relative intensities from β decay are shown. Only the band in ^{99}Y was reported (see Ref. 11) prior to the present work.

It is the purpose of the present Letter to present new data on rotational bands for four deformed odd-A nuclei with $A \sim 100$. Using the mass-separator facility TRISTAN on line to the high flux beam reactor at Brookhaven National Laboratory and a high-temperature surface-ionization ion source,[13] we have studied decays of ^{99}Sr, ^{101}Sr, ^{99}Rb, and ^{101}Y and found half-lives (in milliseconds) of 266 ± 5, 121 ± 6, 52 ± 5, and 500 ± 50, respectively. Figure 2 gives our deduced low-lying rotational bands in the daughter nuclei ^{99}Y, ^{101}Y, ^{99}Sr, and ^{101}Zr. (Detailed reports of each decay are in preparation.) The similarity in the levels of the two Y isotopes is striking, as is that of the two $N=61$ isotones.

Guided by the nearly rigid rotor with small $\hbar^2/2\mathscr{I}$ for ^{99}Y, we analyzed our deduced level energies with the assumption that the higher-order terms are also negligibly small. This assumption leads to unambiguous assignments of K values for all bands. The six values of $\hbar^2/2\mathscr{I}$ given in Table I are remarkably similar. The two bands in ^{101}Zr have slightly larger values, as would be expected from the even-even systemat-

TABLE I. Rotational band properties.

Nucleus	Bandhead (keV)	$\hbar^2/2\mathscr{I}$ (keV)	$\lvert(g_K - g_R)/Q_0\rvert$ (b^{-1})
^{99}Sr	0	18.0	0.164 ± 0.017
^{101}Zr	0	19.4	0.170 ± 0.012
	216	20.9	0.130 ± 0.021
^{99}Y	0	17.8	0.27 ± 0.03
^{101}Y	0	18.3	0.26 ± 0.03
	590	17.7	0.14 ± 0.02

ics of Fig. 1, where Zr has higher 2_1^+ energies than Sr. The four values for Sr and Y are nearly identical, with only a 1.3% rms deviation in the mean value of 17.96 keV. This value gives $\mathscr{G}/\mathscr{G}_{\rm rigid}$ [$\mathscr{G}_{\rm rigid} = 0.4AM_n(1.2A^{1/3}\ {\rm fm})^2(1+0.31\beta)$] in the range 0.80–0.85 for β in the range 0.3–0.5. In comparison, the deformed rare-earth nuclei have $\mathscr{G}/\mathscr{G}_{\rm rigid} \lesssim 0.5$.[14]

Table I also gives experimental results for $|(g_K - g_R)/Q_0|$, where Q_0 is the intrinsic quadrupole moment for the band and g_K and g_R are, respectively, the intrinsic and collective g factors. The results were obtained from branching ratios for the intraband γ transitions shown in Fig. 2, under the assumption of pure K bands (i.e., the Alaga rules).[15] For bands with four members, the result given in Table I is the average for the top two levels in the band. The ground bands in both $N=61$ isotones give nearly identical values, as do the ground bands in both Y isotopes.

The experimental ratio $(g_K - g_R)/Q_0$ can be used to deduce a Nilsson orbital assignment for the band. With $Q_0 = [3/(5\pi)^{1/2}]ZR^2\beta(1+0.16\beta)$ and $R = 1.2A^{1/3}$ fm, $Q_0 = 9.2\beta(1+0.16\beta)$ b for $Z=39$ and $A \approx 100$. For deformed rare-earth nuclei the odd-N g_R values are uniformly less than the even-even g_R values, which in turn are less than the odd-Z g_R value of Z/A.[14] Assuming similar relative values for the deformed $A \sim 100$ region, we used $g_R = 0.4$ for Z odd and $g_R = 0.2$ for N odd. The tabulation of Browne and Femenia[16] can be used to calculate g_K for a given Nilsson orbital. For $Z = 39$ the only $K=\frac{5}{2}$ orbitals near the Fermi surface for $\beta \sim 0.3$–0.4 are $\frac{5}{2}[422]$ and $\frac{5}{2}[303]$. With use of a Nilsson diagram which is appropriate for neutrons in the neutron-rich $A \sim 100$ region, the only $K=\frac{3}{2}$ orbitals expected near the Fermi surface for $\beta \sim 0.3$–0.4 are $\frac{3}{2}[411]$ and $\frac{3}{2}[541]$.

Calculated values of the ratio $(g_K - g_R)/Q_0$ are given in Fig. 3 for the expected Nilsson orbitals and for various values of f, the ratio of the effective g_s to the free g_s for the odd nucleon. In the deformed rare-earth region, the usual values of f, obtained after accounting for mixing of K bands due to the Coriolis interaction, are ~0.6, although values as low as 0.3 and as high as 0.9 have been obtained.[14] For the new deformed $A \sim 100$ region, in which the Nilsson orbitals have not been established, it is inappropriate to assume an orbital and deduce f. Instead, a range of values of f are used in Fig. 3 to allow deduction of Nilsson orbital assignments, if possible, for the new rotational bands presented here for deformations in the range $\beta \sim 0.3$–0.4. For $\pi\frac{5}{2}$-

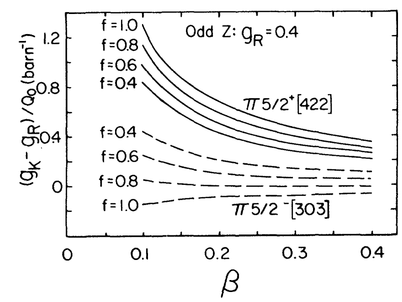

FIG. 3. Calculated values of $(g_K - g_R)/Q_0$ for proton $K=\frac{5}{2}$ Nilsson orbitals expected to lie near the Fermi surface for $Z=39$ and $\beta \sim 0.3$; $f = g_s^{\rm effective}/g_s^{\rm free}$ and $Q_0 = 9.2\beta(1+0.16\beta)$ b.

[303] the large variation of $(g_K - g_R)/Q_0$ with f is due to the near cancellation of g_K and g_R. Indeed, $g_K \approx g_R$ for $f=0.8$.

Comparison of Fig. 3 with Table I shows that $\frac{5}{2}[422]$ is clearly favored for the Y $K=\frac{5}{2}$ ground bands and $\frac{5}{2}[303]$ is favored for the ^{101}Y 590-keV band for $\beta \sim 0.3$–0.4. A low f value of ~0.4 is indicated for the 590-keV band. However, since the $\frac{3}{2}[301]$ orbital, which has a large (i.e., ~0.4) value of $(g_K - g_R)/Q_0$, is also expected near the Fermi surface, Coriolis mixing of the $\frac{5}{2}[303]$ and $\frac{3}{2}[301]$ bands and an f value of ~0.6 could also account for the observed 590-keV $(g_K - g_R)/Q_0$ value.

For the $N=61$ isotones the situation is unclear. Calculations similar to those of Fig. 3 were made for the $N=61$ isotones. The curves for $\nu\frac{3}{2}[411]$ and $\nu\frac{3}{2}[541]$ have appreciable overlap. However, the validity of these curves is questionable since the Nilsson parameters used in Ref. 16 are not appropriate for $N=61$ isotones. Our data cannot determine the $N=61$ Nilsson assignments. Perhaps the present ambiguity for $N=61$ could be removed by a direct measurement of the magnetic moment of the $K=\frac{3}{2}$ ground band or by angular-correlation measurements of intraband cascades.

An interesting feature that emerges from this analysis is that the unique-parity orbitals $\pi\frac{5}{2}[422]$ and $\nu\frac{3}{2}[541]$ seem to be virtually unaffected by Coriolis mixing, which is usually much larger for unique-parity orbitals. The nearly identical values of $\hbar^2/2\mathscr{G}$ for all bands, whether unique parity or not, indicate weak Coriolis mixing. The Coriolis mixing appears to be suppressed in these nuclei as a result of the large energy separation of adjacent unique-parity orbitals.

In summary, we have presented evidence for rotational structure in four odd-A nuclei in the $A \sim 100$ region. All six bands have similar values of $\hbar^2/2\mathscr{I}$ whose magnitudes imply deformations $\beta > 0.3$. These odd-A results thus indicate large deformations that occur only one or two nucleons beyond the onset of deformation. The magnitude of $\hbar^2/2\mathscr{I}$, if scaled by $A^{-5/3}$, makes this region one of the most deformed known. For the two Y isotopes, the $K=\frac{5}{2}$ ground bands are well described by the $\frac{5}{2}[422]$ Nilsson state, and the 590-keV $K=\frac{5}{2}$ band in ^{101}Y is most likely $\frac{5}{2}[303]$. The present data for $N=61$ cannot determine which $K=\frac{3}{2}$ bands are $\frac{3}{2}[411]$ and which are $\frac{3}{2}[541]$. Finally, the existence of the $\pi\frac{5}{2}[422]$ and $\nu\frac{3}{2}[411]$ orbitals near the Fermi surface for highly deformed $A \sim 100$ nuclei is consistent with the concept[9,10] that deformation is strongly promoted by partial occupancy of neutron and proton orbitals with strong spatial overlap.

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