Quark and Gluon Latent Heats at the Deconfinement Phase Transition in SU(3) Gauge Theory

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The authors have run computer simulations of the quark and gluon internal energies in SU(3) lattice gauge theory neglecting dynamical fermion loops. At the first-order deconfinement—chiral symmetry restoration transition the internal energies display large discontinuities. With finite-size effects estimated by a procedure suggested by the Bielefeld group, the total latent heat per unit volume is found to be roughly 1.50 ± 0.50 GeV/fm^3, if the transition temperature is 190 ± 20 MeV. Timing estimates for fermion computer simulation methods are presented.

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The study of hadronic matter under extreme conditions has attracted renewed interest lately because of the prospect of producing densities several times greater than that of nuclear matter in heavy-ion collisions.[1] In particular, the deconfinement transition in quantum chromodynamics may be discovered and the space-time evolution of a hot plasma of quarks and gluons into hadrons may be explored. It is the purpose of this Letter to report a calculation of the latent heat per unit volume, L, at the first-order deconfinement transition, which finds $L = 1.50 \pm 0.50$ GeV/fm^3. Our calculations suggest that hydrodynamic models of the quark-gluon plasma and models of hadronization should incorporate a hard, first-order transition.

Theoretical arguments for the first-order character of the SU(3) deconfining transition are well known.[2] Calculational evidence from computer simulations support those ideas. Hysteresis, coexisting states, and abrupt quantitative changes in various thermodynamic functions have been observed.[3-5] The temperature of the transition T_c has been extracted from the lattice calculations,[3-5]

$$T_c = (0.50 \pm 0.05)\sqrt{\sigma} = (76 \pm 3)\Lambda_L, \tag{1}$$

where $\sqrt{\sigma}$ is the string tension ($\sqrt{\sigma} \approx 420$ MeV) and Λ_L is the scale parameter of the lattice-regulated gauge theory (Λ_L is related to the momentum-space scale parameter of continuum quantum chromodynamics,[6] $\Lambda^{\text{mom}} = 83.5\Lambda_L$). Equation (1) gives $T_c \approx 180$–220 MeV. The uncertainties in these predictions account for the statistical fluctuations in computer simulations and do not include systematic uncertainties, which workers in the field believe could be as large as 50%. A recent calculation[5] considered the influence of this transition on the behavior of the massless quarks of quantum chromodynamics by studying the restoration of chiral symmetry as a function of T. It was found that chiral symmetry is broken spontaneously for $T < T_c$ but is restored at T_c where deconfinement occurs and where the gluons form a plasma. The order parameter $\langle\bar{\psi}\psi\rangle$ and thus the quark's dynamical mass appeared to vanish discontinuously at T_c. Since the transition is first order one naively expects these results which ignore dynamical fermion loops to survive in a more complete calculation.

To begin our discussion of the latent heat we remind the reader of our lattice fermion technique and our computational methods.[5] We use the "staggered" fermion method to describe massless Dirac fermions on the lattice. This method has a remnant of continuous chiral symmetry which breaks spontaneously because of the gauge field interaction and it has various discrete axial and axial-flavor symmetries which forbid the appearance of mass counterterms in the theory's Lagrangian. On a four-dimensional hypercubic lattice it yields four massless Dirac fermions in the continuum limit. In the approximation which neglects internal fermion loops (the "quenched" approximation[7]), results for a massless isodoublet of quarks are obtained from the matrix elements we calculate below by a trivial division by two. Since chiral symmetry is restored at T_c, a lattice fermion method which gives massless fermions is essential to estimate reliably the thermodynamic functions for $T \approx T_c$. Our calculations complement work by other groups[4] which have used expansion methods and fermion techniques where chiral symmetry is not realized in a natural fashion but where a massless Goldstone pion is obtained in the confining phase by fine tuning a parameter in the Lagrangian.

To calculate fermion matrix elements such as $\langle\bar{\psi}\psi\rangle$ and the internal energy, we implemented the pseudofermion Monte Carlo procedure.[8] Recall the basic idea of this method. The fermion piece of the lattice action is

$$S_f = \sum_{i,j} \bar{\psi}_i [\not{D}(U) + m]_{ij} \psi_j , \qquad (2)$$

where $\not{D}(U)$ is the covariant lattice derivative for staggered fermions in the gauge field $\{U\}$, ψ_i are fermion fields residing on the sites i, and m is a bare fermion mass which is taken to zero after the calculations are complete. We need the fermion Green's function $G_{ij}(U) = [\not{D}(U) + m]_{ij}^{-1}$. This can be obtained from a boson Monte Carlo problem which uses the positive-definite action,

$$S_b = \sum_{i,j} \varphi_i^* [-\not{D}^2(U) + m^2]_{ij} \varphi_j , \qquad (3)$$

by computation of the boson correlation function,

$$\langle \varphi_i \varphi_j^* \rangle = \frac{\int [d\varphi^*][d\varphi] \varphi_i \varphi_j^* \exp(-S_b)}{\int [d\varphi^*][d\varphi] \exp(-S_b)} , \qquad (4)$$

by an ordinary heat-bath algorithm and application of $[-\not{D}(U) + m]$:

$$G_{ij}(U) = \sum_k \langle \varphi_i \varphi_k^* \rangle [-\not{D}(U) + m]_{kj} . \qquad (5)$$

For a given gauge field configuration $G_{ij}(U)$ is computed, and a final average over an ensemble of gauge field configurations produced by a pure SU(3) gauge field heat-bath program[9] yields the matrix elements we need. It is crucial to know how quickly the pseudofermion method produces accurate estimates of $G_{ij}(U)$ in order to compute our statistical uncertainties and to judge if the method can be generalized to calculations which include fermion loops in the dynamics.[8] For a given gauge field configuration we typically computed $\langle \varphi_i \varphi_j^* \rangle$ and $G_{ij}(U)$ for several small fermion masses m, constructed fermion matrix elements of interest, and extrapolated the results to $m = 0$. A detailed study[10] showed that calculations at $m = 0.10$, 0.08, and 0.06 (in units of the inverse lattice spacing a^{-1}) and an essentially linear extrapolation to $m = 0$ are required. Larger values of m do not extrapolate linearly to $m = 0$ and smaller values yield matrix elements which suffer from significant finite-size effects. In Fig. 1 we show $\langle\bar{\psi}\psi\rangle$ computed in a given gauge field configuration $\{U\}$ as an average over all the sites of a 2×8^3 lattice at coupling $\beta = 6/g^2 = 5.10$ and bare fermion mass $m = 0.06$ plotted against the number of sweeps through the pseudofermion Monte Carlo program. To obtain $\langle\bar{\psi}\psi\rangle$ accurately (better than 10%, say) at least a thousand sweeps are needed. Since the pseudofermion program runs considerably faster than the gauge field program (a factor of 5 in speed is typical), the method is practical in a supercomputer environment. The convergence is much better for larger bare-fermion mass values (in our production runs 800, 1600, and 2000 pseudofermion sweeps were made for fermion masses of 0.10, 0.08, and 0.06, respectively, and the first 300 sweeps were discarded in each case). However, for quantities which are not translationally invariant, such as specific elements of $G_{ij}(U)$, needed in calculations which incorporate loops into the dynamics,[8] and where the added statistical accuracy obtained by averaging the matrix element over the entire

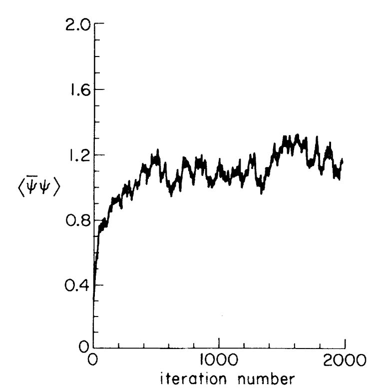

FIG. 1. The lattice average of $\bar{\psi}\psi$ calculated by a pseudofermion Monte Carlo program vs the number of sweeps through the lattice. The lattice is 2×8^3 and $\beta = 5.10$.

lattice is not possible, many more pseudofermion sweeps are needed to obtain 10% accuracy. At $m=0.10$ at least 1000 sweeps are needed to obtain $G_{ij}(U)$ for nearest-neighbor i and j to 10% accuracy and the number of sweeps grows to at least 3000–5000 at $m=0.06$. These estimates cast doubt on the utility of this calculational technique in proceeding beyond the quenched approximation.

The matrix elements of interest in this work are the gluon and fermion internal energies, the lattice analogs of $\vec{E}^2+\vec{B}^2$ and $\bar{\psi}\gamma_0 D_0\psi$. In the weak-coupling $g^2\sim 0$ region the leading terms of the lattice construction of the gluon internal energy density are[11]

$$\epsilon_g=(\beta/V)\Big\{\sum_{\text{space}}(1-\tfrac{1}{3}\operatorname{Re}\operatorname{tr}UUUU)-\sum_{\text{time}}(1-\tfrac{1}{3}\operatorname{Re}\operatorname{tr}UUUU)\Big\}, \qquad (6)$$

where the sums are over spatial and temporal plaquettes, respectively, and $V=N_t N_s^3 a^4$ is the volume of the four-dimensional lattice. There are finite-coupling, $O(\beta^0)$, corrections to Eq. (6) which have been recorded by the Bielefeld group,[11] but our statistical accuracy is not sufficiently good to warrant their study here. The fermion internal energy density is[11]

$$\epsilon_f=V^{-1}\sum_{\substack{\text{time}\\ \text{links}}}\operatorname{tr}[\not{D}_0(U)G(U)]-\tfrac{3}{4}, \qquad (7)$$

where the term $-\frac{3}{4}$ guarantees that ϵ_f vanishes on a symmetric lattice in the limit of vanishing quark mass m.

The Monte Carlo evaluation of Eqs. (6) and (7) proceeded as follows on 2×8^3 and 4×8^3 lattices. Equilibrated gauge configurations at various couplings had been saved from previous studies.[5] $\bar{\psi}\psi$ and ϵ_f were measured in these configurations by means of the pseudofermion program as described above. Then new gauge field configurations were generated by sweeping 100 times through the gauge field Monte Carlo program, and the Wilson line W and ϵ_g were calculated. Then measurements of the fermion matrix elements were made again and the entire procedure was repeated until the statistical uncertainties in the average quantities were less than 10%–15% (7–11 times were required). The internal energy data are shown in Fig. 2. These results must be converted to physical units. Using the relation between the temporal lattice size N_t, the lattice spacing a, and the temperature T, $aT=N_t^{-1}$, and recalling asymptotic freedom,

$$a\Lambda_L=[(8\pi^2/33)\beta]^{51/121}\exp(-4\pi^2\beta/33), \qquad (8)$$

we generate Fig. 3 from the 4×8^3 data. That plot shows ϵ_f/T^4 and ϵ_g/T^4 vs T/Λ_L. We estimate that the phase transition occurs at $T_c=(76\pm 3)\Lambda_L=190\pm 20$ MeV in good agreement with other recent measurements.[3-5]

The most intriguing aspect of Fig. 3 is the fact that both ϵ_f/T^4 and ϵ_g/T^4 turn on abruptly at T_c and are only weakly dependent on temperature for $T>T_c$. The abruptness supports the claim

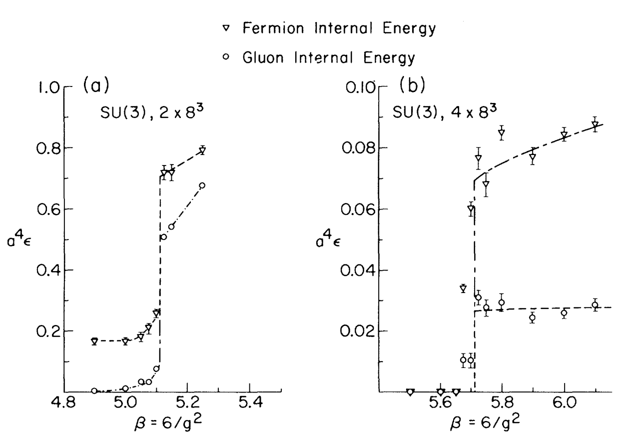

FIG. 2. (a) Data for the quark and gluon internal energies on a 2×8^3 lattice. (b) Same as (a) except the lattice is 4×8^3.

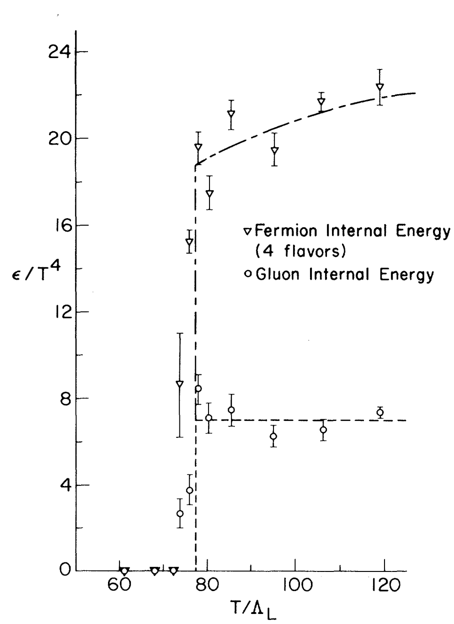

FIG. 3. Gluon and quark internal energies, ϵ_g/T^4 and ϵ_f/T^4, vs T/Λ_L. The free-field limits on a 4×8^3 lattice are approximately 7.5 for gluons and 24.5 for quarks.

that the deconfining–chiral symmetry restoring transition is first order.[3-5] The finite width of the temperature interval where the densities rise from zero, $\Delta T \approx 4\Lambda_L \approx 10$ MeV, is presumably a finite-size effect. To estimate the size of the latent heat we follow the work of the Bielefeld group[12] and compare ϵ_f/T^4 and ϵ_g/T^4 with the free-field calculations of these quantities also made on 4×8^3 lattices. For staggered fermions (four flavors) $\epsilon_f^{\text{free}}/T^4 \approx 24.5$ and for eight massless gluons $\epsilon_g^{\text{free}}/T^4 \approx 7.5$.[12] From Fig. 3 we see that the internal energy for the interacting theory at T just above T_c is 75%–85% of the internal energy of the free constituents of the field theory. This is the central result of our study. An estimate of the latent heat results if we assume, again following Refs. 11 and 12, that the ratio $(\epsilon_g + \epsilon_f)/(\epsilon_g^{\text{free}} + \epsilon_f^{\text{free}})$ computed on a 4×8^3 lattice is the same as its continuum limit. In this limit

$$\epsilon_g^{\text{free}} + \epsilon_f^{\text{free}} = (8\pi^2/15 + 7\pi^2/10)T^4,$$

the classic Stefan-Boltzmann result for two fermion flavors and eight SU(3) gauge fields. Putting these ingredients together with $T_c = 190 \pm 20$ MeV, we estimate the latent heat, $\epsilon_g + \epsilon_f$ just above the transition, to be 1.50 ± 0.50 GeV/fm^3.

It should be clear that our quantitative estimate of the latent heat depends sensitively on the procedure for estimating finite-size effects introduced in Refs. 11 and 12. Our results are, therefore, subject to considerable systematic uncertainty. More sophisticated, time-consuming calculations on larger lattices which use lattice actions that give better continuum fits are necessary to obtain more controlled, reliable results.

The clarity of Fig. 3 has encouraged us to pursue the thermodynamics of the deconfining transition. In addition, we are running simulations on lattices of various sizes to check the universality of the physical quantities T_c and $\epsilon_g + \epsilon_f$.

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