Limits to the Number of Neutrinos: A Comment on the Z^0 Discovery

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The observation of four Z's in a sample of events containing 32 charged W's improves terrestrial bounds on the number of neutrinos by roughly one order of magnitude. Also a bound on the production cross section of Z's relative to W's is obtained. These bounds do not depend on the determination of the Z width.

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Counting the number of massless neutrinos in terrestrial experiments has been an extremely difficult task. The upper limit $N_\nu < 6000$ from the nonobservation of the decay $K \to \pi\nu\bar{\nu}$ was for some time the most successful result.[1] Recently this limit has been improved[2] to $N_\nu < 1400$. Also Jarlskog and Yndurain[3] have obtained the bound $N_\nu < 137$ from a study of the effects of neutrinos on the radiative corrections to the ρ parameter and the $e^+e^- \to \mu^+\mu^-$ cross section. It is well known that the Z boson is a powerful experimental counter of (nearly) massless neutrinos. In this paper we discuss the implications of the observation[4] of 4 $Z \to e^+e^-$ decays in a sample of events containing 32 $W \to e\nu$ events at the CERN $\bar{p}p$ collider.

The UA(1) experiment[4] measures the observable

$$R_{\rm exp} = \sigma_Z B(Z \to e^+e^-)/\sigma_W B(W \to e\nu), \tag{1}$$

where $\sigma_{W,Z}$ stand for the hadronic production cross sections of W, Z bosons, with

$$\sigma_W = \sigma_{W^+} + \sigma_{W^-} \tag{2}$$

Under the assumption of universal weak couplings, the electronic branching ratios of W and Z in Eq. (1) are given by

$$B(W \to e\nu) = 1/N_l + 3N_q, \tag{3}$$

$$B(Z \to e^+e^-) = a_W/(N_\nu + a_W N_l + 3b_W N_q). \tag{4}$$

Here N_l, N_q are the number of charged leptons and quark doublets into which the weak bosons can decay (the case of a heavy t quark is discussed below), and a_W, b_W are known functions of $x_W \equiv \sin^2\theta_W$:

$$a_W = 1 - 4x_W + 8x_W^2, \tag{5}$$

$$b_W = 2 - 4x_W + (40/g)x_W^2. \tag{6}$$

From measurement of M_W and M_Z one obtains[4] $x_W = 0.226$. Equations (1)–(4) can be solved for N_ν,

$$N_\nu = [a_W(N_l + 3N_q)/R_{\rm exp}](\sigma_Z/\sigma_W) - a_W N_l - 3b_W N_q. \tag{7}$$

We now obtain an upper bound on (σ_Z/σ_W) which, via Eq. (7), can be translated into an upper bound on the number of neutrinos N_ν. The hadronic production cross section of a vector meson V is given by[5]

$$\sigma_V = [\Gamma(V \to \text{hadrons})/M^3]\, F(M/\sqrt{s}), \tag{8}$$

where M and Γ are its mass and hadronic width. F is a universal function of the scaling variable $M/\sqrt{s}$ and $\sqrt{s}$ is the center-of-mass energy of the colliding hadrons. From Eq. (8) we obtain

$$\frac{\sigma_Z}{\sigma_W} = \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(W \to \text{hadrons})}\left(\frac{M_W}{M_Z}\right)^3 \frac{F(M_Z/\sqrt{s})}{F(M_W/\sqrt{s})}, \tag{9}$$

where we used the fact that $\sigma_{W^+} = \sigma_{W^-}$ in $\bar{p}p$ interactions. [Notice that the statistical counting factor in (9) is 1 and not $\frac{1}{2}$ as for every formation channel $q_1\bar{q}_2$, $\bar{q}_1 q_2$ for W^+, W^- there are two corresponding formation channels for Z $(q_1\bar{q}_1, q_2\bar{q}_2)$.] If one assumes, as before, universal couplings of W, Z to quarks, Eq. (9) can be written

$$\frac{\sigma_Z}{\sigma_W} = \frac{b_W}{2}\frac{F(M_Z/\sqrt{s})}{F(M_W/\sqrt{s})}. \tag{10}$$

As F is of course a decreasing function[5] of M at fixed $\sqrt{s}$, $M_Z > M_W$ implies that

$$\sigma_Z/\sigma_W < b_W/2. \tag{11}$$

We will later discuss the validity of Eq. (8) from which inequality (11) is obtained. However, the issue is not important as inequality (11) basically follows from phase space. *It states that, after adjusting for the different universal weak coupling of W, Z to the annihilating quark pairs in the $\bar{p}p$ initial state* [i.e., the factor $(b_W/2)$ in Eq. (10)], *the heavier Z is less frequently produced than*

TABLE I. $(N_\nu)_{\max}$ and $(\sigma_Z/\sigma_W)_{\min}$ for various values of $R_{\exp}$ defined by Eq. (1). We assume that the number of charged leptons and quark doublets into which the W and Z decay is equal to $N_l = N_q = 3$ $(M_t < M_Z/2)$. The corresponding result for $N_l = N_q = 4$ is shown in brackets.

$R_{\exp}$	$(N_\nu)_{\max}$	$(\sigma_Z/\sigma_W)_{\min}$
8/32	2(3)	0.64(0.61)
4/32	18(24)	0.32(0.31)
4/64	50(67)	0.16(0.15)

TABLE II. $(N_\nu)_{\max}$ and $(\sigma_Z/\sigma_W)_{\min}$ for the case of $M_t > M_W$. The numbers in brackets correspond to one extra light charged lepton and down quark ($N_l = 4$, $N_d = 4$, $N_u = 2$).

$R_{\exp}$	$(N_\nu)_{\max}$	$(\sigma_Z/\sigma_W)_{\min}$
8/32	0(<0)	0.75(0.81)
4/32	12(12)	0.37(0.40)
4/64	36(39)	0.19(0.20)

the W. Combining Eqs. (7) and (11) we obtain the bound

$$N_\nu < [(a_W b_W/2R_{\exp})(N_l + 3N_q) - a_W N_l - 3b_W N_q]. \tag{12}$$

Also notice the fact that $N_\nu > 2$ implies a lower bound on the cross-section ratio

$$\sigma_Z/\sigma_W > [R_{\exp}/a_W(3N_q + N_l)] \times [2 + a_W N_l + 3b_W N_q]. \tag{13}$$

By use of Eqs. (5) and (6) and taking $N_q = N_l = 3$, the observation $R_{\exp} = \frac{1}{8}$ implies via (12) that $N_\nu < 18$. From Eq. (13) we obtain $\sigma_Z \gtrsim \frac{1}{3}\sigma_W$. The sensitivity of this result to experimental ambiguities is shown in Table I. The first row illustrates how the bound would be significantly improved if some $Z \to e^+e^-$ events went undetected ($N_\nu < 3$ for $R_{\exp} = \frac{1}{4}$!). The bound is of course weakened if some W's were missed by the detector. However, even if one out of two W's went undetected $N_\nu < 50$ as shown in the last row of Table I.

As the experimental accuracy on $R_{\exp}$ will be significantly improved in the near future, it is perhaps more important to discuss possible theoretical ambiguities in the derivation of the bounds (12) and (13). It is easy to show that the results are not sensitive to errors on $\sin^2\theta_W$. The correct treatment of quark masses or mixing angles does not change the results significantly. Furthermore, if $M_t > M_W$ slightly stronger bounds would ensue, see Table II. The ultimate question is therefore: can the bound (11) possibly be violated? Firstly, it follows directly from Eq. (8) for which the empirical evidence is overwhelming. Equation (8) describes[5] ω, ρ, φ, ψ, ψ' production at different energies and correctly predicted the γ[6] and W[7] cross sections. Its application to Z is very safe considering the range over which it has been established. Secondly, QCD calculations satisfy (11). In the high-energy limit, where sea quarks dominate the production of weak bosons, the bound (11) is actually saturated. At $\sqrt{s} = 540$ GeV explicit calculations[8] predict $\sigma_Z/\sigma_W = 0.3-0.4$. This is consistent with $R_{\exp} = \frac{1}{8}$ for $N_\nu = N_l = N_q = 3$. It is also difficult to imagine how higher-order corrections can change this *ratio* significantly.

A significant violation of the bound on N_ν established in this fashion can only occur by a total breakdown of the Drell-Yan picture in QCD or by an explosion of the number of charged leptons and quarks below M_W. Finally our result is consistent with the fact that within 2σ the experiment[4] restricts the total width of the Z to roughly 6 GeV, allowing not more than about 20 neutrinos.

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[1]R. L. Kelly *et al.* (Particle Data Group), Rev. Mod. Phys. 52, S1 (1980).
[2]Y. Asano *et al.*, Phys. Lett. 107B, 159 (1981).
[3]C. Jarlskog and F. J. Yndurâin, Phys. Lett. 120B, 361 (1983).
[4]G. Arnison *et al.*, Phys. Lett. 126B, 398 (1983).
[5]T. K. Gaisser *et al.*, Phys. Rev. D 15, 2572 (1977).
[6]F. Halzen and S. Matsuda, Phys. Rev. D 17, 1344 (1978).
[7]H. Muirhead, Technical Report No. UA1/TN83-01, 1983 (unpublished).
[8]See, e.g., *Proton-Antiproton Collider Physics—1981*, AIP Conference Proceedings No. 85 (American Institute of Physics, New York, 1983).