Quantum Chromodynamic Predictions for the Deuteron Form Factor

Stanley J. Brodsky and Chueng-Ryong Ji
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

G. P. Lepage
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853
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The asymptotic large-momentum-transfer behavior of the deuteron form factor and the form of the deuteron distribution amplitude at short distances are derived from perturbative quantum chromodynamics. The fact that the six-quark state is 80% hidden color at small transverse separation implies that the deuteron form factors cannot be described at large Q^2 by meson-nucleon degrees of freedom, and that the nucleon-nucleon potential is repulsive at short distances.

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If quantum chromodynamics (QCD) is the theory of the strong interactions, then by extension it must also provide a fundamental description of the nuclear force and nuclear physics. Since the basic scale of QCD, $\Lambda_{\overline{\rm MS}}$, is of order of a few hundred megaelectronvolts or less, one expects a transition from the traditional meson and nucleon degrees of freedom of nuclear physics to the quark and gluon degrees of freedom of QCD at internucleon separations of a fermi or less.[1]

In this Letter we give the first exact results for a nuclear amplitude as predicted by perturbative QCD. The asymptotic behavior of the deuteron form factor at large momentum transfer and the evolution of the deuteron six-quark distribution amplitude at short distances are computed to leading order in $\alpha_s(Q^2)$. The QCD predictions appear to be in excellent agreement with experiment for $Q^2 \gtrsim 1$ GeV^2 when expressed in terms of the deuteron reduced form factor. This provides a good check on the six-quark description of the deuteron at short distances as well as the scale invariance of the elastic quark-quark scattering amplitude. The dominance of the hidden-color amplitudes at short distances also provides an explanation for the repulsive behavior of the nucleon-nucleon potential at small internucleon separation.

The hadronic form factors[2] in QCD at large momentum transfer $Q^2 = \vec{q}^2 - q_0^2$ can be written in a factorized form where all nonperturbative effects are incorporated into process-independent distribution amplitudes $\varphi_H(x_i, Q)$, computed from the equal $\tau = t+z$, six-quark valence wave function at small relative quark transverse separation $b_\perp{}^i \sim O(1/Q)$. The $x_i = (k^0+k^3)_i/(p^0+p^3)$ are the light-cone longitudinal momentum fractions with $\Sigma_{i=1}^n x_i = 1$. In the case of the deuteron, only the six-quark Fock state needs to be considered since in a physical gauge any additional quark or gluon forced to absorb large momentum transfer yields a power-law-suppressed contribution to the form factor. The deuteron form factor can then be written as a convolution (see Fig. 1),

$$F_d(Q^2) = \int_0^1 [dx][dy] \varphi_d^\dagger(y,Q) \times T_H{}^{6q+\gamma^*\to 6q}(x,y,Q)\varphi_d(x,Q), \qquad (1)$$

where the hard-scattering amplitude

$$T_H{}^{6q+\gamma^*\to 6q} = [\alpha_s(Q^2)/Q^2]^5 t(x,y) \times [1+O(\alpha_s(Q^2))] \qquad (2)$$

gives the probability amplitude for scattering six quarks collinear with the initial to the final deuteron momentum and

$$\varphi_d(x_i,Q) \propto \int^{k_{\perp i}<Q} [d^2k_\perp]\psi_{qqq\,qqq}(x_i,\vec{k}_{\perp i}) \qquad (3)$$

gives the probability amplitude for finding the quarks with longitudinal momentum fractions x_i in the deuteron wave function collinear up to the scale Q. Because the coupling of the gauge gluon is helicity conserving and because of the fact that $\varphi_d(x_i, Q)$ is the $L_z = 0$ projection of the deuteron wave function, hadron helicity is conserved[3]: The dominant form factor corresponds to $\sqrt{A}(Q^2)$;

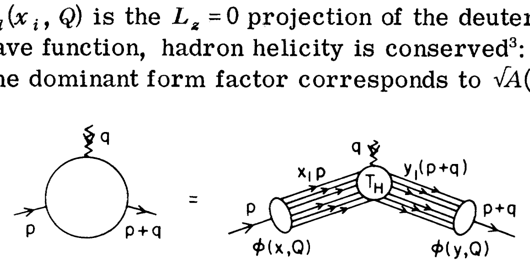

FIG. 1. The general structure of the deuteron form factor at large Q^2.

i.e., $h=h'=0$.

The distribution amplitude $\varphi_d(x_i,Q)$ is the basic deuteron wave function which controls high-momentum-transfer exclusive reactions in QCD. The logarithmic Q^2 dependence of φ_d is determined by an evolution equation computed from perturbative quark-quark scattering kernels at large momentum transfer, or equivalently, by the operator-product expansion at short distances and the renormalization group.[2]

The QCD prediction for the leading helicity-zero deuteron form factor then has the form[3,4]

$$F_d(Q^2)=\left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5\sum_{m,n} d_{mn}\left(\ln\frac{Q^2}{\Lambda^2}\right)^{-\gamma_n{}^d-\gamma_m{}^d}\left[1+O\left(\alpha_s(Q^2),\frac{m}{Q}\right)\right], \tag{4}$$

where the main dependence $[\alpha_s(Q^2)/Q^2]^5$ comes from the hard-gluon exchange amplitude T_H. The anomalous dimensions $\gamma_n{}^d$ are calculated from the evolution equations for $\varphi_d(x_i,Q)$.

The evolution equation for six-quark systems in which the constituents have the light-cone longitudinal momentum fractions x_i $(i=1,2,\ldots,6)$ can be obtained from a generalization of the proton (three-quark) case.[2] A nontrivial extension is the calculation of the color factor, C_d, of six-quark systems[5] (see below). Since in leading order only pairwise interactions, with transverse momentum Q, occur between quarks, the evolution equation for the six-quark system becomes $\{[dy]=\delta(1-\sum_{i=1}^6 y_i)\prod_{i=1}^6 dy_i$, $C_F=(n_c{}^2-1)/2n_c=\frac{4}{3}$, $\beta=11-\frac{2}{3}n_f$, and n_f is the effective number of flavors$\}$

$$\prod_{k=1}^{6} x_k\left[\frac{\partial}{\partial\xi}+\frac{3C_F}{\beta}\right]\tilde{\Phi}(x_i,Q)=-\frac{C_d}{\beta}\int_0^1[dy]V(x_i,y_i)\tilde{\Phi}(y_i,Q), \tag{5}$$

where the factor 3 in the square brackets comes from the renormalization of the six-quark field. In Eq. (5) we have defined $\Phi(x_i,Q)=\prod_{k=1}^6 x_k\tilde{\Phi}(x_i,Q)$. The evolution is in the variable

$$\xi(Q^2)=\frac{\beta}{4\pi}\int_{Q_0{}^2}^{Q^2}\frac{dk^2}{k^2}\alpha_s(k^2)\sim\ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0{}^2/\Lambda^2)}\right). \tag{6}$$

By summation over interactions between quark pairs $\{i,j\}$ due to exchange of a single gluon, $V(x_i,y_i)=V(y_i,x_i)$ is given by

$$V(x_i,y_i)=2\prod_{k=1}^{6}x_k\sum_{i\neq j}^{6}\theta(y_i-x_i)\prod_{l\neq i,j}^{6}\delta(x_l-y_l)\frac{y_j}{x_j}\left(\frac{\delta_{h_i\bar{h}_j}}{x_i+x_j}+\frac{\Delta}{y_i-x_i}\right), \tag{7}$$

where $\delta_{h_i\bar{h}_j}=1$ (0) when the helicities of the constituents $\{i,j\}$ are antiparallel (parallel). The infrared singularity at $x_i=y_i$ is cancelled by the factor $\Delta\tilde{\Phi}(y_i,Q)=\tilde{\Phi}(y_i,Q)-\tilde{\Phi}(x_i,Q)$ since the deuteron is a color singlet.

The six-quark bound states have five independent color-singlet components $(\underline{3}\times\underline{3}\times\underline{3}\times\underline{3}\times\underline{3}\times\underline{3}\supset\underline{1}+\underline{1}+\underline{1}+\underline{1}+\underline{1})$. It can be shown in general[5] that the color factor C_d is given by

$$C_d=\tfrac{1}{5}S_{ijklmn}{}^{\alpha}(\tfrac{1}{2}\lambda_a)_{i'}{}^{i}(\tfrac{1}{2}\lambda_a)_{j'}{}^{j}S_{\alpha}{}^{i'j'klmn}, \tag{8}$$

where λ_a $(a=1,2,\ldots,8)$ are Gell-Mann matrices in the SU(3)^c group and $S_{ijklmn}{}^{\alpha}$ $(\alpha=1,2,\ldots,5)$ are the five independent color-singlet representations. Here we shall focus on results for the leading contribution to the distribution amplitude and form factor at large Q. Since the leading eigensolution to the evolution equation (5) turns out to be completely symmetric in its orbital dependence, the dominant asymptotic deuteron wave function is fixed by overall antisymmetry[6] to have spin-isospin symmetry $\{33\}_{TS}$ which is dual to its color symmetry $[222]_c$. Thus the coefficient for each c (and TS) component has equal weights:

$$\varphi_{6q}([222]_c\otimes\{33\}_{TS})=(1/\sqrt{5})\sum_{\alpha=1}^{5}(-1)^{\alpha}[222]_c{}^{\alpha}\{33\}_{TS}{}^{\alpha}. \tag{9}$$

Since the evolution potential is diagonal in isospin and spin, C_d is computed by the trace of the color representation. The color factor[5,6] is $-\frac{2}{3}$ for the color-antisymmetric pair $\{i,j\}$ and $+\frac{1}{3}$ for the color-symmetric pair $\{i,j\}$. Since three color-antisymmetric pairs $\{i,j\}$ exist in this state, the color factor is[7]

$$C_d=\tfrac{1}{5}(-\tfrac{2}{3}\times3+\tfrac{1}{3}\times2)=-C_F/5. \tag{10}$$

To solve the evolution equation (5), we factorize the Q^2 dependence of $\tilde{\Phi}(x_i, Q)$ as

$$\tilde{\Phi}(x_i, Q) = \tilde{\Phi}(x_i)e^{-\gamma\xi} = \tilde{\Phi}(x_i)[\ln(Q^2/\Lambda^2)]^{-\gamma}, \quad (11)$$

where the eigenvalues of γ will provide the anomalous dimensions γ_n. The general matrix representations of γ_n with bases $|\prod_{i=1}^{5} x_i^{m_i}\rangle$ are given in Ref. 5. The leading anomalous dimension γ_0 [corresponding to the eigenfunction $\Phi(x_i) = 1$] is

$$\gamma_0 = \frac{3C_F}{\beta} + \frac{C_d}{\beta}\sum_{i \neq j}^{6} \delta_{h_i \bar{h}_j}, \quad (12)$$

so that the asymptotically dominant result for the helicity-zero deuteron is given by $\gamma_0 = \frac{6}{5} C_F/\beta$.

In order to make more detailed and experimentally accessible predictions, we define the "reduced" nuclear form factor[8] to remove the effects of nucleon compositeness:

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N^2(Q^2/4)}. \quad (13)$$

The arguments for the nucleon form factors (F_N) are $Q^2/4$ since in the limit of zero binding energy each nucleon must change momentum $Q/2$ because of the electromagnetic interaction. Since the leading anomalous dimension of the nucleon distribution amplitude is $C_F/2\beta$, the QCD prediction for the asymptotic Q^2 behavior of $f_d(Q^2)$ is[9]

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2}\left(\ln\frac{Q^2}{\Lambda^2}\right)^{-(2/5)\,C_F/\beta}, \quad (14)$$

where $\frac{2}{5} C_F/\beta = -\frac{8}{145}$ for $n_f = 2$.

Although the QCD prediction is for asymptotic momentum transfer, it is interesting to compare (14) directly with the available high-Q^2 data[10] (see Fig. 2). In general one would expect corrections from higher-twist effects (e.g., mass and $k_\perp$ smearing) and higher-order contributions in $\alpha_s(Q^2)$, as well as nonleading anomalous dimensions. However, the agreement of the data with simple $Q^2 f_d(Q^2) \sim$ const behavior for $Q^2 > \frac{1}{2}$ GeV^2 implies that, unless there is a fortuitous cancellation, all of the scale-breaking effects are small, and the present QCD perturbation calculations are viable and applicable even in the nuclear physics domain. The lack of deviation from the QCD parametrization suggests that the parameter Λ in (14) is small. A comparison with a standard definition such as $\Lambda_{\overline{MS}}$ would require a calculation of next to leading effects. A more definitive check of QCD can be made by calculating the normalization of $f_d(Q^2)$ from T_H and the evolution of the deuteron wave function to short distances. It is also important to confirm experimentally that the $h = h' = 0$ form factor is indeed dominant.

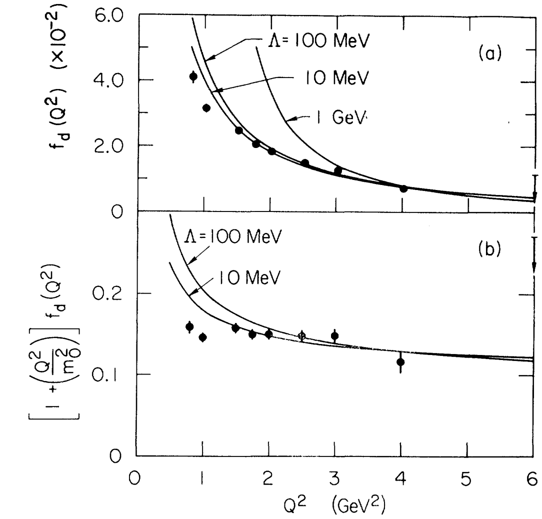

FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71\ \text{GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4$ GeV^2 data point. (b) Comparison of the prediction $[1 + (Q^2/m_0^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0^2 = 0.28$ GeV^2 is used (Ref. 8).

We note that the deuteron wave function which contributes to the asymptotic limit of the form factor is the totally antisymmetric wave function corresponding to the orbital Young symmetry given by [6] and isospin (T) + spin (S) Young symmetry given by $\{33\}$. The deuteron state with this symmetry is related to the NN, $\Delta\Delta$, and hidden-color (CC) physical bases, for both the $(TS) = (01)$ and (10) cases, by the formula[6]

$$\psi_{[6]\{33\}} = (\tfrac{1}{9})^{1/2}\psi_{NN} + (\tfrac{4}{45})^{1/2}\psi_{\Delta\Delta} + (\tfrac{4}{5})^{1/2}\psi_{CC}. \quad (15)$$

Thus the physical deuteron state, which is mostly ψ_{NN} at large distance, must evolve to the $\psi_{[6]\{33\}}$ state when the six-quark transverse separations $b_\perp^i \leq O(1/Q) \to 0$. Since this state is 80% hidden color, the deuteron wave function cannot be described by the meson-nucleon isobar degrees of freedom in this domain. The fact that the six-quark color-singlet state inevitably evolves in QCD to a dominantly hidden-color configuration at small transverse separation also has implications for the form of the nucleon-nucleon potential, which can be considered as one interaction component in a coupled-channel system. As the two nucleons approach each other, the system must do work in order to change the six-quark state to a dominantly hidden-color configuration; i.e., QCD requires that the nucleon-nucleon potential must be repulsive at short distances.[11] The evolution equation (5) for the six-quark system suggests that the distance where this change occurs is in the domain where $\alpha_s(Q^2)$ most strongly varies.

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[9]More generally, we can obtain the anomalous dimension for the reduced form factor of a general nucleus:

$$\Gamma_A = \begin{cases} (C_F/2\beta)(A-1), & A \text{ odd}, \quad h = h' = \pm\frac{1}{2}; \\ (C_F/2\beta)A(3A-4)/(3A-1), & A \text{ even}, \quad h = h' = 0. \end{cases}$$

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