Calculations of Pion Excess in Nuclei

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The authors construct, in the static potential approximation, the number operator for excess pions present in nuclei due to the pion-exchange forces between nucleons. Expectation values are calculated with use of variational wave functions obtained from a realistic Hamiltonian. Results are presented for systems ranging from the deuteron to nuclear matter: For example ^{56}Fe is estimated to have ~7 extra pions, over and above the pion clouds associated with the individual nucleons.

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Nucleons are surrounded by pion fields; hence the expectation value $\langle n^\pi\rangle_N$ of the pion number operator

$$n^\pi = \int d^3k(2\pi)^{-3} a_\pi^\dagger(k)a_\pi(k) \tag{1}$$

in the state describing an isolated nucleon[1] is nonzero. When A nucleons are brought together to form a nucleus, they interact by exchanging pions, and $\langle n^\pi\rangle_A$ for the nucleus is greater than $A\langle n^\pi\rangle_N$. It has been argued in the literature[2] that a fraction of the observed photon scattering off nuclei is due to the pion excess, $\langle \delta n^\pi\rangle_A = \langle n^\pi\rangle_A - A\langle n^\pi\rangle_N$. Part of the exchange currents[3] in nuclear electromagnetic processes can be attributed to the pion excess, and recently it has been suggested that the differences[4] between deep-inelastic lepton scattering from ^{56}Fe and ^{2}H may have a similar source.[5] In this paper we present an initial estimate of $\langle \delta n^\pi\rangle_A$ by means of a many-body calculation with an appropriate operator, and report the momentum distribution $\delta n^\pi(k)$ in the rest frame of the nucleus.

In conventional nuclear many-body theory the pion degrees of freedom are eliminated in favor of a static one-pion-exchange potential V_{ij}^π. The many-body Schrödinger equation is solved for a Hamiltonian containing V_{ij}^π to obtain the ground state $|A\rangle$ of a system with A nucleons. We will construct the analogous static two-body operator δn_{ij}^π whose expectation value taken with the state $|A\rangle$ gives an estimate of the pion excess.

Let H_i' be the interaction responsible for creating and annihilating pions of momentum k on the nucleon i. Because the $\Delta(1232)$ resonance plays an important role in the πN interaction, we include $\pi N\Delta$ as well as πNN couplings:

$$H_i' = \sum_k \{(f/m_\pi)(\vec\sigma_i\cdot\vec{k})\vec\tau_i + (f^*/m_\pi)[(\vec{S}_i\cdot\vec{k})\vec{T}_i + \text{H.c.}]\}\cdot\vec\varphi_\pi(k)\Lambda(k^2), \tag{2}$$

where $\vec{S}$ and $\vec{T}$ are transition spin and isospin operators,[6] $\vec\varphi_\pi(k)$ is the pion field, and $\Lambda(k^2)$ is a form factor to allow for nucleon and Δ finite-size effects as well as vertex corrections. H' connects states $|A\rangle$ to intermediate states $|I\pi\rangle$ containing a pion; V_{ij}^π is defined such that

$$\langle A|\sum_{i<j} V_{ij}^\pi|A\rangle = -\sum_{i\neq j}\sum_{I\pi}\langle A|H_i'|I\pi\rangle\omega_\pi^{-1}\langle I\pi|H_j'|A\rangle. \tag{3}$$

Since $|A\rangle$ contains the effect of V_{ij}^π to all orders, this calculation of the pion-exchange interaction energy is not only of second order in H'; it sums all orders of H' in the static potential approximation (SPA). In (3), the $\sum_j \langle I\pi|H_j'|A\rangle/\omega_\pi$ is the amplitude for the nucleus to be in the state $|I\pi\rangle$. Hence the corresponding operator for the number of excess pions in SPA is defined by

$$\langle A|\sum_{i<j}\delta n_{ij}^\pi|A\rangle = \sum_{i\neq j}\sum_{I\pi}\frac{\langle A|H_i'|I\pi\rangle}{\omega_\pi}\,\frac{\langle I\pi|H_j'|A\rangle}{\omega_\pi}. \tag{4}$$

Equations (3) and (4) lead to the following momentum-space definitions:

$$V_{ij}^{\pi}(k)=\frac{-1}{m_\pi^2(m_\pi^2+k^2)}\{f(\vec{\sigma}_i\cdot\vec{k})\vec{\tau}_i+f^*[(\vec{S}_i\cdot\vec{k})\vec{T}_i+(\vec{S}_i^{\,\dagger}\cdot\vec{k})\vec{T}_i^{\,\dagger}]\}$$
$$\times\{f(\vec{\sigma}_j\cdot\vec{k})\vec{\tau}_j+f^*[(\vec{S}_j\cdot\vec{k})\vec{T}_j+(\vec{S}_j^{\,\dagger}\cdot\vec{k})\vec{T}_j^{\,\dagger}]\}\Lambda^2(k^2) \tag{5}$$

and

$$\delta n_{ij}^{\pi}(k)=-(m_\pi^2+k^2)^{-1/2}V_{ij}^{\pi}(k). \tag{6}$$

We can obtain a better understanding of the operator δn_{ij}^{π} by calculating the pion excess in field theory[7] for a simple model Hamiltonian H_0+H', where H_0 describes free nucleons, pions, and Δ's, and $H'=\sum_i H_i'$:

$$\langle\delta n^\pi\rangle_A^{\mathrm{FT}}=\langle\varphi_0|\sum_{n=0}^{\infty}[(-i)^n/n!]\int_{-\infty}^{\infty}dt_1\cdots\int_{-\infty}^{\infty}dt_n\,T[H'(t_1)\cdots H'(t_n)n^\pi(0)]\,\varphi_0\rangle_{C^*}. \tag{7}$$

Here $|\varphi_0\rangle$ is the Fermi-gas ground state of A nucleons, $n_\pi(t)$ and $H'(t)$ are in the interaction representation, the subscript C denotes a sum over connected diagrams only, and the asterisk indicates that pions emitted and absorbed by the same nucleon should not be counted. Since all pions that are created must be destroyed, only even values of n contribute. In SPA we have

$$\langle\delta n^\pi\rangle_A^{\mathrm{SPA}}=\langle\varphi_0|\sum_{n=0}^{\infty}[(-i)^n/n!]\int_{-\infty}^{\infty}dt_1\cdots\int_{-\infty}^{\infty}dt_n\,T[V^\pi(t_1)\cdots V^\pi(t_n)\delta n^\pi(0)]\,|\varphi_0\rangle_C, \tag{8}$$

where $V^\pi=\sum_{i<j}V_{ij}^{\pi}$ and $\delta n^\pi=\sum_{i<j}\delta n_{ij}^{\pi}$.

Both (7) and (8) are power series in f and f^* with terms proportional to $f^{n_1}f^{*n_2}$, where n_1 and n_2 are even numbers. We may order the terms according to $N=n_1+n_2$. Examples of time-ordered diagrams for $N=2$, 4, and 6 are shown in Fig. 1. Diagrams labeled $F.N$ are generated by (7) while those labeled $P.N$ are generated by (8). In lowest order ($N=2$) only the exchange diagrams $F.2$ and $P.2$ contribute. They give almost identical negative contributions, as may be verified by a simple estimate; these represent the Pauli blocking of nucleon self-energy processes. The contributions of all direct diagrams of order $N=4$ and of ring diagrams of order $N=6$ and 8 to nuclear matter are given in Table I. We see that the SPA overestimates the field-theoretic $\langle\delta n^\pi\rangle$ contributions by ~30%. The phenomenological form factor used here is

$$\Lambda(k^2)=(\lambda^2-m_\pi^2)^2/(\lambda^2+k^2)^2, \tag{9}$$

and the coupling constants are $f^2/4\pi=0.08$, $f^*=2f$.

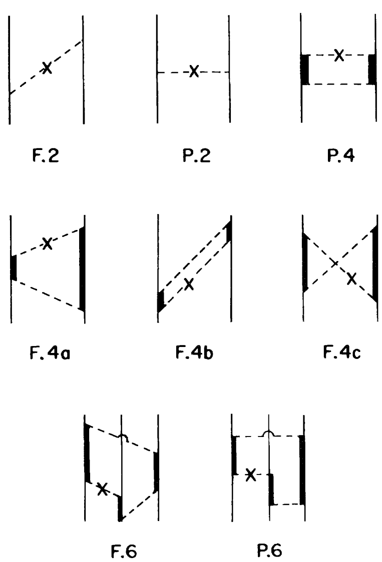

FIG. 1. Diagrammatic representation of contributions to $\langle\delta n^\pi\rangle$. The thin lines are nucleon propagators, while the thick lines can be either nucleon or Δ propagators. The dashed lines in FT (SPA) diagrams are pion propagators (V_{ij}^{π}). The cross in FT diagrams denotes n^π, while the dashed line with a cross in SPA diagrams is the operator δn_{ij}^{π}. Only a few diagrams are shown for the purpose of illustration.

TABLE I. Contributions of selected diagrams in field theory (FT) and SPA to $\langle\delta n^\pi\rangle_A$ in nuclear matter at $k_F=1.33\ \mathrm{fm}^{-1}$, as calculated with the simple model Hamiltonian H_0+H'.

	$\lambda=7\ \mathrm{fm}^{-1}$		$\lambda=4.8\ \mathrm{fm}^{-1}$	
	FT	SPA	FT	SPA
$N=2$	~-0.049	-0.049	~-0.039	-0.039
$N=4$ direct	0.254	0.352	0.100	0.137
$N=6$ rings	0.122	0.144	0.041	0.048
$N=8$ rings	0.080	0.088	0.023	0.025

Realistic nuclear Hamiltonians can be written in the form

$$H=\sum_i\left[-(\hbar^2/2m_i)\nabla_i^{\,2}+m_i-m_N\right]+\sum_{i<j}(V_{ij}^{\;R}+V_{ij}^{\;\pi}), \tag{10}$$

where $V_{ij}^{\;R}$ represents the rest of the interaction (primarily short-range repulsion) between nucleons, and $m_i = m_N$ (m_Δ) when i is in a nucleon (Δ) state. In practice the $\Lambda(k^2)$ and $V_{ij}^{\;R}$ are fitted to the two-nucleon data. In the present work the realistic Argonne National Laboratory v_{28} model[8] of the Hamiltonian (10) is used. The tensor part of $V_{ij}^{\;\pi}$ in this model is consistent with the form factor (9) for $\lambda = 7$ fm^{-1}.

The ground-state wave function is calculated exactly for the deuteron, and by the variational method[9] for nuclear matter. The variational wave functions include Δ components generated by correlation operators[10] containing transition spins and isospins $\vec{S}$ and $\vec{T}$. Techniques for calculating expectation values of two-body operators such as $\delta n_{ij}^{\;\pi}$ are discussed in Refs. 9 and 10.

The $\langle\delta n^\pi\rangle$ calculated in SPA with the full Hamiltonian (10) is 0.18/nucleon in nuclear matter at $k_F = 1.33$ fm^{-1}. This value is much less than the perturbative estimates obtained for the model Hamiltonian in which $V_{ij}^{\;R}$ is neglected (Table I, $\lambda = 7$ fm^{-1} values). The short-range correlations induced by $V_{ij}^{\;R}$ reduce $\langle\delta n^\pi\rangle$ by a large amount, much greater than the uncertainty introduced by using the SPA. The main advantages of the SPA are that (i) models of $V_{ij}^{\;R}$ and $\Lambda(k^2)$ consistent with the two-nucleon data are available, and (ii) the many-body calculations can be done nonperturbatively. The SPA is more accurate for calculating energies than pion excess; the diagrams included in Table I give 31.8 MeV (97.2 MeV) in field theory and 33.0 MeV (113.6 MeV) in SPA for $\lambda = 4.8$ fm^{-1} (7.0 fm^{-1}). It is also a reasonable approximation for calculating the scattering of slow nucleons.[11]

Our results for the pion excess and the momentum distribution of the excess pions $\langle\delta n^\pi(k)\rangle$ are given in Table II and Fig. 2, respectively. The Δ fraction, i.e., the expectation value $\langle n^\Delta\rangle/A$ is also given in Table II. We note that $\langle\delta n^\pi(k)\rangle$ is negative at small k, because of the Pauli blocking of self-energy processes, and has a large peak at $k\sim 2$ fm^{-1}, which is mostly due to tensor contributions through the $N=4$ diagrams. The $\pi N\Delta$ coupling gives the dominant contribution in nuclear matter. When Δ states are neglected, $\langle\delta n^\pi\rangle/A$ at $k_F = 1.33$ fm^{-1} is only 0.04, because of a cancellation between the $N=2$ Pauli blocking term of -0.05 and higher-order terms that give $+0.09$. By contrast, in the deuteron the Δ states give only $\frac{1}{3}$ of the calculated $\langle\delta n^\pi\rangle$.

The results reported in Table II for ^{27}Al, ^{56}Fe, and ^{208}Pb nuclei are obtained in the local density approximation using nuclear matter results from $k_F = 0.93$ to 1.43 fm^{-1}. The fact that these nuclei have unequal numbers of neutrons and protons is ignored. The neutron-proton asymmetry effects are proportional to $|(N-Z)/A|^2$, and are thus negligible in the present context.

For ^{3}He and ^{4}He we have used the three- and four-body wave functions calculated[12,13] with a Hamiltonian containing the Argonne National Laboratory v_{14} two-nucleon potential[8] and the Uni-

TABLE II. Pion excess and Δ fraction in nuclear matter (NM) and nuclei.

	$\langle\delta n^\pi\rangle/A$	$\langle n^\Delta\rangle/A$
NM, $k_F = 0.93$	0.08	0.03
NM, $k_F = 1.13$	0.12	0.04
NM, $k_F = 1.33$	0.18	0.06
^{2}H	0.024	0.005
^{3}He	0.05	0.02
^{4}He	0.09	0.04
^{27}Al	0.11	0.04
^{56}Fe	0.12	0.04
^{208}Pb	0.14	0.05

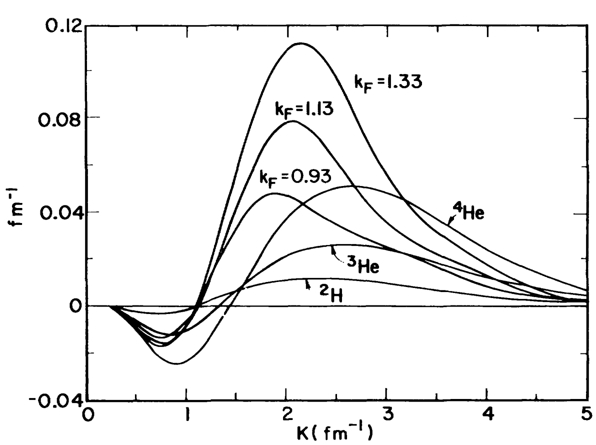

FIG. 2. The calculated values of $k^2\langle\delta n^\pi(k)\rangle/2\pi^2A$ are shown for various systems. $\langle\delta n^\pi\rangle/A$ is the integral over k of this quantity.

versity of Illinois–Urbana model-V three-nucleon interaction.[12] This Hamiltonian does not contain explicit Δ degrees of freedom; to calculate $\langle \delta n^\pi \rangle$ the Δ components in the wave function of these nuclei are estimated with the closure approximation discussed in Ref. 8. This approximation gives the same $\langle \delta n^\pi \rangle$ and $\langle n^\Delta \rangle$ in the deuteron as obtained with the Argonne v_{28} interaction, while in nuclear matter at $k_F = 1.33\ \mathrm{fm}^{-1}$ it gives slightly larger numbers: $\langle \delta n^\pi \rangle / A = 0.21$ and $\langle n^\Delta \rangle / A = 0.07$.

In summary, we have estimated the number of extra pions present in nuclei due to the pion-exchange forces that provide binding. The $\pi N \Delta$ coupling and the short-range repulsion of the NN interaction play important roles in this problem. The SPA may overestimate the pion excess by ~30%, but on the other hand we have neglected contributions from baryon resonances other than the Δ, which would increase $\langle \delta n^\pi \rangle$. The scattering of photons and leptons from nuclei can take place on the excess pions as well as the nucleons, which may help explain anomalies observed in recent experiments.[4,5,14] The relation between $\delta n^\pi(k)$ in the rest frame of the nucleus and the function $f(y)$ giving the probability that a pion carries the fraction y of the momentum in an infinite momentum frame is discussed in Ref. 14.

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