Long-Range Behavior of Nuclear Forces as a Manifestation of Supersymmetry in Nature

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It is shown that each isospin channel in the long-range approximation to the nucleon-nucleon potential (one-pion-exchange potential) corresponds to a realization of a quantum mechanical supersymmetric Hamiltonian. A functional relation between the coordinate-dependent coefficients of the spin-spin and tensor parts of the interaction predicted by supersymmetry is exactly fulfilled.

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Since its discovery,[1] supersymmetry has attracted a great deal of attention because, among other useful properties, it could ultimately provide a natural mechanism for unifying gravity with the strong, electromagnetic, and weak interactions. This new kind of symmetry joins bosons and fermions in irreducible multiplets of the graded Poincaré group. If supersymmetry has something to do with nature, it must be broken, because bosons and fermions with the same mass are not observed. Recently, a lot of effort has been spent in building realistic models of strong, electromagnetic, and weak interactions which incorporate supersymmetry.[2] The preferred way of breaking such symmetry is via the spontaneous symmetry breaking which does not spoil the high-energy behavior which is usually improved for supersymmetric theories. The predictions of almost all contending models remain to be tested experimentally and some of them are at the energy level of currently existing high-energy accelerators. Nevertheless, we might have already observed supersymmetry in nature through some applications of broken supersymmetry in nuclear physics which have predicted, and confirmed, unexpected correlations among energy levels corresponding to even-even and even-odd nuclei.[3]

At the level of a Lagrangian which describes a physical system, supersymmetry manifests itself as a set of transformation rules which mix bosonic and fermionic fields and which leave the action invariant. In other words, the Lagrangian changes at most by a total divergence under such transformations. In this note we consider a supersymmetric field theory where the fields depend only on time, that is, we deal with supersymmetric quantum mechanics. One-dimensional quantum mechanical supersymmetric models were first discussed by Witten[4] and further investigated by Salomonson and van Holten[5] and Cooper and Freedman,[6] mainly with relation to the mechanisms of spontaneous symmetry breaking. Here we discuss an extension to three dimensions of such models which will allow us to interpret the long-range potential between two nucleons as a manifestation of supersymmetry in nature.

The essential idea underlying any quantum mechanical supersymmetric model is that the Hamiltonian of the system can be written as

$$H = \tfrac{1}{2}\{Q, Q^\dagger\} = \tfrac{1}{2}(QQ^\dagger + Q^\dagger Q), \tag{1}$$

where Q and $Q^\dagger$ are basically fermionic (anticommuting) operators which generate the supersymmetry transformations. The expression (1) is a manifestation of the graded extension of the Poincaré group ($H = P^0$) and is the quantum mechanical analog of the fact that supersymmetry transformations are the "square root" of (time) translations.[7]

Let us consider three Hermitian position operators x_k ($k = 1, 2, 3$), together with their canonical momenta p_k, which satisfy the usual commutation relations of quantum mechanics. These variables represent the bosonic degrees of freedom of the model. The fermionic degrees of freedom are provided by non-Hermitian Clifford operators ζ_k and $\zeta_k^\dagger$ which satisfy the algebra ($\hbar = 1$)

$$\{\zeta_k, \zeta_l\} = \{\zeta_k^\dagger, \zeta_l^\dagger\} = 0, \quad \{\zeta_k, \zeta_l^\dagger\} = \delta_{kl}. \tag{2}$$

Also, any bosonic variable commutes with any fermionic variable. We remind the reader that the variables ζ_k appear naturally when quantum mechanics is formulated via the Schwinger action principle. They constitute what Schwinger calls variables of the second kind, and their algebraic properties, together with the properties of their elementary variations $\delta\xi_k$, can be consistently derived from the action principle.[8]

In terms of the dynamical variables defined above we simply extend the one-dimensional construction of the supersymmetry generators (supercharges) to

$$Q=(p_k - iVx_k)\xi_k\,,\qquad Q^\dagger=(p_k+iVx_k)\xi_k^\dagger\,, \tag{3}$$

where we have only added a summation over the new degrees of freedom. The real function V is the so-called superpotential and depends only on $r=(x_kx_k)^{1/2}$. We assume that the indices $k=1$, 2, 3 have vectorial character under three-dimensional rotations in such a way that the supercharges together with the Hamiltonian are totationally invariant. In superspace language the choice (3) for the supersymmetry generators means that we keep the supercoordinates t, θ, θ^* and only attach a vector index k to the superfield.

The algebra of the dynamical variables allows us to show that $Q^2=(Q^\dagger)^2=0$ which together with the definition (1) of the Hamiltonian implies

$$[Q,H]=[Q^\dagger,H]=0\,. \tag{4}$$

In other words, the transformations of the dynamical variables generated by Q and $Q^\dagger$ are symmetries of the system. The infinitesimal supersymmetry transformations among the bosonic and fermionic coordinates are calculated according to the usual prescription $\delta X=i[G,X]$ for the change of any operator X under a transformation generated by G. Here the generator is $G=\epsilon^*Q^\dagger+Q\epsilon$, where ϵ is a complex infinitesimal Grassmann parameter which commutes (anticommutes) with all bosonic (fermionic) degrees of freedom. We do not bother to write explicitly such transformations because they are just the direct generalization of the one-dimensional situation.[4] Calculation of the Hamiltonian according to Eqs. (1) and (3) finally leads to

$$H=\tfrac{1}{2}\{\vec{p}^{\,2}+r^2V^2+V[\zeta_k,\zeta_k^\dagger]+V'(x_kx_l/r)[\zeta_k,\zeta_l^\dagger]\}\,, \tag{5}$$

where $V'=dV(r)/dr$. In order to look for a physical interpretation of the supersymmetric Hamiltonian (5) we need an explicit realization of the algebra (2). To this end it is more convenient to rewrite the Clifford variables ζ_k in terms of Hermitian operators a_k and b_k in such a way that $\zeta_k=\frac{1}{2}\sqrt{2}(a_k+ib_k)$. Such Hermitian operators satisfy the algebra

$$\{a_k,a_l\}=\{b_k,b_l\}=\delta_{kl}\,, \tag{6}$$

$$\{a_k,b_l\}=0\,, \tag{7}$$

which readily imply the relations (2). Equation (6) suggests a realization of the operators $\vec{a}$ and $\vec{b}$ in terms of Pauli spin matrices. The simple assumption that $\vec{a}$ and $\vec{b}$ act on different spaces together with Eq. (7) leads to the following *Ansatz* for the operators:

$$\vec{a}=(1/\sqrt{2})A\otimes\vec{\sigma}^{(1)}\,,\qquad \vec{b}=(1/\sqrt{2})B\otimes\vec{\sigma}^{(2)}\,. \tag{8}$$

Here $\vec{\sigma}^{(1)}$ and $\vec{\sigma}^{(2)}$ are two sets of independent Pauli matrices: $[\sigma_k^{(1)},\sigma_l^{(2)}]=0$. The operators A and B are Hermitian and satisfy

$$A^2=B^2=1\,, \tag{9}$$

$$\{A,B\}=0\,. \tag{10}$$

Condition (9) is imposed in order to satisfy (6) with the representation (8) while condition (10) accounts directly for (7).

Up to this stage we can say that our model Hamiltonian (5) describes the supersymmetric interaction of two spin-$\frac{1}{2}$ particles (with spin operators $\frac{1}{2}\vec{\sigma}^{(1)}$ and $\frac{1}{2}\vec{\sigma}^{(2)}$, respectively) and whose relative coordinates and momenta are represented by the operators $\vec{x}$ and $\vec{p}$, respectively. Also, such particles possess an internal quantum-number space related to the operators A and B. In order to elucidate the meaning of such quantum numbers we now look for a representation of these operators.

Having in mind that we are dealing with a two-particle system, we take the representation

$$A=\rho_1^{(1)}\otimes\rho_3^{(2)}\,,\qquad B=\rho_2^{(1)}\otimes I^{(2)}\,, \tag{11}$$

where $\vec{\rho}^{(1)}$ and $\vec{\rho}^{(2)}$ are another two sets of independent Pauli matrices which operate in the internal space of each particle, respectively. The realization (11) obviously satisfies the requirements (9) and (10) together with the Hermiticity condition. We are now saying that each particle has an extra internal quantum number $C^{(a)}$ ($a=1$, 2), which arises from the internal operators and which we choose to be the eigenvalues ± 1 of $\rho_3^{(a)}$.

In terms of the explicit realization that we have found for the Clifford algebra (2), the Hamiltonian (5) can finally be written as

$$H=\tfrac{1}{2}\{\vec{p}^{\,2}+r^2V^2+C[V\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)}+rV'(\vec{\sigma}^{(1)}\cdot\hat{r})(\vec{\sigma}^{(2)}\cdot\hat{r})]\}. \tag{12}$$

Here we have dropped the direct product notation and $\hat{r}=\vec{r}/r$. The operator C is the only remnant left from the internal space. It is given by $C=-iAB=\rho_3^{(1)}\otimes\rho_3^{(2)}$ and can be effectively written as the product of the internal charges: $C=C^{(1)}C^{(2)}$. Let us remark that the Hamiltonian (12) exhibits central, spin-spin, and tensor interactions with the respective coefficients highly correlated in terms of the superpotential $V(r)$ as a consequence of the underlying supersymmetry of the model.

A complete set of quantum numbers for this two-particle system is given by the energy, the total angular momentum squared $\vec{J}^2$, the z component of the total angular momentum J_z, the total spin squared $S=0,1$, the parity $P=(-1)^l$, and the product of the individual internal charges $C=C^{(1)}C^{(2)}$. A direct calculation shows that the supersymmetry generators Q and $Q^\dagger$ commute with $\vec{J}$ (hence in particular with $\vec{J}^2$ and J_z), which is just the consequence of the supercharges being invariant under rotations. It is easy to verify also that the action of Q and $Q^\dagger$ on the two-particle wave function has the effect of changing $P\to-P$ and $C\to-C$. The action of the supersymmetry generators on the total spin of the system is a bit more subtle: Either they annihilate the wave function or they act as shifting operators that change the total spin of the system by a step of one unit. Further details of the action of the operators Q and $Q^\dagger$ upon the wave functions of the system can be found in Hernández.[9]

Now we come to the main point of this work, which is to show that the long-range nucleon-nucleon potential in nuclear forces is just a particular realization of the supersymmetric Hamiltonian (12). The nucleon-nucleon potential is fairly complicated at short and medium distance ($r<2$ fm) because of many-particle effects together with the fact that many different types of mesons and vector bosons are exchanged between the nucleons. Nevertheless, the long-range behavior of the potential ($r>3$ fm) is due only to pion exchange and has the well established form[10]

$$V_\pi(r)=\pm(\vec{\zeta}^{(1)}\cdot\vec{\zeta}^{(2)})\{V_S(r)\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)}+[3(\vec{\sigma}^{(1)}\cdot\hat{r})(\vec{\sigma}^{(2)}\cdot\hat{r})-\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)}]V_T(r)\}, \tag{13}$$

where $\vec{\zeta}^{(1)}$ and $\vec{\zeta}^{(2)}$ refer to each nucleon isospin. The functions V_S and V_T are given by ($\hbar=c=1$)

$$V_S=\frac{\mu}{3}\left(\frac{g^2}{4\pi}\right)\frac{e^{-x}}{x},$$
$$V_T=\frac{\mu}{3}\left(\frac{g^2}{4\pi}\right)\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x}. \tag{14}$$

Here $x=\mu r$, μ is the pion mass, and g is the pion-nucleon coupling constant. In Eq. (13) the plus sign in front of the right-hand side refers to the potential appropriate to nucleon-nucleon (or antinucleon-antinucleon) interactions, while the minus sign describes the potential for nucleon-antinucleon interactions.[11]

Now we turn to the comparison of the one-pion exchange potential (OPEP) given in Eq. (13) with the corresponding potential arising from the supersymmetric Hamiltonian (12). In the first place we notice that the OPEP really requires two different internal spaces; one related to the leptonic quantum number and the other corresponding to the isospin quantum number. The multiplicative structure of the operator C appearing in (12) leads us to the identification of each charge $C^{(a)}$ with the leptonic number of each nucleon. In its present form our supersymmetric model does not include isospin in a natural way, and any further comparison between the two potentials (12) and (13) will be made for each individual isospin channel: $\zeta\equiv(\zeta^{(1)}\cdot\zeta^{(2)})=-3,1$.

The next step is to identify the coefficients of $\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)}$ and $(\vec{\sigma}^{(1)}\cdot\hat{r})(\vec{\sigma}^{(2)}\cdot\hat{r})$ defined in (12) in terms of the explicit functions (14) and to verify whether or not the relations imposed by supersymmetry are satisfied. A direct comparison shows that

$$\tfrac{1}{2}V=\zeta(V_S-V_T), \tag{15}$$

$$r\,d(\tfrac{1}{2}V)/dr=3\zeta V_T, \tag{16}$$

and the real challenge is to determine whether the last relation, implied by supersymmetry, is true. In terms of the definition (15), relation (16) can be written as

$$r\,d(V_S-V_T)/dr=3V_T. \tag{17}$$

A direct calculation which uses the explicit expressions (14) shows that the equality (17) is indeed satisfied exactly, which comes out as a rather remarkable result.

When comparing the potentials in Eqs. (12) and (13) we are still left with the central term $\frac{1}{2}r^2 V^2$ which is not present in the OPEP. Nevertheless, the choice (15) for the superpotential makes the central term proportional to e^{-2x} which is certainly negligible when compared with the other terms in the potential which go as e^{-x} in the long-range approximation. In this sense, the term $\frac{1}{2}r^2 V^2$ represents the onset of two-pion exchange processes. In the approach presented here supersymmetry is broken, as can be seen because the full nucleon-nucleon potential can bind the deuteron and then cannot be positive definite as implied by (1).

It would be interesting to understand the possible implications of the properties exhibitied here from the field theoretical viewpoint because, if the prediction of relation (17) is not pure coincidence, this equality is saying that supersymmetry is a good symmetry at very low energies, which is not what we expect from the standard approach to the subject.

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