Topological Soliton Bag Model for Baryons

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Furnishing Skyrme's chiral soliton with a quark-bag core appears to generate a qualitative advance in the phenomenology of nucleon structure.

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More than twenty years ago Skyrme[1] suggested a picture of the nucleon as a soliton in the otherwise uniform vacuum configuration of the nonlinear sigma model. Since the soliton "twist" is quantized, he suggested identifying this with baryon number B. Later investigations, including some very recent work,[2-5] have confirmed this definition of B, and even the half-integer spin of the soliton. Other pleasing aspects of this picture include a strong short-range repulsion between nucleons, as indicated by a large increase in energy per baryon for the spherical soliton with $B=2$, and the fact that an underlying chiral symmetry, implied by many phenomena of strong interactions at low energy, is automatically incorporated.

In the past decade, a very different view of the nucleon has received both empirical and theoretical support—the Massachusetts Institute of Technology (MIT) bag model[6] and chiral bag models.[7-10] Here, the baryon number is lodged in three nearly massless quarks, confined in a bubble of abnormal vacuum ($\sigma=0$, or Wigner mode) floating in a chiral vacuum ($\sigma=$const, or Goldstone mode). For some time there have been discussions about how the chiral symmetry, unbroken within the bag, affects the boundary condition on quark wave functions at the bag surface.[7-10] Initial efforts were based on linearized versions of chiral symmetry and amounted to a lowest-order contribution from the pion field.[9-11]

One nonperturbative "hedgehog" solution,[12] apparently a soliton, seemed to be consistent with the qualitative baryon structure obtained in the large-N_c (N_c is the number of colors) limit of quantum chromodynamics (QCD).[13] However, B (like color) was supposed to be confined within the bag.

We wish now to propose a hybrid model, in which the nonlinear, nonperturbative chiral-field degrees of freedom are treated exactly (but classically) outside the bag and, as usual, quark degrees of freedom are recognized explicitly inside. We find that the axial-vector coupling constant g_A comes into good agreement with experiment. Skyrme's result that nucleons have short-range repulsion is maintained as a nonperturbative effect, even taking account of quark degrees of freedom. The bag may be small, but baryon number leaks out into the chiral field region, so that both charge and baryon radii are bigger than the bag radius.

Let us begin with the pure soliton picture, and imagine describing the nucleon with an effective Lagrangian obtained by integrating out quark and gluon fields in favor of effective fields. Such a theory corresponds to the standard current-algebra Lagrangian, namely the nonlinear σ model. In terms of the quaternion $U(x)=F_\pi^{-1}\times[\sigma(x)+i\vec{\tau}\cdot\vec{\pi}(x)]$, where σ is a scalar field and $\vec{\pi}$ is a triplet of pseudoscalar fields, the usual nonlinear σ-model Lagrangian density is

$$\mathcal{L}_2=-\tfrac{1}{4}F_\pi^{\,2}\,\mathrm{Tr}(R_\mu R_\mu), \tag{1}$$

with $R_\mu=(\partial_\mu U)U^\dagger$ and $U^\dagger U=1$. It is known that $\mathcal{L}_2$ cannot support a stable soliton for space dimension $D=3$, but if one adds a term

$$\mathcal{L}_4=-\tfrac{1}{4}\epsilon^2\,\mathrm{Tr}\{[R_\mu,R_\nu]\}^2, \tag{2}$$

then there are solitons.[1,3,4] The term $\mathcal{L}_4$ also insures that $\mathcal{L}_2$ is applied only for wavelengths above some critical value. The resulting dynamics corresponds to a vacuum with constant U, e.g., $U=-1$, and thus spontaneous breaking of chiral symmetry resulting in a triplet of massless Goldstone bosons, the pseudoscalar isovector pion. $\mathcal{L}=\mathcal{L}_2+\mathcal{L}_4+\dots$ is an effective Lagrangian of SU(N_c) quantum chromodynamics valid in the large-N_c limit, with $F_\pi^{\,2}\propto N_c$.

The baryon current postulated by Skyrme and confirmed by Witten[5] is

$$B_\mu\equiv(24\pi^2)^{-1}\epsilon_{\mu\nu\lambda\rho}\,\mathrm{Tr}\{R_\nu R_\lambda R_\rho\} \tag{3}$$

with

$$B=\int B_0(x)d^3x\,. \tag{4}$$

This B is the winding number of the third homotopy group $\pi_3(\mathrm{SU}(2))\approx\pi_3(S^3)=Z$, the additive group of integers.

Take the hedgehog *Ansatz* for the soliton field[1,3-5,12]

$$U(r)e^{i\vec{\tau}\cdot\hat{r}\,\theta(r)} \tag{5}$$

with θ tending to $n\pi$ as $|\vec{r}|\to\infty$ or 0, as required for finite energy. Choose $\theta(r)\to\pi$ as $r\to\infty$, $\theta(r)\to 0$ as $r\to 0$. Equation (5) implies

$$B=\pi^{-1}[\,\theta(\infty)-\theta(0)-\tfrac{1}{2}\{\sin 2\theta(\infty)-\sin 2\theta(0)\}\,], \tag{6}$$

giving $B=1$. Topology has produced a fermion out of Bose fields, a miracle predicted and verified by several workers.[1-5] When quantized, the soliton exhibits correct spin and isospin properties.[2,5]

Imagine inserting into the soliton configuration a bubble or bag of radius R within which quarks may propagate freely. Since by assumption[12] the chiral field U cannot penetrate into the bag, one might expect the defect to alter the topological structure: The bubble with three valence quarks carries unit baryonic charge in the MIT bag model, which has constant $\theta=\pi$ outside. What happens to the baryonic charge distribution as $\theta(R)$ departs from the asymptotic value π?

Since at this stage we neglect interactions among quarks and gluons, the only parameter which could influence the baryon number contained in the bag is the boundary condition on the Dirac wave functions of the massless quarks at $r=R$. The external soliton field is rotationally symmetric under the action of the generator

$$\vec{K}=\vec{J}+\vec{T}, \tag{7}$$

where $\vec{J}$ is ordinary angular momentum and $\vec{T}$ is isospin. Thus we would like our boundary condition to commute with $\vec{K}$. In the linearized chiral bag model, the natural choice is

$$\psi\propto(\cos\tfrac{1}{2}\theta_c-i\gamma_5\vec{\tau}\cdot\hat{r}\sin\tfrac{1}{2}\theta_c)\psi_M\,, \tag{8}$$

$$\vec{\alpha}\cdot\hat{r}\,\psi_M=i\beta\psi_M\,. \tag{9}$$

Boundary conditions of this precise sort have been studied by Yamagishi[14] and by Grossman[15] for the lowest-partial-wave Dirac electrons interacting with a point magnetic monopole. Their formalism may be translated to our case, if we take care to note several technical distinctions. Their condition is applied at $r=0$ to a wave function defined for all r. Ours is at $r=R$ for a function whose behavior at $r=0$ is determined entirely by kinematics. Unlike them, we must examine the effect on all partial waves. Finally, we have an extra degree of freedom, the quark isospin, with associated Pauli matrices $\vec{\tau}$.

The principle which determines the dependence of internal B on θ_c is that $B(r<R)$ for the negative-energy sea changes by a finite amount as θ_c varies. For any fixed Fermi momentum k_F, $\Delta B=B(\theta)-B(0)$ (summed over quark colors) is an integer, but the average value of this difference approaches a constant value as $k_F\to\infty$,

$$\Delta B=\vec{\sigma}\cdot\hat{r}\,\vec{\tau}\cdot\hat{r}\,\theta_c/2\pi\,. \tag{10}$$

For $K=0$ there are two parities, both with $S\equiv\vec{\sigma}\cdot\hat{r}\,\vec{\tau}\cdot\hat{r}=-1$, yielding a total

$$\Delta B_{\rm tot}=-\theta_c/\pi\,. \tag{11}$$

For higher K, states with $S=\pm1$ pair off, so that they make no net contribution to ΔB. This might have been expected to result from the spherical bag boundary conditions, since the higher partial waves would automatically have vanishing radial baryon currents when averaged over all angles.

Comparing (11) with (6), with $\theta(0)$ replaced by $\theta(R)$, we find that if $B(r<R)+B(r>R)$ is to be conserved as $\theta(R)$ varies, then we must fix

$$\theta_c=\pi-\theta(R)+\tfrac{1}{2}\sin 2\theta(R)\,. \tag{12}$$

The nonlinear relation between θ_c and $\theta(R)$ shows that the boundary condition (8) is too rigid to preserve chiral symmetry. A better calculation yet to be done is to make the quark mass outside the bag large but finite, $M=M_0\exp(i\vec{\tau}\cdot\hat{r}\gamma_5\theta)$. Meanwhile, (12) should be a qualitative indicator, exact for $\theta_c=\pi/2$.

For Yamagishi's case,[14] the total energy of the negative-energy sea includes a θ_c-dependent term diverging logarithmically with k_F. For us, the divergent part cancels between the positive and negative parity channels, letting $\theta(r)$ be a well-behaved dynamical variable.

With θ_c tied to $\theta(R)$ by B conservation, our hybrid object is still a soliton. What does the fractional $B(r<R)$ signify? Nothing more nor less than the fractional electron number carried by either half of a hydrogen molecular ion.[16] In both cases, the number distribution is an expected-value or probability distribution. Nevertheless, conservation of B as an operator implies conservation of its expectation value, justifying our link between θ and θ_c. In other words, for an eigenstate of total B, the sum of the expectation values of B in all different regions must be equal to the eigenvalue.

The Skyrme picture[1] gives, as an estimate of the repulsion between nucleons, the difference between the mass of a $B=2$ soliton and twice the mass of a nucleon ($B=1$ soliton). As confirmed by a recent study of Jackson and Rho,[17] this difference is itself one nucleon mass. We now have a second way to make $B=2$, putting six valence quarks into the bag. If θ_c has about the value suggested by a previous investigation,[12] $\theta_c = \frac{1}{2}\pi$, then the first three quarks fill zero-energy levels, each therefore several hundred megaelectronvolts lower than they would have been for $\theta_c = 0$, while the second three are several hundred megaelectronvolts higher. Once again, the result is an energy cost of at least one nucleon mass.[18,19] For smaller θ_c (larger R) the energy will be smaller.

Of course, in a fully quantized theory the $B=2$ system would be a superposition of many different configurations, but it is likely that the repulsion would persist, since the phenomenon considered here is a leading $O(N_c)$ effect in the large-N_c expansion of QCD. This is satisfying in terms of phenomenology derived from nucleon-nucleon scattering, and also seems better than the (perturbative) color-magnetic hyperfine interaction, an $O(1/N_c)$ effect which was used to account for short-range repulsion in the case $\theta_c = 0$.[20] This latter effect may reinforce the $O(N_c)$ repulsion.

The "magic angle" $\theta(R) = \frac{1}{2}\pi$ is particularly interesting for several reasons. The nonlinear term in Eq. (12) vanishes, so that $\theta_c = \theta(R)$. The quark frequency ω for the $K=0$ mode vanishes as $\tan\theta_c(R) = (1-y^2)/2y$, with $y = j_1(\omega R)/j_0(\omega R) \approx \frac{1}{3}\omega R$. There is thus a zero mode, and the baryon charge is shared equally between the bag interior and the soliton sector. The situation is quite analogous to that considered by Jackiw and Rebbi[12] and Goldstone and Wilczek[22] in condensed matter and field theories. Furthermore, the lower component of the quark wave function vanishes, yielding formulas of the nonrelativistic quark model, as noted in Ref. 12, but with quark normalization changed by θ_c.

As an illustration of this change, consider the axial-vector coupling constant $g_A \approx 1.25$. For $\theta(R) = \frac{1}{2}\pi$ the quark wave function is constant within the bag, and the axial charge for N_c quarks in the $K=0$ orbit is directly related to the net baryon charge inside the bag. We find for this configuration

$$g_A{}^H \approx -\tfrac{3}{4} N_c = -\tfrac{9}{4} \,. \tag{13}$$

To obtain g_A for the neutron β decay, projections in angular momentum and isospin must be made. The required operation,[12] which amounts to "first quantizing" the hybrid theory, multiplies $g_A{}^H$ by $-\frac{5}{9}$,

$$g_A \approx -\tfrac{5}{9}\, g_A{}^H = \tfrac{5}{4} \,. \tag{14}$$

This is exactly half of what was found in Ref. 12, the crucial factor coming from the leakage of baryon charge. The result (14), however, is not exact, as the projection procedure used in Ref. 12 is strictly valid just for large bag radii, and contributions from the quartic term are ignored. Values of g_A have not yet been computed for intermediate θ_c, but presumably they change smoothly to the value $g_A = 1.635$ calculated by Jaffe[9] for large bag radii.

The chiral soliton with quark-bag core represents a satisfying advance and a significant challenge for theory. It gives a starting point or "background field" which incorporates both an underlying chiral invariance and its spontaneous breakdown. There is a precise connection between long-wavelength soft-pion physics and short-wavelength QCD with its beautiful property of asymptotic freedom. All the apparatus of bag models may be applied to the quark and gluon degrees of freedom inside the bag, while chiral dynamics govern outside. Consequently, baryon number, but not color, leaks out of the bag. Nucleon-nucleon repulsion is understood as a nonperturbative effect, while earlier bag-model results on mass splittings, magnetic moments, etc., should more or less be preserved with modifications contributed by the chiral fields.

The challenges posed are numerous. While the hybrid picture is more realistic, it is more intricate and thus harder to quantize. As illustrated above for g_A, it is clearly important to work out the relation between states of definite K and states of definite angular momentum and isospin. For the chiral-field contribution, this requires quantization. Even without quantization, already at the leading order in N_c, baryon-baryon interactions are complex and nonlinear. Thus, for example, the possibility of a bound $\Lambda\Lambda$ system stable against dipion emission should be reexamined.

In summary, we believe that the soliton-bag hybrid nucleon fits together so much of what is known that it affords a good foundation for further studies of nucleon structure and interactions.

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Note added.—The standard link of $\theta(R)$ to the quark boundary condition *does* yield shifts of baryon number inside and outside R which balance exactly, once a nonvanishing ΔB from $K > 0$ is included.[23] Therefore, baryon conservation is automatic in the hybrid model.

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