Superfluidity in a Dilute Bose Gas

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Superfluid ^{4}He films adsorbed on porous Vycor glass have been studied in the low-density limit where the superfluid interparticle spacing is more than an order of magnitude greater than the atomic hard-core diameter. For the lowest transition temperatures observed, the influence of critical fluctuations near T_c is much reduced and a range of crossover to behavior more characteristic of the dilute Bose gas is observed.

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Stimulated by recent theoretical and experimental advances in the study of low-density, spin-polarized hydrogen,[1] there has been renewed interest in the phenomenon of Bose condensation and the possibility of observing superfluidity in a dilute weakly interacting Bose gas. In this Letter we report new results from a different approach to the experimental realization of a dilute Bose gas, one that utilizes a low-density ^{4}He system.

Our approach has been to make systematic observations of the temperature dependence of the superfluid mass of low-density ^{4}He films adsorbed on porous Vycor glass. Earlier work on the ^{4}He-Vycor system[2-4] has demonstrated superfluidity for a range of transition temperatures varying over nearly two orders of magnitude. For the lowest transition temperatures previously observed,[3] the effective superfluid coverage was only a few percent of a monolayer, and the average spacing between mobile superfluid particles was considerably greater than the atomic scattering length. We wished to test the proposition that, if the superfluid coverage were sufficiently reduced, superfluid behavior characteristic of the dilute Bose gas might be observed. The data that we report in this Letter give, for the first time, a definite indication that a crossover to a regime of dilute Bose gas behavior has been seen.

The dense three-dimensional (3D) superfluids, bulk ^{4}He, liquid ^{3}He-^{4}He mixtures, and higher coverage superfluid films adsorbed on Vycor, all show superfluid behavior that is strongly influenced by order-parameter fluctuations. The temperature variation for the superfluid mass obeys the well known "two-thirds" power law in the neighborhood of the transition temperature. In contrast, fluctuations are usually assumed to be unimportant in the case of the dilute, weakly interacting Bose gas. For the dilute gas, the superfluid mass is expected to follow a temperature dependence close to that of the condensate density in the ideal Bose gas and thus to vary linearly with $T_c - T$ as the transition temperature, T_c, is approached.

In the present experiment we have determined the temperature dependence of the superfluid mass for an extended range of ^{4}He coverages adsorbed on Vycor glass. As the density of the superfluid was systematically reduced, we have looked for indications of a crossover from the "two-thirds" power-law behavior seen for the dense superfluids to the more linear temperature variation of the superfluid mass expected for the dilute, weakly interacting Bose gas.

The experimental technique employed in the present work is similar to that reported earlier.[2] The ^{4}He sample is adsorbed on the large surface area, estimated at 156 m^2, of a cylinder of porous Vycor glass with a total volume of 1.12 cm^3. The interior channels of the porous glass form a highly interconnected, 3D network. The pore sizes within the Vycor are estimated to range from 40 to 80 Å for the present sample. The Vycor cylinder and the adsorbed ^{4}He sample comprise the inertial element of a high-Q (greater than 10^6 at low temperatures) torsional oscillator. When the oscillator is cooled below the superfluid transition temperature a change in the period of oscillation is observed. This period change, ΔP, provides a signal proportional to the superfluid moment of inertia.

In Fig. 1 we show the superfluid signal, ΔP, as a function of temperature for several different coverages of adsorbed helium. As observed in our earlier measurements,[2-4] the magnitude of the zero temperature superfluid signal and the temperature at which the superfluid transition occurs are both reduced as the total coverage is lowered. The data shown in Fig. 1 display transition temperatures ranging from about 76 mK down to near 5 mK.

In addition to the data shown in Fig. 1, we have also obtained data for a range of superfluid cov-

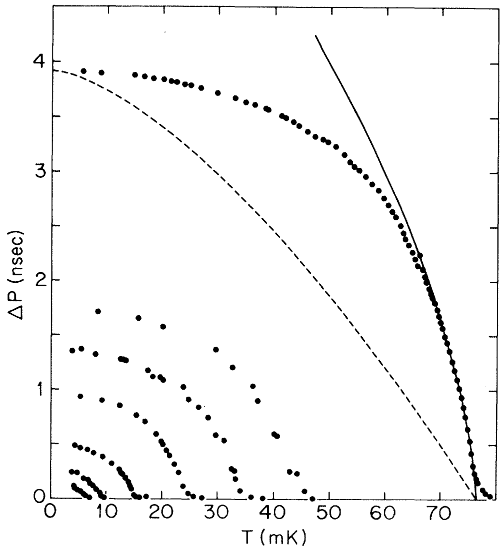

FIG. 1. The measured period shifts, $\Delta P(T)$, proportional to the superfluid mass, are shown as a function of temperature for several different film coverages. The solid line illustrates a "two-thirds" power-law fit to the highest coverage data set, while the dashed line gives the temperature dependence of the condensate density in arbitrary units for an ideal Bose gas with the same T_c as the power-law fit.

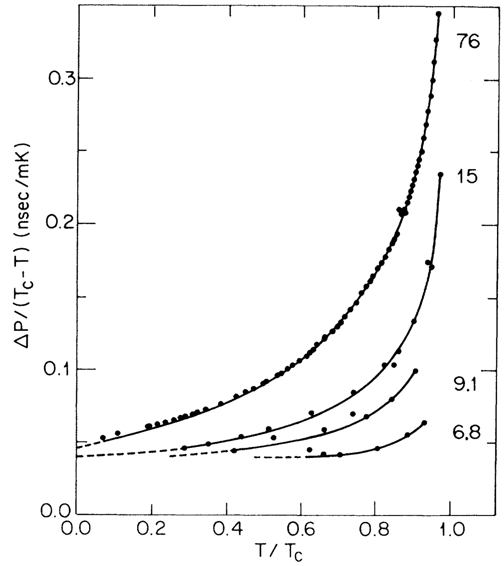

FIG. 2. The period shifts, $\Delta P(T)$, normalized by $(T_c - T)$ are shown as a function of the reduced temperature, T/T_c, for several representative film coverages. The lines are guides for the eye and the T_c for each coverage is indicated in millikelvins.

erages extending to a complete filling of the Vycor pores. The data set in Fig. 1 with a transition temperature near 76 mK is typical of the higher coverage data. Excluding the small rounding in the data above T_c, the temperature variation of the superfluid signal, ΔP, can be fitted with a power-law expression of the form $(T_c - T)^{-\zeta}$. The solid curve shown in Fig. 1 illustrates such a fit to the 76 mk data where $\zeta = 0.63$ gives the best fit. This nearly "two-thirds" power-law behavior for the superfluid mass in the neighborhood of the transition contrasts sharply with the temperature dependence expected for the dilute Bose gas, which is indicated in the figure by the dashed line.

In order to examine our lower coverage data for a possible systematic shift toward the linear temperature dependence expected in the dilute gas case, we have plotted, in Fig. 2, the quantity $\Delta P/(T_c - T)$ as a function of the reduced temperature, T/T_c. Dividing the superfluid signal, ΔP, by $T_c - T$ serves both to normalize the magnitudes of the superfluid signals near zero temperature and to emphasize the contrast between the "two-thirds" power-law behavior of a strongly fluctuating superfluid near T_c and the linear behavior expected for the dilute Bose gas. Thus, for the fluctuating superfluid, the quantity $\Delta P/(T_c - T)$ will diverge approximately as $(T_c - T)^{-1/3}$ as T_c is approached, whereas in the case of the weakly interacting dilute Bose gas the same quantity will vary only slowly as the transition temperature is approached.

The data displayed in Fig. 2 show a clear trend as a function of transition temperature. The data set with the highest transition temperature (76 mK) diverges strongly as the transition temperature is approached. The strength of this divergence diminishes systematically as the superfluid coverage and transition temperature are lowered, until for the lowest coverage film shown, the region of divergence is hardly visible.

We believe these results strongly suggest that a crossover from a regime of a strongly interacting superfluid to one of a dilute-Bose-gas-like behavior is being observed. Although we know of no rigorous quantitative discussion of this crossover behavior for the interacting Bose gas, it can be argued along "Ginzburg criterion" lines that a sufficiently dilute Bose gas will display mean-field behavior where the fluctuation-dominated critical region will be negligibly small.[5,6]

In the standard discussion of the dilute Bose gas, the criterion for diluteness is expressed by the requirement that the mean interparticle spacing be large compared with the scattering length or atomic hard-core diameter. In order to obtain an estimate for the average superfluid interparticle spacing in the present experiment, we will require a model for the structure of the ^{4}He adsorbed within the Vycor.

As has been pointed out previously in the literature,[4] there exists considerable evidence from a number of different types of measurements that an initial coverage of localized or solid helium atoms forms on the substrate and underlies any superfluid layers. Once this localized layer is complete, additional coverage results in the formation of a class of mobile atoms, which can form a superfluid at sufficiently low temperature. In the further analysis of our data we shall assume such a model of a low-density surface gas moving independently of a substrate of localized ^{4}He atoms.

We have obtained data similar to those shown in Fig. 1 for a wide range of adsorbed helium coverages extending to a complete filling of the Vycor pores. In Fig. 3 we have plotted the value for $\Delta P(0)$ obtained by extrapolation of $\Delta P(T)$ vs T data to zero temperature as a function of the mass of helium adsorbed within the Vycor sample. It is clear in this figure that an initial preplating of about 17 mg is required before superfluidity can be observed. For the surface area of the present sample we estimate this initial coverage to be equivalent to 1.5 atomic layers.

At the lowest superfluid coverages, a small curvature is seen in the $\Delta P(0)$ vs coverage plot. We believe that this results from a small growth of the "solid" or localized layer resulting from increased pressure as additional helium is added to the substrate. For most of the range of superfluid coverage, however, the zero temperature superfluid signal increases in a remarkably linear fashion with increasing mass of adsorbed helium. The slope of this linear region is useful in providing a calibration of the sensitivity of our apparatus. We find that a change in the period of one nanosecond corresponds to a superfluid mass of 1×10^{-4} gm cm^{-3}. With our present signal resolution we should be capable of resolving superfluid densities of 10^{17} atoms/cm^3 in our Vycor sample.

We make use of the sensitivity calibration mentioned above and the estimated surface area of the Vycor substrate to obtain a value for the average superfluid interparticle spacing, d_s. For the range of coverages showing crossover behavior the estimated values for d_s are more than an order of magnitude larger than the atomic hard-core diameter.

In Fig. 4 we make a comparison of d_s with the free-particle thermal wavelength, $\lambda = (2\pi\hbar^2/mk_BT)^{1/2}$, at the superfluid transition. On general grounds, degenerate quantum behavior is ex-

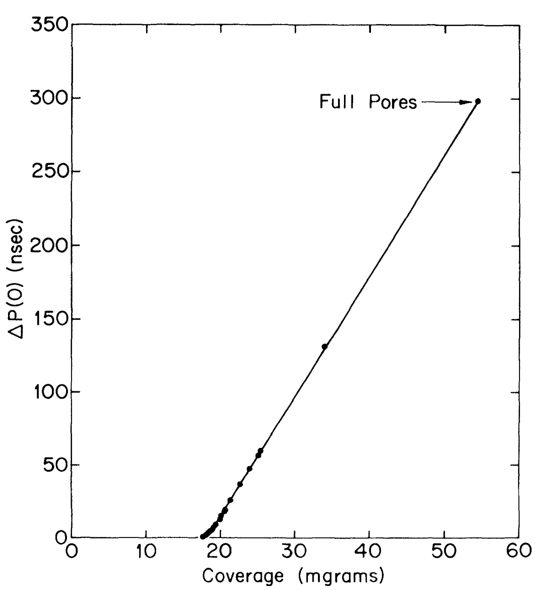

FIG. 3. The zero temperature period shift, $\Delta P(0)$, is shown as a function of the adsorbed helium mass.

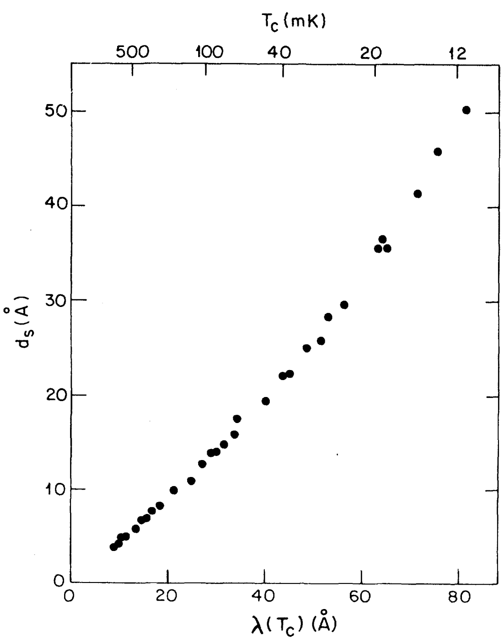

FIG. 4. The estimated superfluid interparticle spacing, d_s, is plotted vs the thermal wavelength, $\lambda(T_c)$, for each film coverage.

pected to appear when the particle spacing is on the order of the thermal wavelength. For instance, in the case of the ideal 3D Bose gas, Bose condensation occurs when $\lambda = 1.38d$, where d is the interparticle spacing. Similarly, a roughly proportional relation seems to hold in Fig. 4 between the interparticle spacing d_s, and $\lambda(T_c)$ for the range of transition temperatures where the thermal wavelength is less than the pore size (about 40 Å). For lower T_c's where $\lambda(T_c)$ becomes greater than the pore size, increasing deviations from a linear relation are seen. This is not surprising, since for larger values of the interparticle spacing and thermal wavelength, the 3D connectivity of the substrate becomes increasingly important. At the lowest superfluid transition temperatures we have observed, the thermal wavelength at T_c is in excess of 100 Å and the superfluid interparticle spacing is estimated to be on the order of the pore size. It should be noted that the large values of the thermal wavelength at low temperatures allow the mobile atoms effectively to average over the substrate heterogeneity for large distances and thus reduce the influence of the details of the substrate potential on the superfluid properties.

In this Letter we have reported a major extension of studies of dilute ^{4}He superfluids to a new range of lower transition temperatures and superfluid particle densities. Our most important finding is a clear indication of a systematic reduction in the relative size of the fluctuation dominated critical region as the transition temperature is decreased. This crossover, from a strongly fluctuating to a more mean-field-like behavior, occurs for transition temperatures on the order of 10 mK and at estimated superfluid particle densities of 1×10^{19} atoms/cm^3. We believe that continued study of the ^{4}He-Vycor system in the low-density limit will provide interesting insights into the subject of Bose condensation and the critical phenomena in a low-density Bose system.

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