Routes to Chaotic Output from a Single-Mode, dc-Excited Laser

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Instabilities in a single-mode, Fabry-Perot, predominantly inhomogeneously broadened xenon laser show progressions through increasingly complicated periodic behavior, ultimately reaching a deterministic chaotic behavior. Period doubling, two-frequency, and intermittency routes to chaos have been observed.

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It has long been predicted that homogeneously broadened single-mode, cw lasers would be susceptible to dynamic instabilities above the threshold for lasing action.[1-5] In the last few years it was discovered by Casperson[6] and confirmed by others[7-9] that inhomogeneously broadened lasers have a remarkably low threshold for the onset of instabilities (requiring gain only a few percent above that necessary for laser action rather than the ninefold increase required for homogeneously broadened lasers). In all cases the single-mode instability requires that the decay rate for the field inside the laser cavity, κ, exceed the relaxation rate of the atomic polarization. This "bad cavity" requirement restricts the suitable laser media to only a few high-gain systems.[5]

In previous reports[10,11] we have examined the thresholds for single-mode instabilities in xenon Fabry-Perot lasers using the high-gain infrared transition at 3.51 μm. Those preliminary studies showed that instabilities depended on the laser cavity tuning, the gain of the active medium, and the degree of homogeneous broadening. Period doubling of the observed pulsations (alternation of the pulse heights) was found,[11] confirming earlier observations by Casperson.[6] The studies reported here include data from systematic variation of parameters, revealing particularly interesting changes of stable periodic behavior and the onset of chaos.

The laser under study employed a dc-excited xenon gas discharge tube with an active region 1 mm in diameter which was fitted with quartz windows at Brewster's angle. (See Refs. 10 and 11 for schematic details.) The laser cavity length of 16.5 cm gave a free-spectral range of 909 MHz as compared with the Doppler-broadened full width at half maximum of the single isotope enriched (91% ^{136}Xe; 9% ^{134}Xe) xenon gas of 110 MHz. The homogeneous linewidth includes both the spontaneous decay rate and pressure broadening.[11] κ could be varied by the insertion of one or more apertures which were sometimes essential to suppress transverse modes and thus to insure single longitudinal mode operation.

Heterodyne detection of the laser signal mixed with a reference laser standard permitted measurements of the optical frequency of the unstable laser to supplement records of the average power output. With use of a fast storage oscilloscope and an rf spectrum analyzer, single-shot real-time intensity traces and time-average power spectra were also recorded. The details of this detection system have been discussed previously[12] and have led to many other interesting observations which will be presented elsewhere.

Most of the general features of the average in-

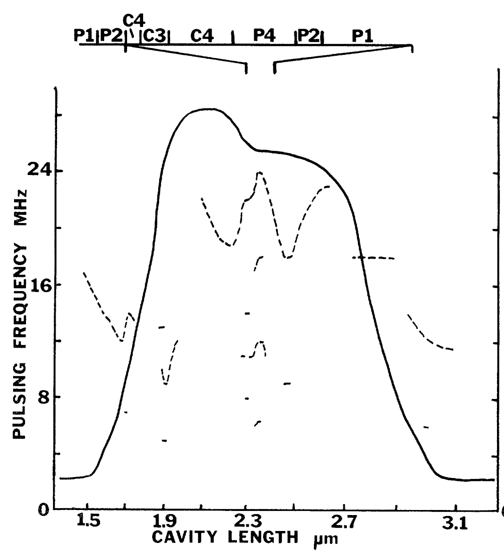

FIG. 1. Laser power output (solid line) and pulsation frequencies vs laser cavity detuning for 175-mTorr Xe, 0.7-Torr He, and 1.43 mA. Total collision-broadened decay rate $\gamma = (6.1 \pm 0.3) \times 10^7$ s^{-1}, $\kappa = (3.3 \pm 0.5) \times 10^8$ s^{-1}. The average mode pulling was a factor of 3.6 and the laser threshold parameter, r, was 2.3. Top inset indicates regions of periodic behavior (period 1, $P1$; period 2, $P2$; period 4, $P4$) and chaotic behavior (chaos with period 4, $C4$; chaos with period 3, $C3$) in a narrow detuning range. See Fig. 2 for details of these regions.

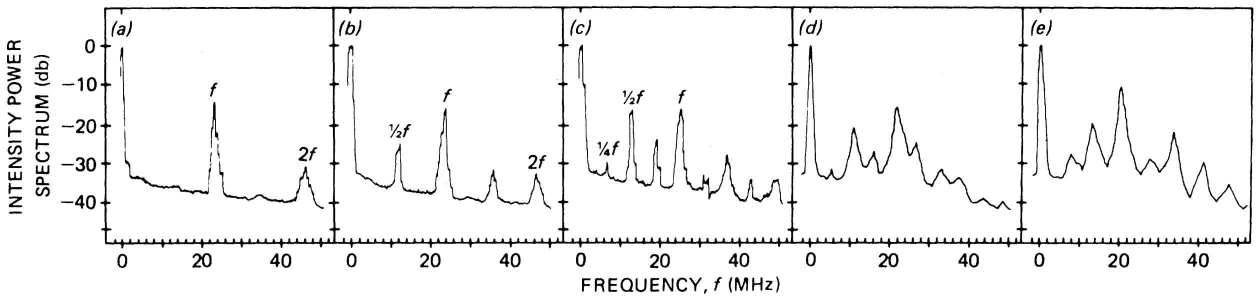

FIG. 2. Power spectra (resolution 1 MHz) of laser output intensity for different detunings in the expanded scale region of Fig. 1 showing period-doubling route to chaos. Spectra are taken from (a) to (e) as samples from the $P1$, $P2$, $P4$, $C4$, and $C3$ regions of Fig. 1.

tensity of a stable inhomogeneously broadened gas laser persisted when the laser showed unstable output. Figure 1, for example, shows a sample power output curve with the expected Lamb dip. The slight asymmetry about the Lamb dip is due primarily to dispersion focusing effects.[13]

Figure 2 shows samples of the power spectrum of the laser intensity for particular parameters. The locations of the spectral peaks are plotted in Fig. 1 for the full range of detuning of the laser. The peaks are narrow but have nonzero widths above the instrumental linewidth of 1 MHz. The minimum linewidth is due in part to the amplitude and phase noise of the residual broadband-amplified spontaneous emission in the laser output.

The pulsation frequencies change rapidly with detuning near the Lamb dip. This can be understood as arising in part from the overlapping of the complicated dispersion of the two spectral holes burned in the Doppler-broadened gain profile by the counter-propagating waves.[14] The fact that the pulsation patterns depend critically on the cavity detuning indicates that the periodic and chaotic phenomena have origins in field-atom dynamics.

Period-doubling route to chaos.—The sequence of spectra in Fig. 2 shows simply periodic behavior [Fig. 2(a)] followed by two period doublings [Figs. 2(b) and 2(c)] and then a chaotic spectrum [Fig. 2(d)] where the broadband noise has filled the spectrum between the originally separated peaks. Above the chaotic threshold a region of period-three behavior [Fig. 2(e)] is also observed. This sequence is representative of many seen at various discharge currents. We have also observed parts of similar sequences for fixed cavity length as the gain was varied. Such scans have been limited by restrictions on the range of the discharge current.

This period-doubling sequence is one of only a few seemingly universal routes to chaos identified in many other nonlinear systems.[15] The truncation of the expected infinite sequence of period doublings may be due to the narrowness of the regions of parameter space which support the higher periodicities but in this case it is more likely to be linked to intrinsic spontaneous-emission noise of the system. It has been previ-

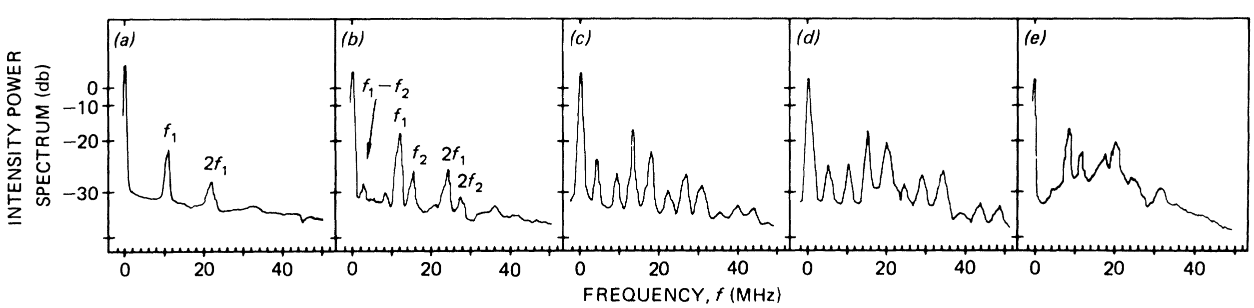

FIG. 3. Power spectra (resolution 1 MHz) of laser output intensity for different values of the discharge current (laser gain variations) for 175-mTorr Xe and 0.3-Torr He [$\gamma=(3.8\pm0.2)\times10^7\ s^{-1}$; $\kappa=(3.3\pm0.5)\times10^8\ s^{-1}$]. Second pulsation frequency f_2 is identified as the strongest new peak to emerge in (b). Secondary combination tones appear in (b) leading to chaos in (e). Currents and relative average power outputs are (a) 0.52 mA, 5.0; (b) 0.67 mA, 6.9; (c) 0.89 mA, 9.0; (d) 1.3 mA, 11.6; (e) 1.6 mA, 12.0 ($r=2.2$).

ously shown that added noise can lead to a truncation of the sequence of period doublings and a premature transition to chaos.[16]

Aside from the analytic comparison with the Lorenz equations as a special case, few detailed predictions have been made for laser equations or for the inhomogeneously broadened case in particular. Recently a dynamical model developed by Casperson has shown evidence of such a period-doubling sequence into chaos for realistic xenon parameters.[5]

Two-frequency route to chaos.—In some regions of detuning the laser would show the coexistence of two initially incommensurate frequencies. Figure 3 shows a sequence of power spectra in such a region taken for fixed cavity detuning as the discharge current (and thereby the gain) was increased. The data show an initial pulsation frequency and its harmonics [Fig. 3(a)] followed by the onset of a second pulsation frequency [Fig. 3(b)]. The second frequency appears to quickly lock to a 4:3 ratio with the first. For higher gain the spectrum is enriched by the harmonics and combination frequencies of the two pulsation frequencies. At sufficiently high gain the broadband noise characteristic of chaos emerges. We have also observed a two-frequency route to chaos for fixed gain by varying the cavity length.

Stability analyses of the laser equations have shown that the steady-state solution will often have two pairs of complex-conjugate eigenvalues which go unstable in turn.[8,9] While dynamical results from these studies are not yet available, (and are required to understand the breakdown of periodic solutions) it appears that the observed two frequencies may reflect the two eigenvalues of the steady-state solution.

This second route to chaos is expected in multivariable systems where the two-frequency behavior is linked to a solution constrained to a torus in parameter space. The fact that the two frequencies lock is not uncommon, but the link between chaos and two locked (commensurate) frequencies has only recently been explained.[17]

These two routes to chaos are strongly analogous to the classic routes to chaos studied recently in fluids.[18] It seems likely that the link of fluids and lasers by the Lorenz equations[19] provides reasons for strong similarities.

Third route to chaotic behavior.—Another distinct route to chaos has been observed for certain laser parameters and is shown in Fig. 4. Here the initial instability is characterized by a narrow spectral peak which is, nevertheless, broader than the instrumental linewidth [Fig. 4(a)]. This periodic behavior becomes increasingly chaotic as evidenced by the significant broadening of the peaks as the gain is increased. Intermittent chaotic behavior near a tangent bifurcation may be at work here.[15] The spectrum analyzer averages over 20–50 ms while the basic pulsation

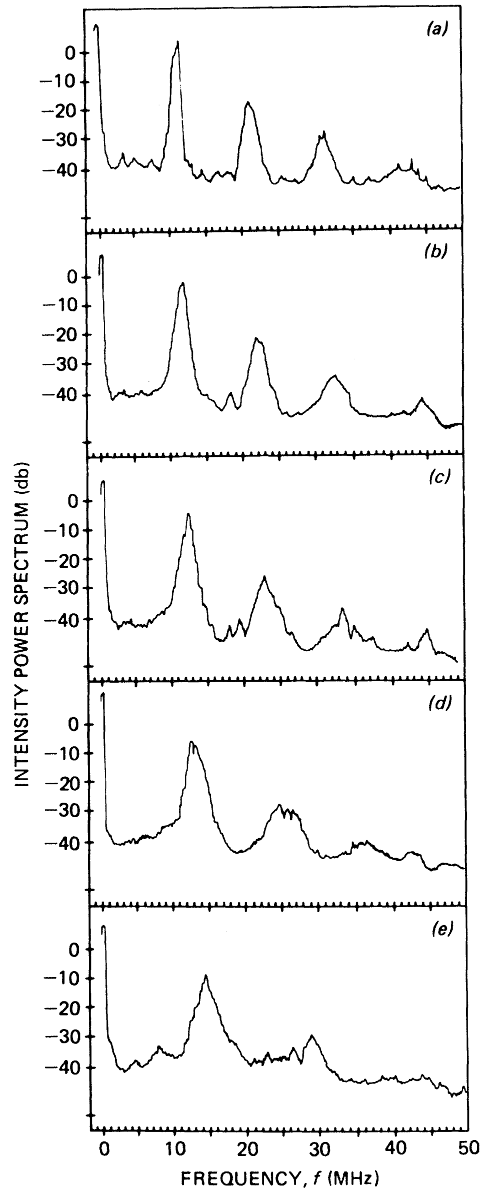

FIG. 4. Broadening of pulsation spectral peaks (resolution 100 kHz) with increasing discharge current for 175-mTorr Xe and no helium [$\gamma = (2.0 \pm 0.1) \times 10^7$ s^{-1}; $\kappa = (3.3 \pm 0.5) \times 10^8$ s^{-1}]. Discharge currents and relative average power outputs are (a) 0.83 mA, 6.2; (b) 0.91 mA, 6.7; (c) 1.0 mA, 7.3; (d) 1.2 mA, 8.3; (e) 1.5 mA, 9.5.

rate is 10 MHz. Increasingly frequent, intermittent bursts of chaos for increasing gain would result in a steady broadening of the peaks.[20]

The single-mode, dc-excited, inhomogeneously broadened laser has shown a richness of instabilities similar to that found recently in fluid dynamics and other nonlinear systems. Recently such phenomena have also been observed in multimode laser systems[21,22] in a related, but theoretically distinct,[3,4] type of dynamical system. The appearance of previously identified routes to chaos adds to the universality of those progressions. However, the Fabry-Perot geometry and other free parameters (particularly cavity detuning) appear to add a variety of phenomena not found in other simpler models. Details of these anomalies will be reported elsewhere.[23]

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