Viscous Damping of Second Sound near the Lambda Point of Liquid ^{4}He

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The theoretical second-sound damping in ^{4}He II is too small in the background region by 40%. Viscous damping in the normal fluid is identified as the origin of the missing attenuation. Because of simple two-fluid kinematics, this damping grows below the λ point as the superfluid density. The strength of the damping is determined by the coefficient of bulk viscosity, which is already known from the measured background first-sound damping above the λ point.

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At the present time there exists a significant discrepancy between the theoretically predicted[1] second-sound damping coefficient $D_2(t)$ [where $t=(T-T_\lambda)/T_\lambda<0$ is the reduced temperature] and the measured values of $D_2(t)$. The former is shown in Fig. 1 by the dashed curve (which merges with the solid curve for $|t|<10^{-3.5}$). The experimental points in Fig. 1 exhibit the recent data of Crooks and Robinson,[2] which span the interval $10^{-5}<|t|<2\times10^{-2}$. For $10^{-5}<|t|\leqslant10^{-3}$ the agreement between theory and experiment is good. The accord deteriorates, however, for $|t|>10^{-3}$. The upper end of the experimental interval corresponds to the furthest point below the λ point, at a temperature difference of $T_\lambda-T\simeq40$ mK. Here it can be discerned from Fig. 1 that the theory predicts a D_2 which is too small by 1.5×10^{-4} cm^2 sec^{-1}, or about 40% below the observed value. Evidently the theory has overlooked an effect which is indeed negligible close to the λ point but which becomes important further below, at temperatures more deeply in the broken-symmetry state where the order parameter is larger. The purpose of this note is to put forward a possible such effect which, if correct, raises the theoretical curve in Fig. 1 from the dashed line to the solid line, resulting in good agreement over the entire experimental range.

But before we proceed it is necessary to discuss the commonly held notion that the second-sound damping is well understood. This impression was generated by two different renormalization-group treatments[4,5] of the problem. Dohm and Folk[6] have made a thorough comparison of these two approaches. From our viewpoint the alleged agreement is impossible for $D_2(t)$, because the discrepancy exhibited in Fig. 1 occurs in the noncritical region far from the critical point. In this region of relatively large values of $|t|$ even the most sophisticated scheme of renormalization cannot save the theory because the renormalization cannot have any effect whatsoever until the critical point is approached more closely. At $|t|=2\times10^{-2}$ the *kinetic coefficients assume their nonuniversal background values.* Furthermore, as already explained at some length,[1] these *background coefficients have fixed values* that are already determined for $T>T_\lambda$, in two different ways. It therefore stands, barring some arithmetic error on our part, that the discrepancy of 40% at $|t|=2\times10^{-2}$ is a clear and

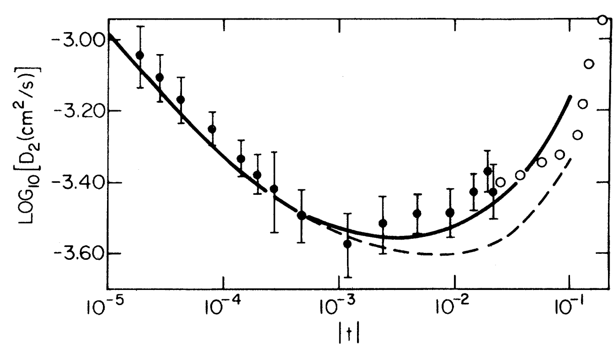

FIG. 1. Second-sound attenuation coefficient D_2 vs reduced temperature $t=(T-T_\lambda)/T_\lambda$. The data are from Crooks and Robinson (Ref. 2) (solid circles) and Hanson and Pellam (Ref. 3) (open circles). The dashed curve shows the prediction of the theory (Ref. 1) omitting normal-fluid bulk viscosity, while the solid curve shows the agreement with experiment that results from its inclusion. This new contribution to D_2 is proportional to ρ_s, the superfluid density, and is therefore greatest at the largest values of $|t|$, in the noncritical background region.

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present failure of the theory, and that the claim of Dohm and Folk[5,6] of agreement in this region applies to the effective ratio R_2^{eff} and not to $D_2(t)$ itself.[7]

The lower (dashed) theoretical curve in Fig. 1 is described by

$$D_2(t)=\Delta\lambda(t)/C_P'(t)+\lambda_B/C_P'(t)+B_\psi\,, \tag{1}$$

where B_ψ describes the background order-parameter relaxation and $C_P'(t)$ is the experimental constant-pressure specific heat *below* the λ point. $\Delta\lambda(t)=\lambda(t)-\lambda_B$ is the experimental increase in the thermal conductivity as the λ point is approached from *above*. It is only in this term that there is any opportunity for renormalization effects to enter. We found that the sum of the critical contributions from the entropy and order-parameter relaxation rates could be approximately identified numerically with the rise in the entropy relaxation rate that occurs for $t>0$. This identification leads to the simple and convenient formula of Eq. (1). But here we want to concentrate on the last two terms of Eq. (1), which depend upon

$$\lambda_B=0.153 \text{ mW/K}, \tag{2a}$$

$$B_\psi=1.05\times10^{-4}\text{ cm}^2\text{ sec}^{-1}, \tag{2b}$$

determined from thermal conductivity and ultrasonic attenuation measurements *above* the λ point.

Our contention has been that Eqs. (2a) and (2b) fix the level of the background second-sound attenuation completely. But we have recently come to realize that, because the superfluid state has the broken symmetry characterized by the complex quantum mechanical order parameter ψ_0, there can be additional contributions to B_ψ, say B_ψ', which are proportional to $|\psi_0|^2=\rho_s(t)$, the superfluid density. Since ρ_s vanishes above the λ point the postulated additional order-parameter damping tends to evade us when we use the $t>0$ data to fix the background coefficients. Consequently, taking the critical exponent of ρ_s to be $\frac{2}{3}$, we must admit the possibility of some additional damping with the temperature dependence

$$B_\psi'(t)=\text{const}\times\rho_s(t)\propto|t|^{2/3}. \tag{3}$$

Such a contribution will also improve the agreement with the light-scattering spectrum.[8] We now proceed to show that the noncritical relaxation processes within the normal fluid contribute precisely in this way. The normal-fluid velocity field $\vec{v}_n$ shown in Fig. 2 for a standing wave of second sound of wavelength $\lambda=2\pi/k$ results in alternate compressions and rarefactions of the normal-fluid density, as described by the nonvanishing value of $\operatorname{div}\vec{v}_n$. These density changes evoke quasiparticle redistributions within the normal fluid leading to a rate of dissipation of energy density

$$\overline{P}=\zeta\langle\operatorname{div}\vec{v}_n\rangle^2=\zeta k^2\langle v_n^2\rangle\,, \tag{4}$$

where ζ is the coefficient of bulk viscosity for the normal fluid. The angular brackets denote space-time averaging.

The calculation of the damping coefficient follows elementary and familiar lines. In standard notation the kinetic energy density is

$$T=\tfrac{1}{2}\rho_s v_s^2+\tfrac{1}{2}\rho_n v_n^2. \tag{5}$$

Imposition of the condition of complete backflow on the mass current density,

$$\vec{J}=\rho_s\vec{v}_s+\rho_n\vec{v}_n=0, \tag{6}$$

gives for the average energy density of the second-sound vibration

$$\langle U_2\rangle=2\langle T_2\rangle=(\rho_n/\rho_s)\rho v_n^2. \tag{7}$$

The damping coefficient is therefore

$$D_2(t)=k^{-2}\frac{\langle P\rangle}{\langle U_2\rangle}=\frac{\rho_s}{\rho_n}\frac{\zeta}{\rho}. \tag{8}$$

It is useful to carry out the same calculation for the damping of first sound. Fixing our eyes on the normal fluid we see that the very same bunching that is shown in Fig. 2 takes place also in a first-sound standing wave. We therefore take over Eq. (4) for first sound, without any modification and with precisely *the same value of* ζ. Because now the superfluid and normal fluid move together the energy density in the first-sound vibration is

$$\langle U_1\rangle=2\langle T_1\rangle=\rho\langle v_n^2\rangle\,, \tag{9}$$

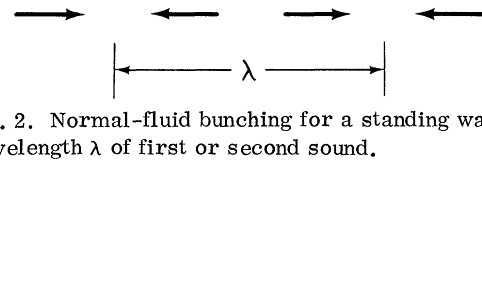

FIG. 2. Normal-fluid bunching for a standing wave of wavelength λ of first or second sound.

and lacks the factor ρ_n/ρ_s. Thus we find

$$D_1 = k^{-2}\langle P\rangle/\langle U_1\rangle = \zeta/\rho. \tag{10}$$

Eliminating the bulk viscosity from Eqs. (8) and (10), which are contained within the framework of Khalatnikov's[9] two-fluid hydrodynamics, we find

$$D_2(t) = (\rho_s/\rho_n)D_1 = 2.4(\rho/\rho_n)D_1|t|^{2/3}. \tag{11}$$

Here we have substituted the Clow-Reppy[10] formula for ρ_s/ρ.

Equation (11) has the temperature dependence anticipated in Eq. (3). Treating $D_1\rho/\rho_n$ as approximately constant, we find its magnitude from the λ-point value of

$$D_1 = 2c_1^3\alpha_1^B/\omega^2, \tag{12}$$

where c_1 and ω are the velocity and angular frequency of first sound, respectively, and α_1^B is the noncritical background amplitude attenuation coefficient in nepers per centimeter at the λ point. Lamberg, Legros, and Salin[11] have found $\alpha_1^B = 980$ Np cm^{-1} at $\omega/2\pi = 1.1$ GHz. With $c_1 \simeq 230$ m/sec we find $k=\omega/c_1 = 3.0\times10^5$ cm^{-1} so that Eqs. (12) and (11) become $D_1 = 5.0\times10^{-4}$ cm^2 sec^{-1} and

$$D_2(t) = B_\psi'(t) = 1.2\times10^{-3}|t|^{2/3}\ \text{cm}^2/\text{sec}. \tag{13}$$

We have added this contribution to that shown by the dashed curve to arrive at the solid curve in Fig. 1, thereby bringing the theory into satisfactory agreement with experiment in the background region.[12] (The open circles exhibit the data of Hanson and Pellam[3] in the temperature range not covered by Crooks and Robinson.[2])

In summary, we have identified the "missing" second-sound attenuation at temperatures well below the λ point and outside the dynamic critical region as coming from viscous damping in the normal fluid. There is no free parameter in our calculation, which we feel is all the more compelling because of the very simple physical picture on which it is based.

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[1]Richard A. Ferrell and Jayanta K. Bhattacharjee, Phys. Rev. B 24, 5071 (1981).

[2]M. J. Crooks and B. J. Robinson, Physica (Utrecht) 107B, 339 (1981), and to be published.

[3]W. B. Hanson and J. R. Pellam, Phys. Rev. 95, 321 (1954).

[4]G. Ahlers, P. C. Hohenberg, and A. Kornblit, Phys. Rev. B 25, 3136 (1982).

[5]V. Dohm and R. Folk, Phys. Rev. Lett. 46, 348 (1981).

[6]V. Dohm and R. Folk, Z. Phys. B 45, 129 (1981), and Physics (Utrecht) 109&110B, 1549 (1982), and in *Festkörperprobleme: Advances in Solid State Physics*, edited by P. Grosse (Vieweg, Braunschweig, 1982), Vol. 22, p. 1.

[7]As explained in Refs. 5 and 6, good agreement is claimed for the effective ratio R_λ^{eff} (essentially the fractional linewidth of the second-sound doublet). The Dohm-Folk $D_2(t)$ background values differ from ours by $c_2^{\text{expt}}/c_2^{\text{theor}}$, the ratio of the experimental to theoretical second-sound velocities.

[8]J. A. Tarvin, F. Vidal, and T. J. Greytak, Phys. Rev. B 15, 4193 (1977).

[9]I. M. Khalatnikov, *Introduction to the Theory of Superfluidity* (Benjamin, New York, 1965).

[10]L. R. Clow and J. D. Reppy, Phys. Rev. Lett. 16, 887 (1966).

[11]B. Lambert, P. Legros, and D. Salin, J. Phys. (Paris), Lett. 41, L-507 (1980).

[12]As explained in Ref. 1 there is a weak contribution to D_2 in the dynamic critical region from longitudinal relaxation of the order parameter. Inclusion of this in Fig. 1. causes the solid curve to pass somewhat higher, but still within the experimental error bars. Longitudinal relaxation has no effect in the background region which is our principal concern here.