Cylindrical $\bar{s}\bar{p}$-Boson Model for Large Deformed Nuclei

Hsi-Tseng Chen
Physics Department, University of Tokyo, Tokyo, Japan, and Pelletron Laboratory, Instituto de Fisica, Universidade de São Paulo, São Paulo, Brazil

and

A. Arima
Physics Department, University of Tokyo, Tokyo, Japan
(Received 7 February 1983)

It is proposed that the intrinsic states in a well deformed nucleus may be described as those of a system of nonspherical bosons, each possessing quasispin 1, $\bar{p}$, or quasispin 0, $\bar{s}$. The model is tested by the experimental data with analytic expressions which have been derived for the bandheads and the $E2$ transitions.

PACS numbers: 21.60.Ev, 21.60.Fw, 21.60.Gx

A recent analysis[1] of the collective states of ^{168}Er with the interacting boson model[2] has led to many interesting discussions and investigations into the nature of the intrinsic states in well deformed nuclei.[3] We propose a possible description of these states in terms of a cylindrical representation of the boson SU(6) group, in which a large deformed nucleus, in its intrinsic frame, may be considered as a system of nonspherical bosons, each possessing quasispin 1, $\bar{p}$, or quasispin 0, $\bar{s}$. These quasispins, which we refer to as Δ spin, are the SU(2) symmetry labels of an intrinsic system expressed mathematically by the group chain $SU(3) \supset SU(2) \otimes U(1)$. Since SU(2) is the symmetry group of the two-dimensional oscillator, we force a cylindrical symmetry by imposing a Δ-spin invariance to the boson system, which generates an energy spectrum equal to the two-dimensional harmonic oscillator. The degenerate boson states with the same value of Δ can be specified by the component K of the angular momentum along the symmetry axis. In the present picture, such a cylindrical symmetry is temporarily imposed on the modes of β and γ vibrations by incorporating them into a $\bar{p}$ boson whose presence represents a single boson excitation from a ground state consisting of N $\bar{s}$ bosons. With excitations of more than one boson, we hope to reach the other higher excited states typical of the observed intrinsic spectra of large deformed nuclei. The imposed energy degeneracy of β and γ vibrations is removed by the interactions among these $\bar{s}\bar{p}$ bosons, which then include Δ-spin–nonconserving terms. In so doing, we expect to explain naturally the anharmonicity of many-phonon states, such as that of the two-γ-phonon states in ^{168}Er.

To introduce Δ spin and thus the cylindrical bosons, let us begin with the SU(3) limit of the sd-boson model. Instead of using the group chain, $SU(3) \supset R(3)$, as used for spherical sd bosons, we will use the scheme $SU(3) \supset SU(2) \otimes U(1)$, which has been studied extensively in the context of fermions by Elliot in his SU(3) model.[4] It is known that the SU(2) of this latter group chain can be generated by the following operators, which define our Δ spin:

$$\Delta_{+1} = \tfrac{4}{3}\sqrt{3}\{(d_{-2}{}^\dagger s + s^\dagger \tilde{d}_{-2}) - \tfrac{1}{2}7^{1/2}(d^\dagger \times \tilde{d})_{-2}{}^2\},$$
$$\Delta_{-1} = -\tfrac{4}{3}\sqrt{3}\{(d_2{}^\dagger s + s^\dagger \tilde{d}_2) - \tfrac{1}{2}7^{1/2}(d^\dagger \times \tilde{d})_2{}^2\}, \quad (1)$$
$$\Delta_0 = -\tfrac{1}{2}10^{1/2}(d^\dagger \times \tilde{d})_0{}^1.$$

We then have cylindrical bosons such as $\bar{s}$ bosons, $\bar{p}$ bosons, and spinor bosons, which we call Λ bosons, as created by the following spherical tensors in the Δ-spin space, respectively:

$$\bar{s}^\dagger = (1/\sqrt{3})(s^\dagger + \sqrt{2}d_0{}^\dagger), \quad \Delta = 0; \quad p_1{}^\dagger = d_{-2}{}^\dagger, \quad p_0{}^\dagger = (1/\sqrt{3})(-\sqrt{2}s^\dagger + d_0{}^\dagger), \quad p_{-1}{}^\dagger = d_2{}^\dagger, \quad \Delta = 1;$$
$$\Lambda_{1/2}{}^\dagger = d_{-1}{}^\dagger, \quad \Lambda_{-1/2}{}^\dagger = d_1{}^\dagger, \quad \Delta = \tfrac{1}{2}. \quad (2)$$

Following Elliot, we choose the intrinsic wave function for a given SU(3) symmetry (λ, μ) to be that with the maximum value of the intrinsic quadrupole moment Q_0 and we see that, in general, more than one boson configuration shares the same Q_0. In a study of this configuration mixing in an intrinsic state, we have proved, using standard group-theoretical techniques, that the configuration

$$(\bar{s}^\dagger)^{(\lambda+\mu+f_3)/2}(p^\dagger)^{(\mu+2f_3)/2}|0\rangle \quad (3)$$

is always dominant over the other components in the states $(\lambda+\mu+f_3,\ \mu+f_3,\ f_3)$. Taking the physically most interesting state, $(\lambda,\mu)=(2N-4,2)$, as an example, we see that the coefficient for the configuration that has two spinor bosons is $-[1/(2N-1)]^{1/2}$, indeed small compared to that of $(\bar{s}^\dagger)^{N-1}p^\dagger|0\rangle$, which is $[2(N-1)/(2N-1)]^{1/2}$. For the next higher band, $(\lambda,\mu)=(2N-8,4)$, the ratio of the intensities of the configurations, with and without spinor bosons, is of the order $1/N$. This behavior persists generally, even with higher SU(3) representations, and thus is closer to the Bohr and Mottelson geometrical picture, where, in the discussion of β and γ vibrations, the $L=2$, $K=1$ mode of excitation is absent in large deformed nuclei.

In every intrinsic state, keeping only *one* configuration as given by (3) leads to the scheme $SU(6)\supset SU(4)$.

If we take the intrinsic energies to be proportional to the eigenvalues of the Casimir operator of SU(3), namely,

$$-\kappa C(\lambda,\mu)=-\kappa(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu),$$

the resulting spectrum can be grouped into bands (not to be confused with the usual rotational bands) labeled by the number of $\bar{p}$ bosons, N_p, which is common to all the following representations in the band:

$$(\lambda,\mu)=(2N-4N_p+2\nu-2,\ 2N_p-4\nu+4);\quad \nu=1,2,3,\ldots,\begin{cases}\frac{1}{2}N_p+1, & \text{even } N_p,\\ \frac{1}{2}N_p+\frac{1}{2}, & \text{odd } N_p.\end{cases} \tag{4}$$

All of them have the same intrinsic quadrupole moment $Q_0=4N-6N_p$, and therefore the same dominant spinor-boson–free configuration $(\bar{s}^\dagger)^{N-N_p}\times(p^\dagger)^{N_p}|0\rangle$. All of these representations have three rows in their corresponding Young's diagram except the energetically lowest member of the band, which has two rows. The latter representation has the symmetry $(\lambda,\mu)=(2N-4N_p,2N_p)$, and serves as the bandhead. Such a grouping of bands becomes reasonable since the energy spacing between bandheads goes as N while that between band members goes as unity. Since the given symmetry (λ,μ) has $\Delta=\frac{1}{2}\mu$, each member within a band can now be assigned a Δ spin according to (4), namely, $\Delta=N_p,\ N_p-2,\ \ldots,\ 1$, or 0. For a given number N, the SU(3) energies relative to the ground state can be written down in terms of the Δ spins; for the N_p band

$$\Delta E=3\kappa\{2N_p(2N-2N_p+1)+(N_p-\Delta)(N_p+\Delta+1)\}. \tag{5}$$

In the limit of large N, we see that all the members of a band N_p become degenerate in energy and the energy spacings between any two neighboring bandheads become equal. The level scheme of this type spells out immediately the underlying $\overline{sp}$-boson picture in its $SU(4)\supset SU(3)\otimes U(1)$ limit. The energy expression (5) is then exactly that of $\overline{sp}$ bosons according to the group chain $SU(4)\supset SU(3)\supset R(3)$, and has the closed form, orginally given by Racah,[5]

$$\Delta E=N_p\epsilon+\tfrac{1}{2}aN_p(N_p-1)+b[\tfrac{1}{2}\Delta(\Delta+1)-N_p], \tag{6}$$

which immediately fixes an interacting p-boson Hamiltonian, provided that $\epsilon=6\kappa(2N-1)$, $a=-18\kappa$, and $b=-6\kappa$.

In the cylindrical $\overline{sp}$-boson model, the model Hamiltonian is, in general, no longer rotational invariant in Δ-spin space as with good L in the case of spherical *sd* bosons. In order to preserve the axial symmetry and time-reversal invariance, the most general form of the Hamiltonian is the mixture of the zero-z-component Δ-spin tensors of rank zero, two, and four, namely, $H=H_0^0+H_0^2+H_0^4$. In terms of the SU(3) generators $\overline{Q}_\mu$ $[=-6^{1/2}(p^\dagger\times\tilde{p})_\mu^2]$ and the R(4) Runge-Lenz vector A_μ $(=p_\mu^\dagger\bar{s}+\bar{s}^\dagger\tilde{p}_\mu)$, the Δ-spin–nonconserving part of the Hamiltonian is chosen to contain the terms $(\overline{Q}\times\overline{Q})_0^2$, $(\overline{Q}\times\overline{Q})_0^4$, and $(A\times A)_0^2$. This choice is adequate, since the source of these terms may be traced back to the pairing, octupole-octupole, and hexadecupole-hexadecupole interaction terms in the *sd*-boson Hamiltonian. Actually, we have performed a complete mapping of the most general *sd*-boson Hamiltonian onto an axial-symmetric one in the $\overline{sp}$ space and thus obtained a linkage between both Hamiltonians, enabling us to make the source tracing mentioned above. The details of this mapping will be reported elsewhere. We focus our attention on cases not far from SU(3)

TABLE I. Bandhead energies of ^{168}Er (in megaelectronvolts).

N_p	K	Δ	E_{theor}	E_{expt} (Ref. 6)
1	2	1	0.88	0.82
	0	1	1.24	1.22
2	4	2	1.94	2.06
	2	2	1.89	1.85
	0	2 + 0	1.36[a]	1.42
	0	2 + 0	2.35[a]	

[a]A Δ-spin mixing calculation.

symmetry, where our model Hamiltonian is chosen to contain the scalar term, H_0^0, which is the sum of a term $\gamma\Delta^2$ and a Hamiltonian whose parameters are adjusted to reproduce the SU(3) energies (5), and the Δ-spin–nonconserving term given by $\alpha(\overline{Q}\times\overline{Q})_0^2+\beta(\overline{Q}\times\overline{Q})_0^4$. A simple first-order perturbation treatment leads to the following energy expression relative to the ground state:

$$
\begin{aligned}
E &= 3\kappa[2N_p(2N-2N_p+1)+(N_p-\Delta)(N_p+\Delta+1)]+\gamma\Delta(\Delta+1)\\
&+\frac{3\alpha}{14^{1/2}}\,\frac{3K^2-4\Delta(\Delta+1)}{2\Delta+1}\left[\frac{\Delta+2}{(2\Delta+3)^2}(N_p-\Delta)(N_p+\Delta+3)+\frac{\Delta-1}{(2\Delta-1)^2}(N_p+\Delta+1)(N_p-\Delta+2)\right.\\
&\qquad\left.-\frac{(2\Delta-3)(2\Delta+5)(2\Delta+1)}{12(2\Delta+3)^2(2\Delta-1)^2}(2N_p+3)^2\right]\\
&+\frac{6\beta}{70^{1/2}}(-1)^{\Delta+K/2}\begin{pmatrix}\Delta & 4 & \Delta\\ K/2 & 0 & -K/2\end{pmatrix}\left[\frac{\Delta(\Delta-1)(\Delta+1)(\Delta+2)(2\Delta+3)(2\Delta-3)(2\Delta-1)}{(2\Delta+1)(2\Delta+5)}\right]^{1/2}\\
&\times\left[\frac{1}{(2\Delta+3)^2}(N_p-\Delta)(N_p+\Delta+3)+\frac{2\Delta+5}{(2\Delta-1)^2(2\Delta-3)}(N_p+\Delta+1)(N_p-\Delta+2)\right.\\
&\qquad\left.+\frac{(2\Delta+1)(2\Delta+5)}{(2\Delta+3)^2(2\Delta-1)^2}(2N_p+3)^2\right].
\end{aligned}
\tag{7}
$$

This last expression has been used to reproduce various bandheads in ^{168}Er with $3\kappa=0.11$, $\alpha=-0.065$, $\beta=-0.2$, and $\gamma=0.1$. The results, shown in Table I, are quite encouraging. The reproduction of the two-γ-phonon $K=4$ band seems to be very instructive in relation to its anharmonicity. In Table I, the reported 1.36-MeV state, compared to the empirical 1.42-MeV one, was obtained by a Δ-spin mixing calculation, within the $N_p=2$ band, with the mixture given by $|0_3\rangle=-0.76|N_p=2,\Delta=2,K=0\rangle+0.65|N_p=2,\Delta=0,K=0\rangle$. This mixing will be used afterwards to analyze the $E2$ transition data and leads to interesting results.

To test the model with $E2$ transitions, we have constructed closed expressions for intrinsic matrix elements in terms of Δ spin. To assimilate the situation in the vicinity of SU(3) symmetry, we use the most general $E2$ operator,

$$T_\mu=(d_\mu^\dagger s+s^\dagger\tilde{d}_\mu)+7^{1/2}\eta(d^\dagger\times\tilde{d})_\mu^2,$$

with a value not necessarily its SU(3) one, $-\frac{1}{2}$. After decomposing it into Δ-spin tensors of various ranks and defining

$$|(N_p\Delta)_K\rangle=[2(1+\delta_{K0})]^{-1/2}\{|N_p\Delta K\rangle+|N_p\Delta-K\rangle\},$$

we reach expressions which reveal the general trends of the transitions among the first few excited N_p bands, near SU(3) symmetry, for a series of nuclei in the rare-earth region, where the values of η are in the range $-0.085>\eta>-0.203$, as determined by Warner and Casten.[1] The results, presented in Fig. 1, are as follows: (i) a dominance of the intraband transitions between the members of the same N_p band over some of the neighboring interband transitions between different N_p bands, and (ii) a constant value of the ratio $\langle N_p\Delta K\pm\nu|T|N_p-1,\Delta\pm1,K\pm\nu\rangle/\langle N_p\Delta K\pm2|T|N_p-1,\Delta\pm1,K\rangle$, where $\nu=0,2$, independent of N_p, N, and η. In the special case of $N_p=1$, these features are exactly those obtained recent-

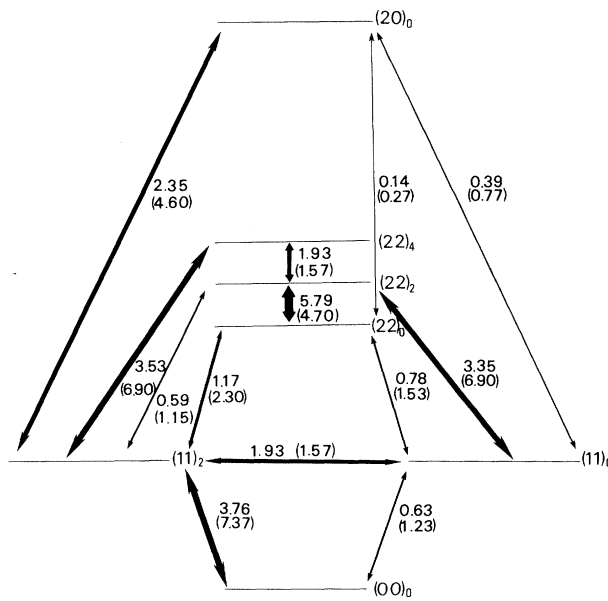

FIG. 1. Interband and intraband $E2$ transitions among the intrinsic N_p bands for $N=16$. The transition rates are calculated for both end values of η in the range $-0.085>\eta>-0.203$ for the rare-earth nuclei. The numbers in the parentheses besides the arrows are the rates for $\eta=-0.085$ and otherwise are for $\eta=-0.203$.

ly by Bijker and Dieperink[7] from which they demonstrate (i) the dominance of $\beta \to \gamma$ over $\beta \to g$ and (ii) $\langle \beta | T | g \rangle^2 / \langle \gamma | T | g \rangle^2 = \frac{1}{6}$. This last ratio can be obtained easily in the present work from the simple expression $\Delta/3(\Delta+1)$.

To compare the available $E2$ transition data[6] of ^{168}Er with our model we have calculated the quantities $B(E2, I_i K_i \to I_f K_f)$ in the adiabatic limit, using the derived intrinsic matrix elements with Casten's value of η, -0.115. The results of $B(E2)$ branching ratios from the γ and the β band are very close to those of Casten and Warner,[1] showing the goodness of the adiabatic approximation. After extracting the empirical values for the intrinsic $B(E2)$ branching ratios in the adiabatic limit, we compare the calculated ones, 2.57:100 and 2.6:400 for $(11)_2 \to (00)_0/(11)_2 \to (11)_2$ and $(11)_0 \to (11)_2/(11)_0 \to (11)_0$, with the experimental values 2.60:100 and 3.0:400. The dominance of $(11)_0 \to (11)_2$ over $(11)_0 \to (00)_0$ is demonstrated by its ratio 8.66:5.5 compared with its empirical estimate, 28.0:5.5. In calculating the $B(E2)$ branching ratios from the 0_3 band, we recall the fact that the intrinsic 0_3 state, reproduced in the present calculation, is a Δ-spin-impure one, given by $|0_3\rangle = -0.76|(22)_0\rangle + 0.65 \times |(20)_0\rangle$, and we obtain the value 38.5 for $\langle 0_3 | T \times |(11)_0\rangle^2 / \langle 0_3 | T | (11)_2\rangle^2$ compared with its empirical value, < 59.0.

This work was supported in part by the Fundação de Amparo à Pesquisa do Estado de São Paulo. One of the authors (H.T.C.) wishes to thank the Nuclear Theory Group of the University of Tokyo for its hospitality.

[1]D. D. Warner, R. F. Casten, and W. F. Davidson, Phys. Rev. C 24, 1713 (1981); D. D. Warner and R. F. Casten, Phys. Rev. C 25, 2019 (1982).

[2]A. Arima and F. Iachello, Ann. Phys. (N.Y.) 99, 253 (1976), and 111, 201 (1978), and 123, 468 (1979).

[3]A. Bohr and B. R. Mottelson, Phys. Scr. 25, 28 (1982); T. S. Dumitrescu and I. Hamamoto, Nucl. Phys. 383, 205 (1982).

[4]J. P. Elliot, Proc. Roy. Soc. London, Ser. A 245, 128, 562 (1958).

[5]G. Racah, "Group Theory and Spectroscopy," in *Springer Tracts in Modern Physics, Vol. 37* (Springer, Berlin, 1965).

[6]W. F. Davidson, D. D. Warner, R. F. Casten, K. Schreckenbach, H. G. Borner, J. Simic, M. Stojanovic, M. Bogdanovic, S. Koicki, W. Gelletly, G. B. Orr, and M. L. Stelts, J. Phys. G 7, 455, 843 (1981).

[7]R. Bijker and A. E. L. Dieperink, Phys. Rev. C 26, 2688 (1982).