Unlimited Electron Acceleration in Laser-Driven Plasma Waves

T. Katsouleas and J. M. Dawson
University of California, Los Angeles, California 90024
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It is shown that the limitation to the energy gain of $2(\omega/\omega_p)^2mc^2$ of an electron in the laser-plasma beat-wave accelerator can be overcome by imposing a magnetic field of appropriate strength perpendicular to the plasma wave. This accelerates particles parallel to the phase fronts of the accelerating wave which keeps them in phase with it. Arbitrarily large energy is theoretically possible.

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Recently there has been a great deal of interest in using laser-plasma interactions to accelerate particles to high energies more rapidly than the 20 MeV/m to which linear accelerators are currently limited.[1] The beat-wave accelerator is one scheme proposed by Tajima and Dawson[2] to excite electrostatic plasma waves which can accelerate particles; the attraction of the method is the extremely large electric fields which can be generated (order 10^9 V/cm). Whereas particles in the beat-wave accelerator can gain only a finite amount of energy before they get out of phase with the beat wave, by introduction of a perpendicular magnetic field the particles are deflected across the wave front and thereby prevented from outrunning the wave. The particles may be accelerated to arbitrarily high energy as they ride across the wave fronts like surfers cutting across the face of an ocean wave (see Fig. 1).

Sugihara and Midzuno[3] and Dawson *et al.*[4] have shown that classical particles trapped by a perpendicularly propagating electrostatic wave are accelerated until they detrap near the $E\times B$ velocity (cE/B). In this Letter we consider the relativistic effects introduced when the $E\times B$ velocity is greater than the speed of light (i.e., $E>B$) and when the wave's phase velocity is not small compared to c.

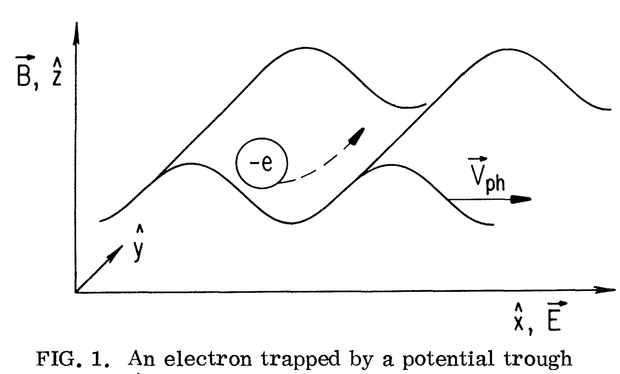

FIG. 1. An electron trapped by a potential trough moving at $\vec{V}_{ph}$ sees an electric field from the Lorentz transformation $\gamma_{ph}\vec{V}_{ph}\times\vec{B}/c$ which accelerates it across the wave front.

We begin by giving a general treatment of the trapped-particle motion analytically and numerically, followed by application of these results to the beat-wave example. We consider a longitudinal plane-wave electric field and uniform magnetic field,

$$\vec{E}=E_0\sin(kx-\omega t)\hat{x},\quad \vec{B}=B\hat{z}.$$

The equations of motion for a particle of charge q and rest mass m are given by

$$d(\gamma V_x)/dt=(qE_0/m)\sin(kx-\omega t)+\omega_c V_y, \tag{1}$$

$$d(\gamma V_y)/dt=-\omega_c V_x, \tag{2}$$

$$\gamma=(1-V_x^2/c^2-V_y^2/c^2)^{-1/2}, \tag{3}$$

where ω_c is the nonrelativistic cyclotron frequency qB/mc and V_x and V_y are velocities in the x and y directions, respectively. To solve for the particle's motion we assume that it is trapped by the wave. The criterion for the particle to be trapped can be obtained by examining the x component of the force on the particle in the *wave* frame:

$$F_x=q(E_0\sin kx_1+\gamma_{ph}V_y'B/c),$$

where $\gamma_{ph}=(1-V_{ph}^2/c^2)^{-1/2}$, $V_{ph}=\omega/k$, $x_1=x-V_{ph}t$, and V_y' is the y velocity in the wave frame. The first term of the Lorentz force is the trapping term and the second is the gyratory or detrapping term. Therefore, an initially trapped particle can never detrap if

$$\gamma_{ph}B<E_0. \tag{4}$$

For the zeroth-order motion we assume that (4) is satisfied so that we may take $V_x=V_{ph}$. Integrating equation (2) and substituting from (3) gives

$$V_y=\frac{-\omega_c V_{ph}t}{\gamma_{ph}(1+\omega_c^2t^2V_{ph}^2/c^2)^{1/2}} \tag{5}$$

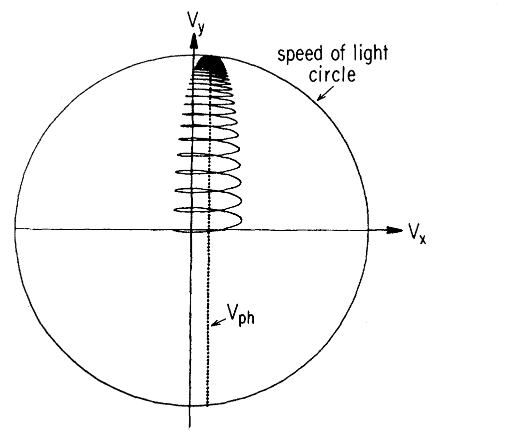

FIG. 2. Velocity-space trajectory of a particle in a low–phase-velocity wave ($V_{ph} = 0.1c$, $E_0/B = 1.5$, $\omega/\omega_c = 2$).

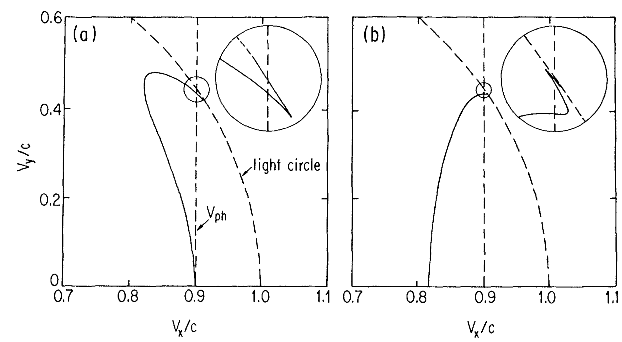

FIG. 3. Velocity-space trajectories of particles in a high–phase-velocity wave ($V_{ph} = 0.9c$, $E_0/B = 2.5$, $\omega/\omega_c = 9$) for initial velocities (a) equal to and (b) slightly below the phase velocity.

for the acceleration across the wave front. Figures 2 and 3 show the velocity-space trajectories obtained numerically for negatively charged particles trapped in low– and high–phase-velocity waves, respectively. In both cases, the particles' total velocity approaches asymptotically the speed-of-light circle as predicted by Eq. (5).

The higher-order motion observed in Figs. 2 and 3 can be represented by the first-order expression for Eq. (1):

$$\ddot{x}_1 + (\omega_B{}^2/\gamma)x_1 \approx -(\gamma_{ph}{}^2/\gamma^2)\omega_c{}^2 V_{ph}t, \tag{6}$$

where $\omega_B = (eE_0k/m)^{1/2}$ is the nonrelativistic bounce frequency. This driven-oscillator equation describes the bounce motion of a particle in the potential trough of the wave and its shift out of the bottom of the potential well due to the relativistic mass increase and the $V_y \times B$ force. From the decreasing bounce frequency and adiabatic invariance of the x motion we obtain the following expression for the bounce amplitude in velocity space:

$$\Delta V = \Delta V_0\left(1 - \frac{V_y{}^2}{c^2}\right)^{3/8}\left[1 - \frac{1}{8}\left(\frac{\gamma_{ph}{}^2 V_y B}{cE_0}\right)^2\right],$$

where V_0 is the initial velocity bounce amplitude and $(\gamma_{ph}{}^2 V_y B/cE_0)^2 \ll 1$. This accurately describes the bounce amplitude observed in Fig. 2. In the high–phase-velocity examples of Fig. 3 the acceleration is so rapid that only after the particles have neared their asymptotic values does a slow bounce motion appear. However, an initial velocity shift is visible in Fig. 3(a) as the particle falls behind the wave because of its relativistic mass increase, and can be shown from Eq. (6) to be roughly

$$V_{x1} \approx \frac{-V_{ph}{}^2/c}{(\omega/\omega_c)(E_0/\gamma_{ph}B)}.$$

For the parameters of Fig. 3(a), $V_{x1} = -0.08c$ in agreement with the figure. For particles which start out slightly slower than the wave the acceleration is more nearly monotonic as shown by Fig. 3(b).

Although the velocity of the particles is asymptotic to c, their energy continues to increase indefinitely. The total energy as a function of distance traversed across the wave front can be found by integrating (5) and eliminating t in favor of y in expression (3) for γ. Thus,

$$\gamma(y) = \gamma_{ph}{}^2 y\omega_c V_{ph}/c^2 + \gamma_{ph}. \tag{7a}$$

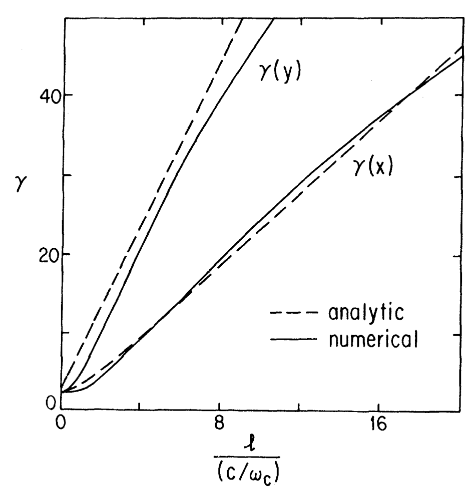

FIG. 4. Total particle energy (γmc^2) vs distance traveled (l) in the direction of the wave (x) or across the wave front (y).

Alternatively, in terms of distance in the direction of the wave, we have

$$\gamma(x)=\gamma_{\rm ph}(1+\omega_c^2x^2/c^2)^{1/2}. \qquad (7b)$$

These are plotted in Fig. 4 along with the numerical results corresponding to the particle of Fig. 3(a). It is clear from Eq. (7) that a high-phase-velocity wave is advantageous for rapidly accelerating particles in addition to minimizing the damping of the trapping wave by the thermal plasma.

We now apply our acceleration results to the example of the fast electrostatic (upper hybrid) wave which may be created by the beat-wave technique[2] or by forward Raman scattering of a single incident laser beam.[5] In this case, the phase velocity of the electrostatic wave is the group velocity of the incident wave; namely, $V_{\rm ph}=c(1-\omega_p^2/\omega_0^2)^{1/2}\simeq\omega_{\rm UH}/k\simeq\omega_p/k$ [ω_0 is the angular frequency of the incident laser; ω_p is the plasma frequency ($\omega_p^2=4\pi n_0e^2/m_e$); e is the electronic charge; $\omega_{\rm UH}^2=\omega_p^2+\omega_{ce}^2$; n_0 is the plasma density; and the subscript e denotes electron quantities]. By approximating $\omega_{\rm UH}\approx\omega_p$ we have neglected the effect of the magnetic field on the dispersion properties of the plasma wave. This is justified by the trapping inequality (4) which takes the form

$$\omega_p/\omega_{ce}>\gamma_{\rm ph}/\epsilon, \qquad (8)$$

where we have taken E_0 to be a fraction ϵ of the field given by the cold wave-breaking limit[6] ($4\pi e n_0/k$). Normally one would take ϵ to be enough less than 1 that the cold background plasma cannot be trapped [order $(1-3)V_{\rm th}/c$; $V_{\rm th}$ is the plasma thermal velocity]. In this way injected high-energy particles can be preferentially accelerated.

Finally, with $V_{\rm ph}\approx\omega_0/\omega_p$ we obtain the change in γ per unit distance from Eq. (7):

$$\Delta\gamma/\Delta y=\omega_0^2\omega_c/\omega_p^2c, \quad \Delta\gamma/\Delta x\simeq\omega_0\omega_c/\omega_p c,$$

where the latter expression is valid for $\omega_c t\gg 1$. If we multiply by the rest energy (mc^2) the mass dependences cancel, and we obtain from these and inequality (8) the following handy formulas for the rate of energy gain of either electrons or protons:

$$B_{\rm kG}/n_{16}\lambda_\mu<\epsilon, \qquad (9)$$

$$\Delta U/\Delta y=(30\ {\rm GeV/cm})(B_{\rm kG}/n_{16}\lambda_\mu)1/\lambda_\mu, \qquad (10a)$$

$$\Delta U/\Delta x=(0.1\ {\rm GeV/cm})(B_{\rm kG}/n_{16}\lambda_\mu)\sqrt{n_{16}}, \qquad (10b)$$

where $B_{\rm kG}$ is the magnetic field in units of kilogauss, n_{16} is plasma density in $10^{16}/{\rm cm}^3$, and λ_μ is the wavelength of the incident laser in microns.

Several examples are presented in Table I with length (Δx), width (Δy), and incident laser power (P_i) requirements for accelerating particles to 1 TeV. The power requirement is given in watts per square centimeter instead of watts since the total power depends on Δy and how small the laser can be focused in the z direction. The length requirements of the unmagnetized beat-wave accelerator ($\Delta x_{\rm BWA}$) corresponding to the same wavelength laser and same ϵ are also given. In contrast to the unmagnetized case, arbitrarily higher energies can (theoretically) be reached with our arrangement, the Surfatron, by merely extending the device.

An advantage of the Surfatron acceleration mechanism is that the power radiated is negligible. If $V_{\rm ph}$ is nearly c, a trapped particle's velocity is primarily in the direction of the wave while its acceleration is primarily perpendicular to the wave. Thus, the power radiated will be[7]

$$P=\frac{2}{3}\frac{e^2}{c}\gamma^6[(\dot{\vec\beta})^2-(\vec\beta\times\dot{\vec\beta})^2]\approx\frac{2}{3}\frac{e^2}{c}\gamma^6\dot{\vec\beta}^2/\gamma_{\rm ph}^2$$

($\vec\beta=\vec V/c$) which is less by the factor $\gamma_{\rm ph}^2$ than that of a conventional linear accelerator.

The realizability of a Surfatron accelerator depends on the successful extension of current technologies. To keep the laser beam from spreading, one might use a plasma wave guide with the plasma density slightly lower at the center of the channel than at the edge. To confine a diffraction-limited beam of width Δ one needs an in-

TABLE I. Sample parameters to reach 1 TeV.

n (cm^{-3})	λ (μm)	ϵ	$B_{\rm kG}$	Δy (m)	Δx (m)	$\Delta x_{\rm BWA}$ (m)	P_i (W/cm^2)
10^{17}	10	0.9	90	3	35	3500	10^{15}
10^{18}	1	0.5	50	0.6	20	850	10^{16}
10^{20}	0.3	0.2	600	0.5	5	1000	5×10^{16}

crease of plasma density at the edge of the channel by $3\times10^{11}\pi^2/\Delta^2$ cm^{-3}. For a 30-μm channel width the density rise must exceed 3×10^{17} cm^{-3}. Such confinement of a 10-μm CO_2 laser beam over a distance of 3 m has been demonstrated experimentally[8] for a beam power of 10^{12} W/cm^2 and channel width of a few millimeters. Since the accelerating electrons keep up with the light pulse, the accelerating wave needs to remain coherent only in the vicinity of the light pulse and not over the entire length of the plasma. The y width of the beam might be provided by a cylindrical lens.

Of particular interest to the beat-wave accelerator is the recent development of femtosecond laser pulses.[9] If these can be generated with sufficient intensity it should be possible to guide them over the required distance to accelerate particles. However, for such short pulses the plasma should not be subject to many of the instabilities which will degrade the performance of longer pulsed devices.

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