Observation of Sub-Poissonian Photon Statistics

R. Short and L. Mandel
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 29 April 1983)

The number of photons emitted in a short time interval by a single atom in the process of resonance fluorescence is measured, and it is shown that the probability distribution of this number is sub-Poissonian.

PACS numbers: 42.50.+q, 12.20.Fv, 32.50.+d

Among the possible quantum states of the electromagnetic field there exist numerous states for which the probability distribution $p(n)$ of the photon number n is narrower than Poissonian, or the variance $\langle(\Delta n)^2\rangle$ is smaller than the mean $\langle n\rangle$. Although one tends to think of sub-Poissonian statistics as associated more with fermions than bosons, photons can exhibit a great variety of different probability distributions when they are not in thermal equilibrium. Unlike states of the field with $\langle(\Delta n)^2\rangle \geq \langle n\rangle$, those exhibiting sub-Poissonian statistics have no classical analog, and are describable only quantum mechanically.

If $\hat{n}$ is the photon number operator,[1] it may be shown that the dispersion of $\hat{n}$ is given by[2-4]

$$\langle(\Delta\hat{n})^2\rangle = \langle\hat{n}\rangle + \int (\Delta U)^2 \varphi(\{v_{\vec{k},s}\}) d\{v_{\vec{k},s}\}, \qquad (1)$$

where $\varphi(\{v_{\vec{k},s}\})$ is the diagonal representation of the density of the operator of the field in terms of coherent states $|\{v_{\vec{k},s}\}\rangle$ ($\vec{k},s$ labels the modes), and $U \equiv \sum_{\vec{k},s} |v_{\vec{k},s}|^2$ is the c-number corresponding to the total photon number operator $\hat{n}$. It is clear from this equation that $\varphi(\{v_{\vec{k},s}\})$ cannot be a probability density whenever $\langle(\Delta\hat{n})^2\rangle < \langle\hat{n}\rangle$, so that we are dealing with an optical field without classical analog. Examples of such fields are provided by the so-called squeezed states,[5-8] and by the fields produced in harmonic generation,[9,10] in parametric processes,[11,12] and in resonance fluorescence from a coherently driven atom.[13-16]

Despite the fact that photons are readily counted with photomultipliers, no direct measurement of sub-Poissonian photon statistics has so far been reported, although the photon antibunching exhibited in the process of atomic resonance fluorescence, i.e., the tendency for fewer photons to be emitted close together in time than further apart, has been demonstrated.[17] The reason is that the statistical narrowing is usually very small, and in the atomic beam experiments fluctuations in the number of atoms cause photon fluctuations that tend to mask the sub-Poissonian character of the photons from one atom. Although they are often associated, antibunching and sub-Poissonian statistics are distinct effects that need not necessarily occur together.[18]

We wish to report the results of measurements in which sub-Poissonian photon statistics in the emission from a coherently excited atom were observed directly, both before and after certain counting corrections were made.

The apparatus has a good deal in common with that used in the earlier experiments.[17] A weak beam of sodium atoms from a thermal equilibrium oven, so weak that the individual atoms are separated on the average by about 10 μsec in time and about 1 cm in distance, provides single atoms. The beam is crossed at right angles by the circularly polarized light from a dye laser tuned to the $3^2S_{1/2}, F=2$ to $3^2P_{3/2}, F=3$ transition of sodium. The laser is stabilized in intensity to a few percent, and in frequency to 1 or 2 MHz. A prior crossing light beam prepares each sodium atom to be in the $3^2S_{1/2}, F=2$, $m_F=2$ magnetic sublevel by optical pumping in a weak magnetic field, and from here the second light beam can excite it only to the $3^2P_{3/2}, F=3$, $m_F=3$ quantum state.[17] The excited atom can spontaneously emit a photon, return to the $3^2S_{1/2}, F=2$, $m_F=2$ state, and then become excited again, and this cycle may be repeated several times during the microsecond or so that the atom takes to pass through the laser field. Some of the emitted photons are collected by an $f/0.55$ microscope objective, and imaged on photomultiplier B, where they are counted during a 200-nsec counting interval. What makes the measurement difficult is the low detection probability, and the need to keep the exciting field from becoming too strong, when the departures from Poisson statistics become difficult to observe. This means that the photon emission rate has to be kept low. As a result, a great deal of care has to be taken to reduce background counts produced by elastically scattered photons from the exciting laser beam. This is achieved by placing light traps close to the interaction region to absorb the main laser beam and some of the scattered light, and by limiting the photon acceptance angle with apertures. The phototubes are cooled to $-30°$C to reduce dark current.

Fewer than 1% of the photon counts were due to scattered light or other background in our measurements.

The essential features of the experiment, that made the sub-Poissonian statistics observable, were (a) the use of an atomic beam so weak that with high probability only one atom is present in the field of view of the microscope objective at one time, and (b) the use of an auxiliary detector for registering the arrival of the atom and gating-on the counting electronics. The latter is achieved by splitting the image formed by the microscope objective into two portions. The light from the first part, corresponding to a 50-μm-long region of the atomic beam, is directed to photomultiplier A, while the light from the second part, corresponding to an adjacent 425-μm region of the same beam, is sent to photomultiplier B. When A registers a photoelectric count, after a 90-nsec delay to allow the atom time to leave region A and enter B, the counter associated with detector B is activated for a 200-nsec-long counting interval T. It can be shown that the atom remains in region B during the measurement about 98% of the time. The number of counts n registered during the interval T is recorded by treating n as an address in a computer memory, and incrementing the number $N(n)$ stored at address n by unity. After many similar counting cycles, the numbers $N(n)$ of events n ($n=0,1,2,\ldots$) provide a measure of the probability $p(n)$ of detecting n photons through the relation $p(n)=N(n)/N$, $N\equiv\sum_{n=0}^{\infty}p(n)$. In practice, the photon counting rates were always so low that even in many hours of data acquisition $N(n)$ and $p(n)$ remained indistinguishable from zero for $n>3$, and only one event $n=3$ was observed. $N(2)$ therefore plays the dominant role in determining the width of the probability distribution $p(n)$.

The results of 11 million separate counting measurements are presented in Table I. The strength of the exciting field was such that the ratio of the atomic Rabi frequency Ω to half the natural linewidth $\beta=3.1\times10^{7}$ sec^{-1} for the transition was close to unity. The uncertainties are standard deviations based on the observed numbers $N(n)$. If the probability $p(n)$ were Poissonian with the same mean $\langle n\rangle$, the expected numbers $N_e(n)$ would be given by

$$N_e(n)=N\langle n\rangle^n e^{-\langle n\rangle}/n!\,, \qquad (2)$$

and these are shown in column 2 of Table I. It will be seen that $N_e(2)$ is significantly larger than the observed number $N(2)$ by five standard deviations, so that the observed distribution is evidently narrower than Poissonian. A convenient parameter that measures the narrowing as compared with a Poisson distribution is[15] $Q\equiv[\langle(\Delta n)^2\rangle-\langle n\rangle]/\langle n\rangle$, which is negative for $\langle(\Delta n)^2\rangle<\langle n\rangle$. From the raw data in column 1 of Table I we readily find that $Q=-0.001\,83\pm0.000\,38$, which provides statistically significant evidence for a sub-Poissonian distribution.

TABLE I. Observed and corrected experimental results for photon statistics and comparison with a Poisson distribution.

	Observed	Poisson	Corrected	Poisson
N	11 025 000	11 025 000	10 927 000	10 927 000
$N(0)$	10 953 136	10 953 201	10 859 079	10 859 165
$N(1)$	71 695	71 565	67 797	67 624
$N(2)$	168	234	123	211
$N(3)$	1	0.5	1	0.4
$\langle n\rangle$	0.006 53 ±0.000 03	0.006 53	0.006 23 ±0.000 03	0.006 23
Q	−0.001 83 ±0.000 38	0	−0.002 52 ±0.000 40	0

However, a number of corrections should be applied to the measured data. These include the correction for dead time of the counting electronics, the correction for spurious multiple counts of the photodetector, and the corrections for counts produced by background and by the occasional presence of an unwanted atom in the field of view.

Our counting circuits had a dead time of $T_D\approx 8$ nsec after each count. If the photons arrived completely at random (and of course they do not), this would cause the numbers $N(2)$ to be too small by the factor $1+2T_D/T$. It is worth noting that even if $N(2)$ were increased by this factor the probability $p(n)$ would still be clearly sub-Poissonian. However, because of the nonrandom photon emissions, this factor substantially overestimates the effect of dead time. A more realistic assessment of the correction can be obtained from the conditional probability $\mathcal{P}_c$ that given one photon count at some time t, a second photon count arrives during the subsequent time interval T_D when the counter does not respond. $\mathcal{P}_c$ is expressible in the form[4]

$$\mathcal{P}_c=\alpha\langle I\rangle[T_D+\int_0^{T_D}\lambda(\tau)d\tau], \qquad (3)$$

where $\langle I\rangle=\langle n\rangle/\alpha T$ is the average intensity or the photon arrival rate at the detector, α is the detector quantum efficiency, and $\lambda(\tau)$ is the normalized, normally ordered intensity correlation function, whose form has been calculated.[13] The first term in Eq. (3) gives the probability associated with random events, while the second term corrects this for the nonrandom statistics, and we find that

$$\mathcal{P}_c = \langle n\rangle (T_D/T)[\tfrac{1}{6}(\Omega^2/\beta^2+2)(\beta T_D)^2 + O(\beta T_D)^3]. \quad (4)$$

The number of events $N(n)$, corrected for those lost due to dead time, is then obtained by increasing the observed $N(n)$ by $\mathcal{P}_c N(n-1)$ $(n=2,3,\ldots)$. Numerical estimates show that $\mathcal{P}_c \approx 8\times10^{-6}$, so that very few events are actually expected to be lost because of counter dead time.

The photon-counting corrections for spurious afterpulsing, for background, and for the presence of an unwanted atom in the field of view act in the opposite direction, in that they tend to make the distribution narrower and Q more negative. An auxiliary experiment with white light showed that any given photoelectric pulse has a probability $q\approx 5\times10^{-4}$ of being followed by a spurious pulse within a time interval $T=200$ nsec. It follows that a certain number of events $N(2), N(3)$ are therefore expected to be spurious, and $N(n)$ should be reduced by $(q/n)N(n-1)$ for $n=2,3$, while $N(1)$ is increased by $\frac{1}{2}qN(1)$.

In order to correct for background, we compare the average counting rates $R_A \approx 41\,000$ sec^{-1}, $R_B\approx 32\,670$ sec^{-1} of the two detectors when there is an atom in the corresponding field of view with the rates $R_A' \approx 270$ sec^{-1}, $R_B'\approx 1827$ sec^{-1} of the same two counters averaged over a long time, and with the background rates $R_A''\approx 2.4$ sec^{-1}, $R_B''\approx 15.8$ sec^{-1} measured with the atomic beam turned off. We first note that the starting sequence initiated by counter A has a probability $\mathcal{P}_s = R_A''/R_A' \approx 8.9\times10^{-3}$ of being due to a background count. The counts registered by B during these false starts would then also be caused by background and should be subtracted. If they are distributed in an approximately Poissonian manner, with mean $R_B''T\approx 3.2\times10^{-6}$, then the measured $N(n)$ should be reduced by $NP_s(R_B''T)^n/n!$ Actually, there is evidence that the background is non-Poissonian, but the correction is negligible in any case for $n\geqslant 1$. Secondly, some counts registered by counter B are attributable to background even when the counting sequence is initiated by an atomic photon, and this leads to the further reduction of $N(n)$ by $R_B''TN(n-1) + (R_B''T)^2N(n-2)$ [with $N(n)=0$ for $n<0$].

In order to correct for the counts produced by the occasional presence of an unwanted atom, we start from the probability $P_{at}\equiv(R_B'-R_B'')/(R_B-R_B'')\approx 0.055$ that there is an atom present in the field of view of detector B at any arbitrary moment. This atom has a probability $P_{ct}\equiv(R_B-R_B'')T\approx 6.5\times10^{-3}$ of generating a photon count during the interval T. It follows that some of the events $N(n)$ $(n=1,2,\ldots)$ registered by counter B are attributable to atoms other than those responsible for triggering counter A. We can correct for the counts produced by these unwanted atoms by subtracting $P_{at}P_{ct}N(n-1) + P_{at}^2P_{ct}^2N(n-2)$ $(n=1,2,\ldots)$ from $N(n)$ and adding $P_{at}P_{ct}N(0)$ to $N(0)$. The numbers obtained after the various corrections are made are also shown in Table I, and they clearly demonstrate that the photon counting distribution $p(n)$ is sub-Poissonian.

Finally, we attempt to make some comparisons of the observed numbers with the theory of atomic resonance fluorescence. The average rate $\mathcal{R}$ at which a two-level atom in a resonant coherent field emits photons is given by[13,14]

$$\mathcal{R} = \left(\frac{\frac{1}{4}\Omega^2/\beta^2}{\frac{1}{2}\Omega^2/\beta^2+1}\right)2\beta. \quad (5)$$

Some fraction f of these spontaneously emitted photons is collected by the optical system, a fraction g of the collected photons reaches the counting photomultiplier B, and a fraction α_B of the incident photons gives rise to a photoelectric count. α_B is the quantum efficiency of the detector, which is estimated to be 15% at the 5890-Å wavelength of the atomic transition. It follows that the expected counting rate of detector B, after subtraction of background, is $R_B - R_B'' = \mathcal{R}fg\alpha_B$. f was calculated to be 0.067 for our microscope objective,[17] and from the transmissivities of the various glass windows and lenses we estimate that $g\sim 0.4$. With $\beta = 3.1\times10^7$ sec^{-1}, $\Omega/\beta=1$, this then leads to $R_B-R_B''\approx 40\,000$/sec, with an estimated uncertainty of about 25%, which is in fair agreement with the observed values. The expected value of the parameter Q that measures the departure from Poisson statistics is given by[15]

$$Q = fg\alpha_B\mathcal{R}T\{(2/T^2)\int_0^T dt'\int_0^{t'} dt''[1+\lambda(t'')]-1\}, \quad (6)$$

and evaluation of the integral with $\lambda(\tau)$ given by Refs. 13 and 14 leads to[15]

$$Q\approx(-0.59)fg\alpha_B \approx -0.0023\pm 25\%, \quad (7)$$

and this is also in reasonable agreement with the observed value given in Table I.

We conclude that our results are consistent with the QED theory of resonance fluorescence, and within the somewhat large statistical uncertainties resulting from the low frequency of detecting photon pairs, that the sub-Poissonian character of the emitted photons is confirmed.

This work was supported by the National Science Foundation.

---

[1]We identify Hilbert space operators by the caret, $\hat{O}$.

[2]R. J. Glauber, Phys. Rev. 131, 2766 (1963).

[3]E. C. G. Sudarshan, Phys. Rev. Lett. 10, 277 (1963); J. R. Klauder and E. C. G. Sudarshan, *Fundamentals of Quantum Optics* (Benjamin, New York, 1968).

[4]L. Mandel and E. Wolf, Rev. Mod. Phys. 37, 231 (1965).

[5]D. Stoler, Phys. Rev. D 1, 3217 (1970).

[6]H. P. Yuen, Phys. Rev. A 13, 2226 (1976); H. P. Yuen and J. H. Shapiro, IEEE Trans. Inf. Theory 24, 657 (1978), and 26, 78 (1980).

[7]D. F. Walls and P. Zoller, Phys. Rev. Lett. 47, 709 (1981).

[8]L. Mandel, Phys. Rev. Lett. 49, 136 (1982), and Opt. Commun. 42, 437 (1982).

[9]D. F. Walls and C. T. Tindle, J. Phys. A 5, 543 (1972); H. D. Sinaan and R. Loudon, J. Phys. A 8, 539 (1975).

[10]S. Kielich, M. Kozierowski, and R. Tanas, in *Coherence and Quantum Optics IV,* Proceedings of the Fourth Rochester Conference, edited by L. Mandel and E. Wolf (Plenum, New York, 1978), p. 511.

[11]B. R. Mollow and R. J. Glauber, Phys. Rev. 160, 1076, 1097 (1967).

[12]D. Stoler, Phys. Rev. Lett. 33, 1397 (1974).

[13]H. J. Carmichael and D. F. Walls, J. Phys. B 9, L43, 1199 (1976).

[14]H. J. Kimble and L. Mandel, Phys. Rev. A 13, 2123 (1976).

[15]L. Mandel, Opt. Lett. 4, 205 (1979).

[16]R. J. Cook, Phys. Rev. A 23, 1243 (1981); D. Lenstra, Phys. Rev. A (to be published); Surendra Singh, to be published.

[17]H. J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. 39, 691 (1977), and Phys. Rev. A 18, 201 (1978); M. Dagenais and L. Mandel, Phys. Rev. A 18, 2217 (1978).

[18]A somewhat fuller discussion of this point will be given in the paper by R. Short and L. Mandel, in "Coherence and Quantum Optics V," Proceedings of the Fifth Rochester Conference, edited by L. Mandel and E. Wolf (Plenum, New York, to be published).