Is Magnetic Flux Quantized in a Toroidal Ferromagnet?

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Total magnetic flux trapped in a tiny toroidal magnet is measured by an electron holography technique to test predicted flux quantization. The experimental results show that the magnetic flux is quantized neither by h/e units nor by $h/2e$ units. This fact provides further evidence for the existence of the Aharonov-Bohm effect.

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The significance of the Aharonov-Bohm effect[1] (AB effect) has recently increased, since it gives direct evidence of the validity of local gauge theory in electromagnetism.[2] The AB effect states that a phase difference between two electron beams is produced proportional to the enclosed magnetic flux, even if they never touch the magnetic field. However, the existence of the AB effect has been questioned in both theoretical and experimental aspects.[3] Our recent experiment[4] with electron holography dispelled one of the most caustic criticisms of confirmation experiments in the past,[5] i.e., that leakage magnetic fields from finite solenoids would have produced the imaginary AB effect. A recent experiment by Möllenstedt, Schmid, and Lichte[6] dispelled the criticism that the AB effect may be due to the penetration effect of an electron beam into the solenoid.

A new problem has now come to the fore[7]: the possible quantization of magnetic flux,[8] which would result in a drastic change in the interpretation of the AB effect. An impenetrable toroidal magnet containing a magnetic flux quantized in h/e units produces no physically observable change in electron waves around the magnet. This would mean that the AB effect does not exist if the flux number cannot be counted, while the smaller quantum of $h/2e$[8] makes the AB effect observable. This could not be verified in the former experiment,[4] because leakage flux produced an error of $\pm h/2e$ in the flux measurement.

In this paper, we report on a new experiment to determine whether magnetic flux is quantized or not in toroidal magnets. Experimental procedures were almost the same as before,[4] but special attention was paid to the accuracy requirement.

The ferromagnetic samples of toroidal geometry were fabricated with use of photolithography. To minimize leakage flux, the toroidal magnets were made circular instead of square. Photoresist 0.75 μm thick was coated on a glass substrate that had previously been covered with double layers of evaporated NaCl and carbon [Fig. 1(a)]. Negative patterns were formed in the photoresist, and then Permalloy films 50–100 Å thick were evaporated onto them. When the photoresist was dissolved, only the positive patterns of Permalloy remained on the substrate [Fig. 1(b)]. These patterns on carbon film were floated off on a warm water surface by dissolution of the NaCl layer and placed on a supporting mesh.

The samples were observed by both ordinary and Lorentz electron microscopy. Only magnets in which magnetization rotated smoothly were selected as samples. Two examples of the Lorentz micrographs are shown in Fig. 2. The black ring near the inner circumference of the toroid

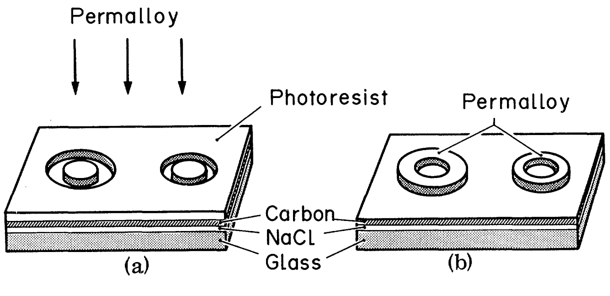

FIG. 1. Method of fabricating toroidal magnets. (a) Negative photoresist pattern. (b) Positive permalloy pattern.

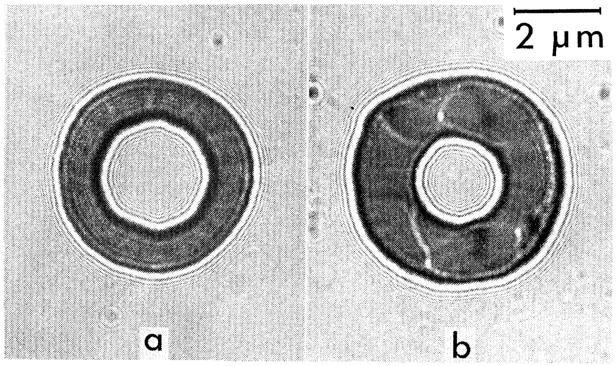

FIG. 2. Lorentz micrographs of toroidal magnets (a) without leakage flux, (b) with leakage flux.

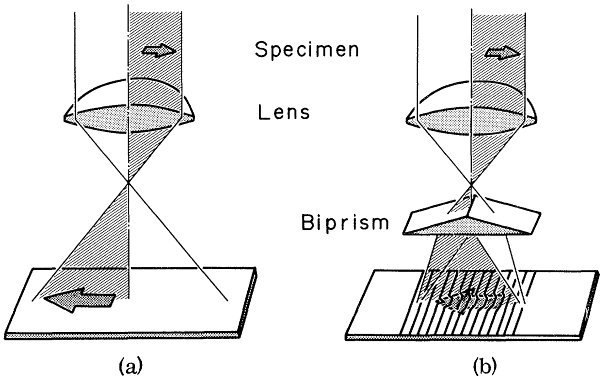

FIG. 3. Schematic of hologram formation with an electron biprism. (a) Electron microscopic image. (b) Off-axis electron hologram.

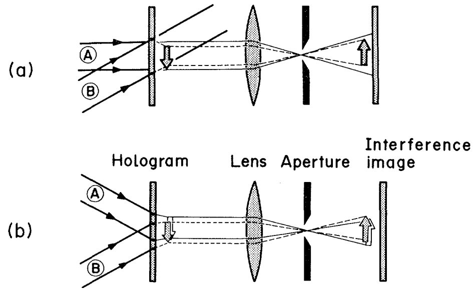

FIG. 4. Schematic of optical reconstruction for interference microscopy. (a) Normal interference micrograph. (b) Doubly amplified interference micrograph.

in Fig. 2(a) shows that magnetization rotates clockwise. However, in Fig. 2(b), the black and white streaks indicate the existence of several domains.

Off-axis holograms of the samples were formed in a 100-kV field-emission electron microscope.[9] The principle of hologram formation is shown in Fig. 3. A sample is situated in only one-half of the specimen plane. A collimated electron beam illuminates it, and the image is formed through a lens. A reference beam passing close to the sample reaches the image plane directly [Fig. 3(a)]. These two beams are superimposed to form an image hologram with an electron biprism [Fig. 3(b)].

Optical reconstruction for interference microscopy was carried out with a He-Ne laser (wavelength: 6328 Å). A schematic of the reconstruction is shown in Fig. 4. The collimated laser beam is split into two beams by a Mach-Zehnder type interferometer. When they illuminate a hologram, each beam is partly transmitted and partly diffracted in two directions, where the original image and its conjugate are reconstructed. When the directions of the two incident beams, A and B, are adjusted so that the transmitted beam of A and the diffracted beam of B travel along an optical axis to pass through an aperture [Fig. 4(a)], a normal interference micrograph is obtained by superposition of the reconstructed image and the plane wave.

The higher-order diffracted beams generated by a high contrast hologram can be utilized to amplify the phase information; i.e., $2n$-times amplification can be attained by use of the $\pm n$th order diffracted beams [Fig. 4(b)].

First, magnetic flux leaking from a sample was confirmed to be less than the allowable value of $h/4e$. The sample interference micrograph shown in Fig. 5 has a phase amplification of 4. Since the contour lines directly indicate magnetic flux flow in $h/4e$ units,[10] it can be seen from this micrograph that the amount of leakage flux is less than $h/4e$.

Total magnetic fluxes inside the toroidal magnets with negligible leakage fields were measured in interferograms obtained by slightly tilting the two laser beams in the reconstruction system so that parallel fringes could be observed in the flux-free part of the image. Flux quantization in h/e units could then be checked by ascertaining whether the interference fringes are on the same

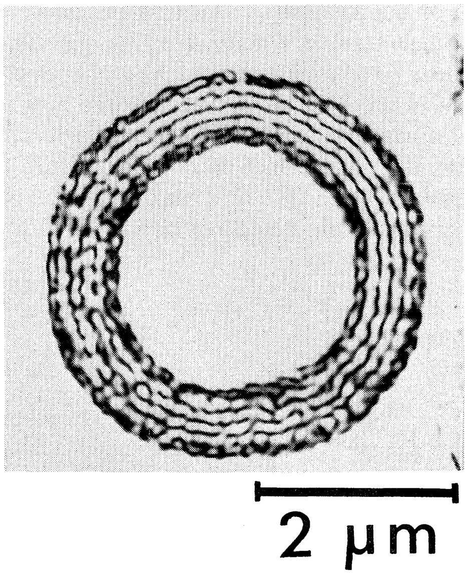

FIG. 5. Interference micrograph of a toroidal magnet (four-times phase amplification). A magnetic flux of $h/4e$ flows between two adjacent contour lines. No leakage flux is observed even in this four-times phase-amplified interference micrograph.

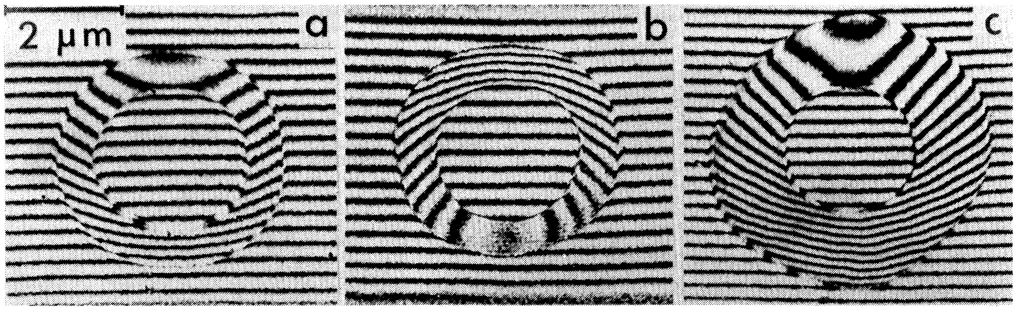

FIG. 6. Interferograms of toroidal magnets. (a) $\Phi = 1.2(h/e)$. (b) $\Phi = 2.0(h/e)$. (c) $\Phi = 2.8(h/e)$. Fringes are, in general, not on the same straight lines in the two spaces inside and outside a toroid, except for accidental coincidence (b). This proves that magnetic flux is not quantized in h/e units.

straight lines in spaces both inside and outside the toroid. Only three examples of different magnets are shown in Fig. 6. In general, interference fringes are not on the same straight lines in the two spaces. Therefore, it is concluded that the magnetic flux is not quantized in h/e units. Fringe steps at the magnet edges are due to refraction effects and have no influence on the measurement, since the fringe shift at the outer edge is always cancelled at the inner edge.

The possibility of the smaller quantum, $h/2e$, can also be checked from these interferograms. However, this can be done more precisely from doubly phase-amplified interference micrographs, such as those shown in Fig. 7. For flux quantization of $h/2e$, the interference fringes must be on the same straight lines in both spaces. The interferograms in Fig. 7 thus show that magnetic flux is not quantized in $h/2e$ units either.

Thus, magnetic flux in a toroidal magnet was determined to have a continuous value within an experimental error of $\pm h/10e$. The error can be estimated from the deviation of the fringes observed in the free spaces in Figs. 6 and 7 from straight lines.

These experimental results negate the possibility of magnetic-flux quantization in a toroidal magnet. Furthermore, this gives further evidence for the existence of the AB effect, even in the case of an impenetrable magnet.

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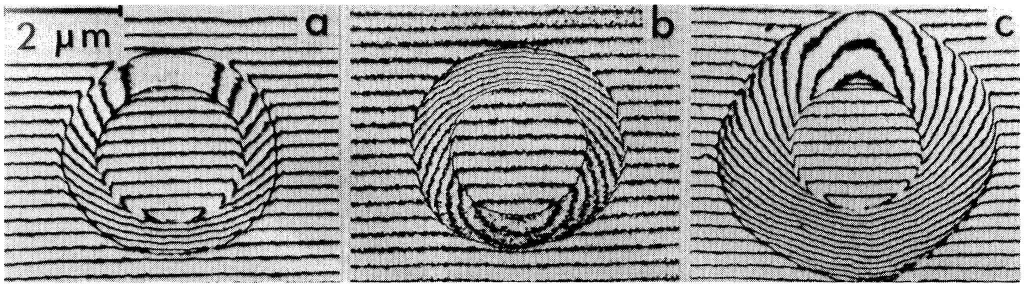

FIG. 7. Interferograms of toroidal magnets (two-times phase amplification). (a) $\Phi = 1.2(h/e)$. (b) $\Phi = 2.0(h/e)$. (c) $\Phi = 2.8(h/e)$. These doubly phase-amplified interferograms show that magnetic flux is not quantized in $h/2e$ units either.

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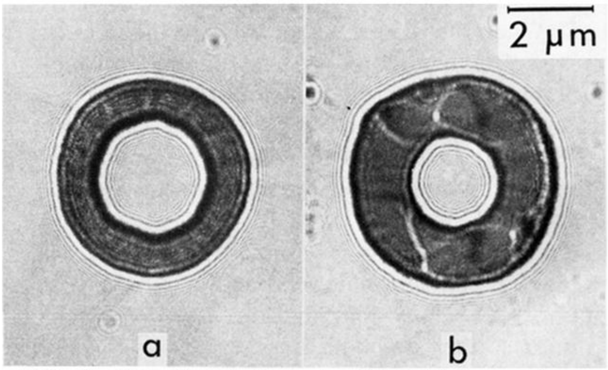
[10] A. Tonomura *et al.*, Phys. Rev. Lett. 44, 1430 (1980). Flux quantization in h/e units means that the total number of the contour lines (magnetic lines of force) in normal interference micrographs is integer. If interference micrographs were to be obtained with a monopole beam, electric lines of force could be observed as every electric flux of e. In this case, their total number is undoubtedly integer.



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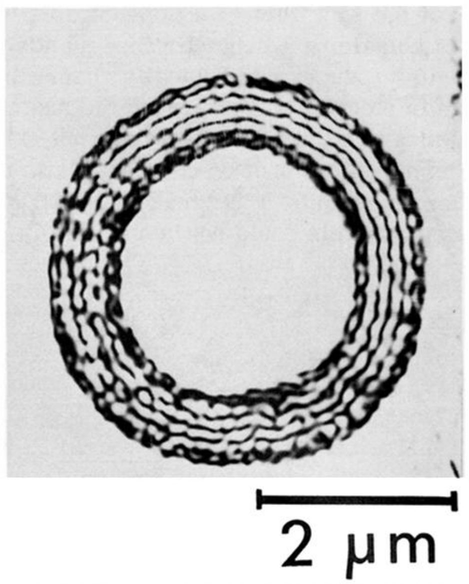

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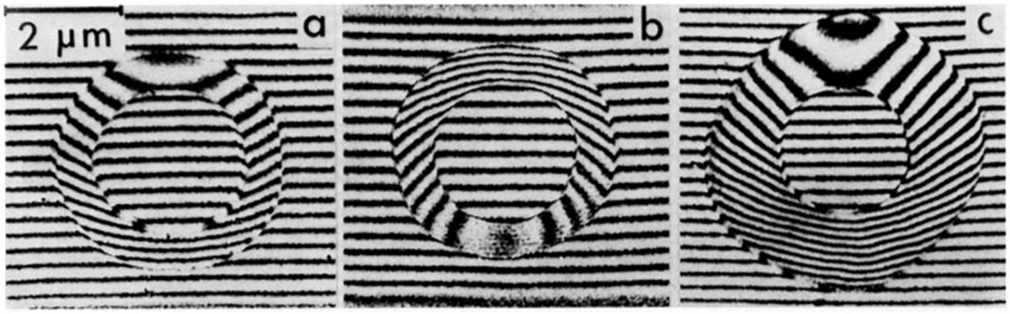

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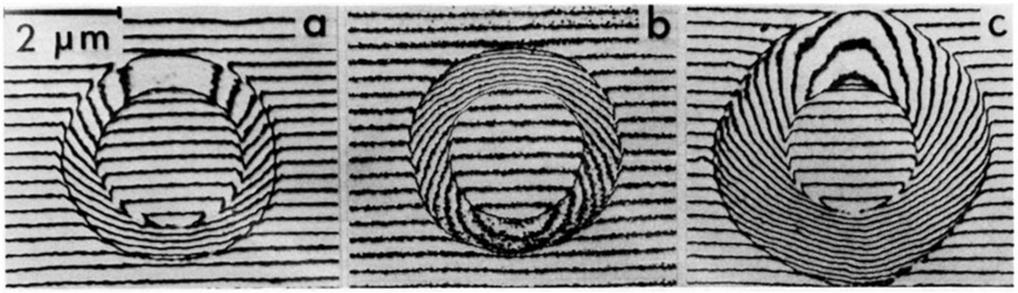

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