PHYSICAL REVIEW LETTERS

VOLUME 51 | 1 AUGUST 1983 | NUMBER 5

$N=1$ Supergravity with Nonminimal Coupling: A Class of Models

Ngee-Pong Chang, Stephane Ouvry,[(a)] and Xizeng Wu
Physics Department, City College of the City University of New York, New York, New York 10031
(Received 6 April 1983)

The component formulation of the Wess-Zumino superspace action leads, when compared with the general results of Cremmer *et al.*, to a nonminimal version of $N=1$ supergravity. A new superpotential, $g_2(z)$, has been found that gives a scalar potential $V(z)$ that is even in z and which breaks supersymmetry inside the circle of radius $\sqrt{6}/\kappa$ in the complex $\langle z\rangle$ plane.

PACS numbers: 04.50.+h

Supersymmetric grand unified theories have had problems with phenonemology that locally supersymmetric theories, calling on the residual effects of gravitation, might well solve. These problems include the degeneracy of the spontaneously broken vacua, the SU(2) ⊗ U(1) symmetry breaking, and the s-quark, s-lepton, and gaugino[1] masses. The mechanism proposed by Ovrut and Wess[2] introduced the effect of the $N=1$ Wess-Zumino supergravity into supersymmetric (SUSY) grand unified theory. An explicit soft SUSY-breaking term is generated as a result of this supergravity. Weinberg and others[3-5] showed that supergravity can lift the degeneracy of the global SUSY vacuum and trigger the SU(2) ⊗ U(1) symmetry breaking.

Phenomenological attempts in this direction have mostly been based on the so-called minimal coupling of matter and supergravity.[4,5] Minimal models may be a promising approach but they have encountered problems with anti-de Sitter vacua[5] and with stability of the gauge hierarchy.[6]

In this paper, we approach the problem from the point of view of the superspace action. Making use of the extensive reduction work of Cremmer *et al.*[4] we are able to go freely between choices of the superspace action and the effective action after integrating out the superspace Grassmann elements. In this way, it has been shown that what is called the minimal action in reality is not simple, while what is simple from the superspace action point of view leads to the so-called nonminimal coupling.

Our nonminimal model is based on the formalism of the $N=1$ Wess-Zumino supergravity.[7] We will show that it generalizes the Ovrut-Wess mechanism and improves upon it in two respects: Instead of a negative infinite cosmological constant, we have a zero cosmological constant and the SUSY-breaking strength in our effective theory is just the mass of the gravitino rather than the arbitrary parameter, Δ, in Ref. 2. It should be emphasized that, in contrast to the Ovrut-Wess mechanism, in our theory the supersymmetry is broken *spontaneously* and not explicitly.

We begin with the general action of the coupled Yang-Mills matter-supergravity system:

$$A=\int d^4x\, d^4\theta\, E\{\Phi(Te^VT)+\mathrm{Re}[g(T)/R]\}+\int d^4x\, d^4\theta\, E\,\mathrm{Re}[f_{\alpha\beta}(T)W^\alpha W^\beta/R], \tag{1}$$

where V is the gauge multiplet of the group G, T_i are chiral superfields, $g(T)$ is the gauge-invariant superpotential, Φ is a gauge-invariant real function of T_i and T^{+i}, E is the vierbein determinant, R is the chiral scalar curvature superfield, W is a gauge-invariant chiral superfield which contains the vector field strength, and $f_{\alpha\beta}$ is a chiral superfield transforming as the symmetric product of the adjoint representation of G. The tree-level potential for the scalar fields, following Cremmer *et al.*, may be written as

$$e^{-1}V = -\frac{1}{\kappa^4}\exp(-\mathcal{G})[3 + (\mathcal{G}''^{-1})_i{}^j\mathcal{G}'^i\mathcal{G}_{,j}] + \tfrac{1}{2}(D^\alpha)^2, \tag{2}$$

where

$$\mathcal{G} = 3\ln(-\tfrac{1}{3}\kappa^2\Phi) - \ln(\tfrac{1}{4}\kappa^6|g|^2) \tag{3}$$

and

$$(D^\alpha)^2 = \mathrm{Re}\tilde{g}^2\frac{1}{\kappa^4}f_{\alpha\beta}{}^{-1}(\mathcal{G}'^i T^\alpha{}_i{}^j z_j)(\mathcal{G}'^k T^\beta{}_k{}^l z_l), \tag{4}$$

where $\tilde{g}$ is the gauge coupling constant, $\mathcal{G}'^i = \partial\mathcal{G}/\partial z_i$, and the z's stand for the scalar components of the chiral superfields.

The popular minimal coupling corresponds to the choice

$$\Phi(z,z^*) = -(3/\kappa^2)\exp(-\tfrac{1}{6}\kappa^2 z_i z^{*i}),\quad f_{\alpha\beta} = +\delta_{\alpha\beta} \tag{5}$$

resulting in the scalar potential

$$e^{-1}V = \tfrac{1}{2}\exp(\tfrac{1}{2}\kappa^2 z_i z^{*i})[\,|\partial g/\partial z_i + \tfrac{1}{2}\kappa^2 z^{*i}g|^2 - \tfrac{3}{2}\kappa^2|g|^2] + \tfrac{1}{2}(D^\alpha)^2. \tag{6}$$

From the point of view of Eq. (1), a far simpler form to take for Φ, one that is consistent with the decoupled gravity and global supersymmetry limit, is[8]

$$\Phi = -(3/\kappa^2)[1 - \tfrac{1}{6}\kappa^2 S^{+i}S_i - \tfrac{1}{6}\kappa^2 T^{+a}e^V T_a - \tfrac{1}{6}\kappa^2(c^{*i}S_i + \mathrm{H.c.})], \tag{7}$$

where we have let T_a be a set of chiral gauge multiplets while S_i refers to the set of chiral gauge-singlet superfields. This choice of Φ leads to a space-time action that is identical with that derived from the $N=1$ Wess-Zumino supergravity.

Upon performing of the necessary Weyl rescaling, the scalar potential now reads[8]

$$e^{-1}V = \frac{1}{2}\frac{1}{I^2}\Bigg[\hat{g}^i\hat{g}_j\left(\delta_j{}^i - \frac{\frac{1}{6}\kappa^2(z^{*j}+c^{*j})(z_i+c_i)}{1+\frac{1}{6}\kappa^2 c_i c^{*i}}\right) + \hat{g}^a\hat{g}_b\left(\delta_a{}^b - \frac{\frac{1}{6}\kappa^2 y_a y^{*b}}{1+\kappa^2 c_i c^{*i}}\right)$$
$$-\left(\tfrac{1}{6}\kappa^2\hat{g}^i\hat{g}_b y^{*b}\frac{(z_i+c_i)}{1+\frac{1}{6}\kappa^2 c_i c^{*i}} + \mathrm{H.c.}\right) - \tfrac{3}{2}|g|^2/I\Bigg] + \tfrac{1}{2}(D^\alpha)^2, \tag{8}$$

where $I = -\frac{1}{3}\kappa^2\Phi(z,y)$ and the z's and y's refer to the scalar components of the chiral superfields S and T. Here

$$\hat{g}^i = \partial g/\partial z_i - 3(\partial\ln I/\partial z_i)g \tag{9}$$

and

$$(D^\alpha)^2 = \tilde{g}^2\kappa^{-4}(\mathcal{G}'^i T^\alpha{}_i{}^j z_j)(\mathcal{G}'^k T^\alpha{}_k{}^l z_l). \tag{10}$$

The coupling between matter and supergravity is a nonminimal one because the kinetic energy term for the z_i has the noncanonical form

$$-\frac{e}{2}\left[\delta_j{}^i\left(1 - \frac{\kappa^2}{6}z_i z^{*i}\right) + \frac{\kappa^2}{6}z_j z^{*i}\right]\frac{D_\mu z^j D^\mu z_j}{(1-\frac{1}{6}\kappa^2 z_i z^{*i})^2}\ . \tag{11}$$

The theory here, thus, makes sense only if $\langle z_i\rangle$ satisfies the constraint

$$\kappa^2|\langle z_i\rangle|^2 < 6. \tag{12}$$

Before we put in explicit GUT fields for model building, which will be done elsewhere, it is instructive to consider a class of superpotential with only one singlet complex field, z:

$$g_2(z) = \sum_{n=0}^{\infty} a_n z^n, \tag{13}$$

where the a_n are complex c numbers. Note that we do not assume *a priori* that g_2 is a cubic polynomial since supergravity theory is in any case nonrenormalizable. Also we should note that the Polony superpotential or the O'Raifertaigh mechanism will lead to scalar potentials that are unbounded from below in this nonminimal model.

A very interesting feature of the potential as given in Eq. (8) is that, in contrast with the minimal case, it can be made invariant under the reflection of the complex field, $z \to -z$, if we set c_i equal to zero and restrict the form of $g_2(z)$. For this analysis it is convenient to rewrite Eq. (8) as[9]

$$e^{-1}V = \frac{1}{(1-\frac{1}{6}\kappa^2 z_i z^{*i})^2}\left(\frac{1}{2}\left|\frac{\partial g}{\partial z_i}\right|^2 - \frac{3}{4}\kappa^2\left|g - \frac{1}{3}z_i\frac{\partial g}{\partial z_i}\right|^2\right). \tag{14}$$

Namely, we have derived the following:

Theorem. If $V(z)=V(-z)$ for z in the complex plane, then

$$g_2(z) = a_0(1+\tfrac{1}{2}\kappa^2 z^2) + a_1 z(1+\tfrac{1}{18}\kappa^2 z^2). \tag{15}$$

If, furthermore, $V=0$ at the minimum then $V(z) = 0$ and

$$g_2(z) = a_0(1 \pm \kappa_z/\sqrt{6})^3. \tag{16}$$

Proof: By direct inspection from Eq. (14), it is easy to derive first that invariance under $z \to -z$ in the complex z plane requires

$$a_n = 0, \quad \text{for } n>3, \quad a_1 a_2^* = \tfrac{1}{2}\kappa^2 a_0 a_1^*, \quad a_2 a_3^* = \tfrac{1}{18}\kappa^2 a_1 a_2^*. \tag{17}$$

Since the overall phase of g_2 is irrelevant, a_0 can always be chosen real. Next, the relative phase of a_1 may be compensated by a choice of the phase of z. Equation (17) then completely fixes the phases of $a_{2,3}$ to be real. The scalar potential then reads

$$e^{-1}V = \tfrac{1}{2}(a_1^2 - \tfrac{3}{2}\kappa^2 a_0^2)\left(1 - \tfrac{2}{3}\kappa^2(\mathrm{Im}z)^2\frac{1}{(1-\frac{1}{6}\kappa^2 zz^*)^2}\right). \tag{18}$$

If $\rho = a_1^2 - \frac{3}{2}\kappa^2 a_0^2 < 0$, the minimum is obtained for $\mathrm{Im}z = 0$. However, at the minimum, V is then negative. If $\rho > 0$ the potential is not bounded from below. For $\rho = 0$, i.e.,

$$a_1^2 = \tfrac{3}{2}\kappa^2 a_0^2, \tag{19}$$

V vanishes *identically* and $g_2(z)$ is given by Eq. (16).

Thus if we require invariance under reflection in the complex z plane and zero cosmological constant, the resulting V is completely flat. Supersymmetry is actually broken by g_2, as may be demonstrated by evaluating Eq. (9),

$$\hat{g}_2(z) = a_1(1 \pm \kappa z/\sqrt{6})^2(1 \pm \kappa z^*/\sqrt{6})/(1 - \tfrac{1}{6}\kappa^2 zz^*). \tag{20}$$

For all z satisfying the constraint Eq. (12), $\hat{g}_2$ is not zero.

The flat V implies that in the effective theory in the limit as $\kappa \to 0$ the z field is absolutely massless. This feature of the theory has, however, no direct phenomenological implications as long as the coupling of the scalar singlet with matter is small. In conventional treatments, where g is set equal to

$$g = g_1(y) + g_2(z), \tag{21}$$

the grand unified theory fields contribute to g_1 while the singlet fields contribute only to g_2 and induced couplings between z and y are indeed small.

The total potential $V(g_1+g_2)$ is then given by

$$e^{-1}V(g_1+g_2) = \tfrac{1}{2}[1 - \tfrac{1}{6}\kappa^2(y_a y^{*a} + zz^*)]^{-2}\{|\partial g_1/\partial y_a|^2 - \tfrac{3}{2}\kappa^2|G_1(y)|^2 - \tfrac{3}{2}\kappa^2[G_1(y)G_2^*(z) + \mathrm{H.c.}]\} + (D^\alpha)^2/2, \tag{22}$$

where

$$G_1(y) = g_1 - \tfrac{1}{3}y_a\,\partial g_1/\partial y_a, \quad G_2(z) = g_2 - \tfrac{1}{3}z\,\partial g_2/\partial z = a_0(1 \pm \kappa z/\sqrt{6})^2. \tag{23}$$

For there to be a residual effect as $\kappa \to 0$, it is necessary that

$$a_0 = m/\kappa^2. \tag{24}$$

It then follows that the gravitino mass is given by

$$m_{3/2}^2 = \kappa^{-2}\exp(-\mathcal{G}) \tag{25}$$

$$= \tfrac{1}{4}m^2(1+a)^3(1+a^*)^3/(1-aa^*)^3, \tag{26}$$

and that the effective theory finally reads

$$\lim_{\kappa\to 0} e^{-1}V(g_1+g_2)=\frac{1}{2}\,\frac{1}{(1-aa^*)^2}\left(\left|\frac{\partial g_1}{\partial y_a}\right|^2 \mp 3m_{3/2}\left[\frac{(1-aa^*)^{3/2}G_1(y)}{(1+a^*)^{-1/2}(1+a)^{3/2}}+\mathrm{H.c.}\right]\right)+\frac{(D^\alpha)^2}{2}, \tag{27}$$

where we have now reparametrized

$$\langle z\rangle = a\sqrt{6}/\kappa \text{ with } |a|<1 \tag{28}$$

and, because a can range over both positive and negative values, it is sufficient to consider only the plus sign in Eq. (23). The residual effect of supergravity is entirely in the parameters a and $m_{3/2}$. In Eq. (27), the sign for the second term is model dependent; for example, the minus sign has been chosen in Ref. 3. It is important to note that the minimum of the potential [Eq. (27)] occurs always for $|a|<1$, whatever the model. This is because the presence of softly SUSY-breaking terms will make at least one of the $\langle|\partial g_1/\partial y_a|\rangle$ not equal to zero.

To conclude, we emphasize again that with this approach from a simple choice of superspace action we have derived a natural generalization of the Ovrut-Wess mechanism. We find that a simple reflection principle leads to a zero cosmological constant and it results in a SUSY-breaking strength in the effective theory that is the mass of the gravitino.

Two of us (N.P.C. and X.W.) acknowledge support from the City University of New York Professional Staff Congress–Board of Higher Education Awards Program. N.P.C. would also like to thank the Eppley Foundation for support. One of us (S.O.) would like to thank the High Energy group at the City College for their warm hospitality. X.W. would like to acknowledge discussions with Professor B. Ovrut while S.O. acknowledges discussions with Professor R. Mohapatra.

[a] On leave from Institut de Physique Nucléaire, Orsay, France.

[1] We use "s-quark," "s-lepton," and "gaugino" to designate the supersymmetry partners of the quark, the lepton, and the gauge boson, respectively.

[2] B. A. Ovrut and J. Wess, Phys. Lett. 112B, 347 (1982).

[3] S. Weinberg, Phys. Rev. Lett. 50, 387 (1983); B. A. Ovrut and J. Wess, Institute for Advanced Study report, 1982 (to be published); X. Z. Wu, City College of New York Report No. CCNY-HEP-82/17 (unpublished).

[4] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello, and P. Van Nieuwenhuizen, Nucl. Phys. B147, 105 (1979); E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, Phys. Lett. 116B, 231 (1982), and Nucl. Phys. B212, 413 (1983). See also J. Bagger and E. Witten, Princeton University report, 1982 (to be published).

[5] A. H. Chamseddine, R. Arnowitt, and Pran Nath, Phys. Rev. Lett. 49, 970 (1982); H. P. Nilles, M. Srednicki, and D. Wyler, CERN Report No. CERN TH-3432 (to be published); R. Barbieri, S. Ferrara, and C. A. Savoy, CERN Report No. CERN TH-3365 (to be published); L. Hall, J. Lykken, and S. Weinberg, University of Texas, Austin, report, 1983 (to be published).

[6] H. P. Nilles, M. Srednicki, and D. Wyler, CERN Report No. CERN TH-3461, 1982 (to be published); J. M. Frere, D. R. T. Jones, and S. Raby, University of Michigan Report No. UMHE-82-58 (to be published).

[7] J. Wess and B. Zumino, Phys. Lett. 74B, 51 (1978). See also J. Wess and J. Bagger, lecture notes, Princeton University, 1982 (unpublished).

[8] X. Z. Wu, thesis, 1983 (unpublished).

[9] This result has also been obtained in a different context by R. Barbieri, S. Ferrara, D. V. Nanopoulos, and K. S. Stelle, Phys. Lett. 113B, 219 (1982). But they did not relate it to the Wess-Zumino supergravity as formulated in superspace action nor did they discuss the choice of $g_2(z)$ that would yield a vanishing cosmological constant and a spontaneous supersymmetry breaking.