Quark Distributions in Nuclei

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Recent deep-inelastic lepton-scattering data from iron and deuterium targets are interpreted from a quark-model viewpoint as evidence for a significant number of six-quark clusters in iron. Standard expectations for the distribution of quarks within a six-quark cluster give some quantitative success in describing the data.

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Recently, Pirner and Vary[1] and Jaffe[2] have considered the effects of quark degrees of freedom upon inelastic electron or muon scattering from nuclei. The data are quite suggestive. Electron scattering upon ^{3}He occurs in kinematic regions not allowed if the scattering is off a single stationary nucleon.[3] Standard wave functions for ^{3}He give insufficient Fermi motion to account for what is seen, but Pirner and Vary[1] show that clustering of nucleons into six- and nine-quark clusters, with standard estimates[4] for the momentum distributions of the quarks within these clusters, gives a convincing account of the data. More recently, inelastic muon[5] and electron[6] scattering upon ^{56}Fe, in a kinematic region easily accessible to individual nucleons as well as the entire nucleus, give data indicating that ^{56}Fe is significantly different from a collection of 56 free nonoverlapping nucleons.

It is the purpose of this note to show that the ^{56}Fe data, both at low x and high x,[7] follow trends that can be expected within a standard quark-gluon picture of nucleons. Quantitatively, the data suggest that nucleons in ^{56}Fe are often, about 30% of the time, subsumed into six-quark or larger clusters.

The F_2 structure function for ^{56}Fe divided by the corresponding quantity for deuterium is shown in Fig. 1, together with a curve to be explained shortly. The deuteron is weakly bound, so that the neutron and proton in it are almost free, and the small chance that they overlap will be neglected here. (Some corrections have been made to the ^{56}Fe data to account for ^{56}Fe having different numbers of neutrons and protons.) The ratio is not constant. Quark clustering can explain the rise both at small x and at large x. Qualitatively, the rise at small x occurs because a cluster of two nucleons allows quarks to spread over a larger region of space, leading one to expect by the uncertainty principle lower quark momenta, or more quarks at low x. In addition, there is some probability in a two-nucleon or six-quark cluster that one single quark can obtain more momentum than a single nucleon[1] without Fermi motion. Hence there must be a rise in the plotted ratio as x approaches 1. (Fermi-motion effects could also explain this rise.[8]) A quantitative analysis follows.

In determining F_2, we will use simple quark distributions dictated by a few principles given below, supplemented by experimental data when available.

(1) At high x, quark distributions will be close to what is expected from the counting rules,[4]

$$f(x) \sim (1-x)^{2n_s - 1 + 2|\Delta s_z|}, \qquad (1)$$

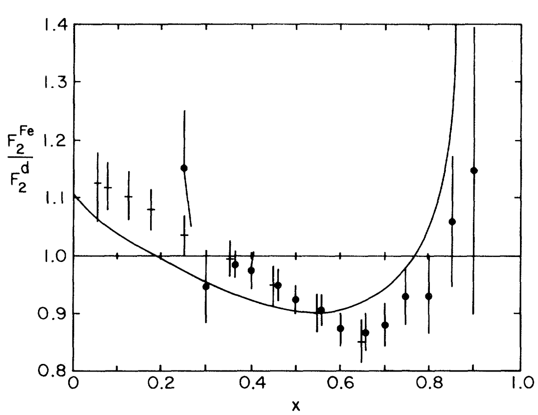

FIG. 1. Data from Ref. 5 (crosses), where Q^2 is from 20 to 60 GeV^2, and Ref. 6 (circles), where Q^2 is from 4 to 19 GeV^2, for $F_2^{Fe}(x)/F_2^d(x)$. Also shown is our result when 30% of the nucleons in ^{56}Fe are subsumed into six-quark clusters.

where n_s is the minimum number of spectators and Δs_z is a spin factor, s_z(target) $- s_z$(quark). At low x, the behavior will be what is expected from Regge arguments,[9] namely, $x^{-1/2}$ for valence quarks and x^{-1} for ocean quarks.

(2) The fraction of momentum carried by gluons at very high Q^2 is given by[10,11]

$$\langle x_g \rangle = N_g/(N_g + \tfrac{3}{2} N_f), \qquad (2)$$

where $N_g = 8$ is the number of colored gluons and N_f is the number of flavors of quarks. The above result is 63% for three flavors, 57% for four flavors, and (55 ± 2)% experimentally.[12] We will take $\langle x_g \rangle = 57\%$.

(3) The distribution functions for quarks are slowly varying functions of Q^2. The variation with Q^2 can be calculated from the Altarelli-Parisi equations or other means,[11] but only if the distribution functions at some fixed Q^2 are known. We do not have detailed information at varying Q^2 to examine what happens in the ^{56}Fe case, and so we will stick to the expected simple distributions, neglecting the slow Q^2 dependence for both ^{56}Fe and the deuteron.

For valence quarks in the proton, we have

$$U_v(x) = x u_v(x) = 2N_u \sqrt{x}\,(1-x)^3, \qquad (3)$$

where $N_u = B^{-1}(\tfrac{1}{2}, 4) \cong 1.094$, and an observation[13] that $d(x)$ falls one power of $1-x$ faster gives

$$D_v(x) = x d_v(x) = N_d \sqrt{x}(1-x)^4, \qquad (4)$$

with $N_d = B^{-1}(\tfrac{1}{2}, 5) = \tfrac{9}{8} N_u$. The ratio agrees well with the experimentally determined[13] $d_v(x)/u_v(x) = 0.57(1-x)$.

For the ocean quarks, observations suggest that all the distributions have the same shape, and that[14]

$$s(x) \cong \tfrac{1}{2}\bar{u}(x) = \tfrac{1}{2}\bar{d}(x). \qquad (5)$$

We then take

$$\bar{U}(x) = x\bar{u}(x) = \bar{N}(1-x)^7. \qquad (6)$$

Momentum normalization requires

$$\langle x_0 \rangle = 1 - \langle x_v \rangle - \langle x_g \rangle = 1 - (\tfrac{2}{9} + \tfrac{1}{11}) - \tfrac{4}{7} = 0.115, \qquad (7)$$

where $\langle x_0 \rangle$ is the momentum fraction carried by all the ocean quarks. This gives $\bar{N} = 0.1857$.

These distributions reproduce credibly the experimental[15] $F_2^{\,d}$ at $Q^2 = 50\ \mathrm{GeV}^2/c^2$, as may be seen in Fig. 2.

The quark distributions for the six-quark cluster are most naturally expressed in terms of z,

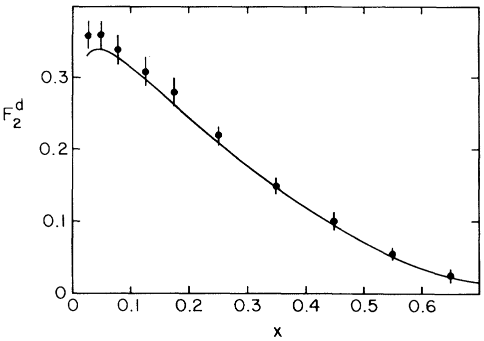

FIG. 2. Our $F_2^{\,d}(x)$ and data of Ref. 15. The data are at $Q^2 = 50\ \mathrm{GeV}^2$ except the first three points which are at $Q^2 = 15\ \mathrm{GeV}^2$ (first point) and 30 GeV^2 (next two).

the fraction of the full momentum of a six-quark cluster (i.e., two nucleons) carried by a given quark. There are no detailed experimental data guiding us in determining the separate quark distributions. Using Eq. (1) for the valence quarks we have

$$V_6(z) = z v_6(z) = N_6 \sqrt{z}\,(1-z)^{10}, \qquad (8)$$

where $N_6 = B^{-1}(\tfrac{1}{2}, 11) \cong 1.850$. In terms of x, the momentum fraction of a single nucleon, $x = 2z$ and $0 \le x \le 2$. For the ocean quarks, we take the same relation between s, $\bar{u}$, $\bar{d}$, etc., quarks as for the proton. Equation (1) gives

$$\bar{U}_6(z) = z\bar{u}_6(z) = \bar{N}_6(1-z)^{14}. \qquad (9)$$

Momentum normalization requires

$$\langle z_0 \rangle = 1 - \langle z_v \rangle - \langle z_g \rangle = 0.168 \qquad (10)$$

or $\bar{N}_6 = 0.5042$. With these distributions, the valence quarks in the six-quark cluster take up a lower fraction of the total momentum than in the proton, and the fraction carried by ocean quarks is correspondingly enhanced. This plays an important role in the rise of $F_2^{\,\mathrm{Fe}}/F_2^{\,d}$ at low x. (We have taken the momentum fraction carried by gluons in ^{56}Fe to be the same as in the nucleus. If it should be less,[2] the momentum carried by ocean quarks and their effects upon $F_2^{\,\mathrm{Fe}}$ would be correspondingly greater.)

The ratio of F_2 for a six-quark cluster to that for a deuteron is

$$R_6(x) = \frac{F_2^{\,6}(x)}{F_2^{\,d}(x)} = \frac{\tfrac{5}{6} V_6(\tfrac{1}{2}x) + \tfrac{11}{18}\bar{U}_6(\tfrac{1}{2}x)}{\tfrac{5}{18}[U(x) + D(x)] + \tfrac{11}{9}\bar{U}(x)}. \qquad (11)$$

We take a simple model where a fraction f of the nucleons in ^{56}Fe have become part of a six-quark cluster, with a fraction $1-f$ remaining single nucleons. The ratio

$$F_2^{\mathrm{Fe}}(x)/F_2^{d}(x) = (1-f) + fR_6(x) \tag{12}$$

is plotted in Fig. 1. We choose $f = 30\%$. We should emphasize that these results follow from standard ways of estimating quark distributions together with a chosen value for f.

One may ask if 30% is a reasonable six-quark fraction. We may answer affirmatively. The radius of iron is given approximately by (1.12 fm) $\times A^{1/3} = 4.3$ fm; then the volume of 56 nucleons of radius 0.75 fm is 30% of the volume of ^{56}Fe. Said differently, a given nucleon within ^{56}Fe has roughly a 30% chance of overlapping with another nucleon. One may also ask how consistent we are with the high-x ^{3}He calculations. If $R_N = 0.75$ fm, then Pirner and Vary[1] calculate (geometrically) 38% six-quark state. They used $\sqrt{z}\,(1-z)^9$ for the valence quark distribution and also a smaller R_N which led to a smaller six-quark fraction[16]; using $\sqrt{z}\,(1-z)^{10}$ instead will account for the ^{3}He data with a larger six-quark fraction. Taking the ^{56}Fe and ^{3}He results together, and taking both quite seriously, gives evidence that the counting with the $2|\Delta s_z|$ in the exponent is right and that the nucleon core radius (bag radius) is around 0.75 fm.

The results obtained here from a quark model follow the trend of the data. One question to be raised is how to mix a quark description with a nuclear-physics nucleon-meson description of the nucleus. A rise of $F_2^{\mathrm{Fe}}/F_2^{d}$ at high x is expected from Fermi motion of nucleons,[8] but this is connected with the nuclear wave function for overlapping nucleons which is here modeled as a six-quark cluster; the two effects should not be added together. Similarly, it may be possible to describe the low-x behavior in terms of meson-exchange currents and pair currents, but again these descriptions are not easily separated from a description of low-x quarks in a nucleus obtained here from the six-quark cluster.

In conclusion, the combination of the quark model and nuclear physics should be valuable to both those who wish more information about the confinement mechanisms of quarks and those who wish to understand nuclear dynamics when the nucleons appear closely spaced. We have not yet explained the $F_2^{\mathrm{Fe}}/F_2^{d}$ data perfectly, but we have a first, simple, quantitative quark picture of the physics and results which suggest that the picture is quite relevant. It would be very useful to have many precise measurements of nuclear structure functions at varying x, Q^2, and nucleon number.

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Noted added.—After submission of this paper, we received a new report from Pirner and Vary, entitled "Sea Quark Distributions in Nuclei," which is similar to the present work.

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[7]Notation: $x = Q^2/2m_N\nu$, where q is the four-momentum of the virtual photon, $Q^2 \equiv -q^2 \geq 0$, $\nu = q_0$ in the laboratory system, and m_N is the nucleon mass; x is also the fraction of a nucleon's momentum carried by a quark, in the infinite-momentum frame.

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[16]Using $\sqrt{z}(1-z)^9$ changes our low-x results very little.