Hadron Electromagnetic Mass Differences and a Prediction of B^+-B^0

Lai-Him Chan

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803

It is proposed that the dynamical contribution to the hadron electromagnetic mass difference induced by the splitting of the up and down quark masses can be extracted phenomenologically from the quark masses by interpolation, in accordance with general QCD assumptions. The results are consistent with all observed electromagnetic mass differences. The model-independent prediction $B^0 - B^+ = 2.3 \pm 0.3$ MeV will serve as a crucial test for these assumptions. A useful mass formula for the s-wave hadron ground states is also presented.

Recently Behrends *et al.* have successfully reconstructed the B decays into a D or D^* meson plus charged pions.[1] The masses of B mesons given by their data are $B^0 = 5274.2 \pm 2.0$ MeV and $B^+ = 5270.8 \pm 2.3$ MeV.[2] The average B mass is 5272.3 ± 2.0 MeV and the mass difference is $B^0 - B^+ = 3.4 \pm 3.0$ MeV. With the open-channel $\bar{B}B$ threshold only 32 MeV below the $\Upsilon(4S)$ mass we should expect that very accurate determination of B masses may be possible in the near future. In particular the precise measurement of $B^+ - B^0$ to within 1 MeV should contribute greatly toward our understanding of the hadron electromagnetic mass differences.[3]

In this paper I would like to present a reliable prediction $B^0 - B^+ = 2.3 \pm 0.3$ MeV extracted phenomenologically from some general properties of quantum chromodynamics (QCD) but otherwise model independent.

The electromagnetic mass difference (EMD) arises from two different sources: (1) the one-photon exchange and (2) the strong isospin violation due to intrinsic up and down quark mass difference. The one-photon exchange representing the electromagnetic interaction energy between the constituent quarks is well understood. In most cases it contributes less than 2 MeV or less than 40% of the EMD. Apart from a possible difference of a fraction of a megaelectronvolt, its contributions to the EMD are not very sensitive to the theoretical model used for such calculations. A nonelectromagnetic isospin-breaking mechanism, necessary to explain $n > p$ and $K^0 > K^+$, has been attributed to the difference in the up and down quark masses. I would like to assert that associated with this mass difference there must be an induced dynamical isospin-breaking effect and that its contribution to the hadron EMD can be estimated very reliably from some general assumptions which are consistent with the QCD hypotheses. The assumptions are

that (1) the only flavor-symmetry–breaking mechanism is the quark mass, and (2) the flavor symmetry violation due to the sea quarks or the quark loop is negligible. If one is further restricted to the nonannihilation (nonzero flavor quantum number) channel, then there should not be any dependence on the flavor quantum numbers other than the implicit dependence through the quark masses. Therefore, it is possible to continue analytically from one flavor to another via a continuous parameter, the quark mass. It is not necessary to specify the definition of the quark mass as long as different types of quark mass are related by analytic functions. It will be assumed that there exists a choice of quark mass such that the physical observables are smooth functions of the quark masses. This will allow meaningful interpolation between quark masses to extract EMD from given hadron masses. As a consequence, if two different models, dynamical or phenomenological, satisfy the above assumptions and agree on their predictions on the hadron strong mass splittings, they must also predict hadron EMD consistent with each other.

The assumptions used are sufficiently general that all dynamical models of QCD, such as potential models and the MIT bag model,[4] would satisfy them. In principle their predictions of the EMD of hadrons are also model independent. In practice the usefulness of their predictions is limited by their ability to fit the observed hadron mass splittings. For the purpose of quark-mass interpolation, it is sufficient to use a phenomenological model versatile enough to fit the observed hadron masses accurately but restrictive enough so that the mass interpolation is sufficiently unique. The latter condition can be checked against a dynamical model such as the MIT bag model for a range of bag parameters. I shall choose the mathematically simplest phenomenological model which satisfies these conditions.

The main advantage is that the masses of hadrons can be expressed in terms of quark masses in a simple mass formula and many of the necessary equations can be solved algebraically. Moreover there exists a consistent physical picture of this model.

The model,[5] motivated by the De Rújula–Georgi–Glashow model,[6] has been proposed for a broken-SU(8) quark model and has successfully predicted $D^+ - D^0$. However, its full potential as a useful tool for quark mass interpolation has not been recognized. In this model, the s-wave hadron masses are determined by the matrix element

$$M = \mu_p + \sum_i m_i + \alpha_s^{(p)} k_p D_p m_u^2 \times 4 \sum_{i>j} \frac{\vec{s}_i}{m_i} \cdot \frac{\vec{s}_j}{m_j} + \alpha C_p \sum_{i>j} Q_i Q_j - \alpha D_p m_u^2 \times 4 \sum_{i>j} Q_i Q_j \frac{\vec{s}_i}{m_i} \cdot \frac{\vec{s}_j}{m_j}, \tag{1}$$

where the index p takes on two values, B for baryon and M for meson. m_i, Q_i, and $\vec{s}_i$ are the effective constituent quark mass, charge, and spin of the ith quark, respectively. According to SU(3) QCD, $k_M = \frac{4}{3}$ and $k_B = \frac{2}{3}$. The $\alpha_s^{(p)}$ term is the color spin-spin interaction and the last two terms are the Coulomb and magnetic interactions. α and $\alpha_s^{(p)}$ are the fine-structure constant and the effective QCD coupling constant, respectively. The zero-point energy μ_p, $C_p = \langle 1/r \rangle_p$, and $D_p = (2\pi/3m_u^2)\langle \delta^3(r)\rangle_p$ are assumed to be constant which is a fair approximation for the mass matrix.[5]

The masses of mesons are given by

$$M_s(m_1, m_2) = \mu_M + m_1 + m_2 + \alpha C_M Q_1 Q_2 + (\tfrac{4}{3}\alpha_s^{(M)} - \alpha Q_1 Q_2)D_M(m_u^2/m_1 m_2)[2s(s+1) - 3],$$

where s is the total spin. Given that the quark masses satisfy $m_d - m_u \ll m_s - m_d \ll m_c - m_s \ll m_b - m_c$, one can always number the three quarks in any baryon so that $|m_1 - m_2| \ll 2m_3 - m_1 - m_2$; then the eigenstates of $s_{12}^2 = (\vec{s}_1 + \vec{s}_2)^2$ and $\vec{s}^2$ are, for all practical purposes, the eigenstates of the mass matrix (1). The masses of baryons are given by

$$B_{s,s_{12}}^Q(m_1, m_2, m_3) = \mu_B + m_1 + m_2 + m_3 + \tfrac{1}{2}\alpha C_B[(Q - Q_u)(Q - Q_d) + 2Q_u Q_d]$$

$$+ \tfrac{2}{3}\alpha_s^{(B)} D_B m_u^2 \left\{ \frac{1}{m_1 m_2}[2s_{12}(s_{12}+1) - 3] + \frac{1}{m_3}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)[s(s+1) - s_{12}(s_{12}+1) - \tfrac{3}{4}]\right\}$$

$$- \alpha D_B m_u^2 \left\{ \frac{Q_1}{m_1} \frac{Q_2}{m_2}[2s_{12}(s_{12}+1) - 3] + \frac{Q_3}{m_3}\left(\frac{Q_1}{m_1} + \frac{Q_2}{m_2}\right)[s(s+1) - s_{12}(s_{12}+1) - \tfrac{3}{4}]\right\},$$

where $Q = Q_1 + Q_2 + Q_3$, $s_{12} = 1$ for $s = \frac{3}{2}$ and $s_{12} = 0$, 1 for $s = \frac{1}{2}$. The off-diagonal matrix element which mixes the $s_{12} = 0$ and $s_{12} = 1$ states is $\frac{2}{3}\sqrt{3}\alpha_s^{(B)} D_B(m_u/m_1 - m_u/m_2)(m_u/m_3)$. The corrections to the mass eigenvalues are negligibly small in all cases. As can be seen clearly, in the limit $\alpha = 0$, the mass of the hadron depends on the flavor only through the smooth continuous function of effective quark masses. It should be

emphasized that in parametrizing the hadron masses in this particular form, one is essentially choosing a convenient mapping of the current-quark mass μ into the effective quark mass $m(\mu)$. For the purpose of interpolation between flavor splittings the detailed functional form of $m(\mu)$ is irrelevant.

By neglecting the isospin violation, one can determine the following:

$$m = m_u = m_d = (\Lambda - N)(\Sigma^* - \Sigma)/(\Delta - N - \Sigma^* + \Sigma) = 335.7 \text{ MeV}, \quad m_s = m(\Delta - N)/(\Sigma^* - \Sigma) = 512.5 \text{ MeV},$$

$$\alpha_s^{(B)} D_B = \tfrac{1}{4}(\Delta - N) = 73.3 \text{ MeV}, \quad \mu_B = \tfrac{1}{2}(\Delta + N) - 3m = 78.2 \text{ MeV}, \tag{2}$$

$$\alpha_s^{(M)} D_M = \tfrac{3}{16}(K^* - K)m_s/m = 113.9 \text{ MeV}, \quad \mu_M = \tfrac{1}{4}(3K^* + K) - m - m_s = -53.7 \text{ MeV}. \tag{3}$$

The charmed-quark mass $m_c = 1674$ MeV is obtained from the equation $D - K = m_c - m_s + \tfrac{3}{4}(K^* - K)(1 - m_s/m_c)$. For the heavy b quark the equation $B - K = m_b - m_s + \tfrac{3}{4}(K^* - K)(1 - m_s/m_b)$ can be well approximated by $m_b = B - 248$ MeV to give $m_b = 5024$ MeV. Except for electromagnetic splittings, the parameters in Eq. (1) are completely

determined. The masses for the ordinary and the charmed hadrons can be found in Table I of Ref. 5. The agreement with the known values is good to the order of 10 MeV.

Alternatively one can obtain mass relations by eliminating the parameters. Relations between

the known hadrons are well satisfied:

$$\Sigma - \Lambda - \tfrac{2}{3}(\Delta - N - \Sigma^* + \Sigma) = 0 \ (10 \text{ MeV}),$$

$$\Xi^* - \Xi - \Sigma^* + \Sigma = 0 \ (23 \text{ MeV}),$$

$$\Omega - \Delta - 3(\Xi^* - \Sigma^*) = 0 \ (-3 \text{ MeV}),$$

$$\Lambda - N - \Xi^* + \Sigma^* + \tfrac{1}{6}(\Delta - N)\left[\left(\frac{\Sigma^* - \Sigma}{\Delta - N}\right)^2 - 1\right]$$
$$= 0 \ (10 \text{ MeV}),$$

$$K^* - p - \Lambda + N + \tfrac{1}{4}(K^* - K)\left(\frac{\Delta - N}{\Sigma^* - \Sigma} - 1\right)$$
$$= 0 \ (-5 \text{ MeV}).$$

The charmed-meson predictions include $D^* = 1987$ MeV, $F = 2073$ MeV, and $F^* = 2153$ MeV.[5] The observed values are $D^* = 2007 \pm 2$ MeV, $F = 2021 \pm 15$ MeV, and $F^* = 2140 \pm 60$ MeV. The mass of the

TABLE I. Electromagnetic mass differences (in megaelectronvolts).

	Predicted	Observed
$p - n$	Input	-1.29
$\Sigma^+ - \Sigma^-$	Input	-7.98 ± 0.08
$\Sigma^0 - \Sigma^-$	Input	-4.88 ± 0.06
$\Xi^0 - \Xi^-$	-6.7	-6.4 ± 0.6
$\Sigma^{*+} - \Sigma^{*-}$	-4.9	-5.2 ± 0.6
$\Sigma^{*0} - \Sigma^{*-}$	-3.3	-5.5 ± 2.5
$\Xi^{*0} - \Xi^{*-}$	-3.6	-3.2 ± 6
$\pi^+ - \pi^0$	5.4 ± 0.8	4.6
$K^+ - K^0$	Input	-4.01 ± 0.13
$D^+ - D^0$	4.7 ± 0.4	4.7 ± 0.3
$D^{*+} - D^{*0}$	3.3 ± 0.4	2.6 ± 1.8
$B^+ - B^0$	-2.3 ± 0.3	-3.4 ± 3.0
$B^{*+} - B^{*0}$	-2.0 ± 0.3	$\cdots$

charmed baryon $C_0 = \Lambda_c = N + m_c - m = 2277$ MeV is in good agreement with the recent value $\Lambda_c = 2282 \pm 3$ MeV.[7] The lowest-lying bottom baryon mass is $\Lambda_b = N + m_b - m = 5627$ MeV.

The complete set of EMD relations for the ordinary and the charmed baryons is given in Eqs. (5)–(8) in Ref. 5. The calculated as well as the observed values of the hadron EMD are listed in Table I. The agreement with measured values is excellent.[8] The values of $p - n$, $\Sigma^+ - \Sigma^0$, and $\Sigma^- - \Sigma^0$ are used for the input to determine

$$m_u - m_d = \frac{n - p - \tfrac{1}{3}(\Sigma^+ + \Sigma^- - 2\Sigma^0)}{1 - (\Delta - N)/3m} = -2.66 \text{ MeV},$$

$$\alpha D_B = 2\frac{m_u - m_d}{m}\alpha_s^{(B)} D_B - \frac{3m_s}{m_s + 2m}(\Sigma^+ - \Sigma^0 - p + n) = 1.18 \pm 0.08 \text{ MeV}, \tag{4}$$

$$\alpha C_B = \alpha D_B + (\Sigma^+ + \Sigma^- - 2\Sigma^0) = 2.96 \text{ MeV}.$$

From Eqs. (2) and (4) we can evaluate $\alpha_s^{(B)} = 0.45 \pm 0.05$ MeV.[5] It is gratifying that D_B, C_B, and α_s are positive as they should be. More importantly the magnitude of $\alpha_s^{(B)}$ is very close to the value expected from QCD.

The value of $K^+ - K^0$ is used to determine the meson parameters:

$$K^+ - K^0 = m_u - m_d + \tfrac{3}{4}[(m_u - m_d)/m_s](K^* - K) + \tfrac{1}{3}\alpha C_M + (m/m_s)\alpha D_M,$$

$$-4.01 \pm 0.13 = -2.66 - 2.36 + \tfrac{1}{3}\alpha C_M + 0.55/\alpha_s^{(M)} \text{ (in megaelectronvolts)}.$$

In the absence of the 2.36-MeV term from the strong spin-spin interaction, either the Coulomb energy would have to be negative or $m_d - m_u > 4.01$ MeV which is inconsistent with $m_d - m_u = 2.66$ MeV obtained from baryons. The large contribution from the spin interaction term has just the right magnitude to make the one-photon exchange contribution very reasonable.[5] We have

$$(K^+ - K^0)_{1\gamma} = \tfrac{1}{3}\alpha C_M + 0.55 \text{ MeV}/\alpha_s^{(M)} = 1.01 \pm 0.13 \text{ MeV}. \tag{5}$$

The positivity of C_M and $\alpha_s^{(M)}$ implies $C_M < 3.03$ MeV and $\alpha_s^{(M)} > 0.54$. Again the inequality of $\alpha_s^{(M)}$ seems to be very reasonable. For the value $\alpha_s^{(M)} = 1.1 \pm 0.5$, one obtains $\alpha C_M = 1.5 \pm 0.8$ MeV and $\alpha D_M = 0.8 \pm 0.3$ MeV.

Since it is unlikely to expect this nonrelativistic model to be applicable to the pion, we shall follow the scheme of Lane and Weinberg.[9] They have combined Dashen's theorem[10] and the nonrelativistic atomic model of the K meson and derived $\pi^+ - \pi^0 = \tfrac{3}{2}(K^+ - K^0)_{1\gamma} m_K/m_\pi$. Putting in the value from Eq. (6), one obtains $\pi^+ - \pi^0 = 5.4 \pm 0.8$ MeV which is consistent with the known value 4.6 MeV.

For the D meson, a simple calculation[5] shows $D^+ - D^0 = 5.4 - 0.77/\alpha_s^{(M)}$ MeV and $D^{*+} - D^{*0} = 4.4 - 1.2/\alpha_s^{(M)}$ MeV. The condition $\alpha_s^{(M)} > 0.54$ imposes the inequalities $4.0 \text{ MeV} < D^+ - D^0 < 5.4$ MeV and $2.2 \text{ MeV} < D^{*+} - D^{*0} < 4.4$ MeV. For the value $\alpha_s^{(M)} = 1.1 \pm 0.5$, one obtains $D^+ - D^0 = 4.7 \pm 0.4$ MeV and $D^{*+} - D^{*0} = 3.3 \pm 0.4$ MeV. The measurements by Peruzzi et al., $D^+ - D^0 = 4.7 \pm 0.3$ MeV and $D^{*+} - D^{*0} = 2.6 \pm 1.8$ MeV, are in excellent agreement with these predictions.[11] They are significantly below 6.7 MeV,

the prediction of Ref. 9.

In the case of the B meson, we have

$$B^+ - B^0 = m_u - m_d + 4\frac{m_u - m_d}{m_b}\alpha_s D_M + \tfrac{1}{3}\alpha C_M + \frac{m}{m_b}\alpha D_M = -1.9 + 0.5/\alpha_s^{(M)} \text{ MeV},$$

$$B^{*+} - B^{*0} = m_u - m_d - \frac{4}{3}\frac{m_u - m_d}{m_b}\alpha_s^{(M)}D_M + \tfrac{1}{3}\alpha C_M - \frac{1}{3}\frac{m}{m_b}\alpha D_M = -1.6 + 0.6/\alpha_s^{(M)} \text{ MeV}.$$

The restraint $\alpha_s^{(M)} > 0.54$ confines the prediction in a very narrow range 1.9 MeV $< B^0 - B^+ < 2.7$ MeV and 1.6 MeV $< B^{*0} - B^{*+} < 2.5$ MeV. $\alpha_s^{(M)} = 1.1 \pm 0.5$ implies $B^0 - B^+ = 2.3 \pm 0.3$ MeV and $B^{*0} - B^{*+} = 2.0 \pm 0.3$ MeV. The prediction of $B^0 - B^+$ is consistent with the CLEO measurement, 3.4 $\pm$ 3.0 MeV.[1] The theoretical prediction of 4.4 MeV quoted in Ref. 1 is the result of a calculation[12] following the method used by Lane and Weinberg.[9] Since the $\Upsilon(4S)$ is only 32.4 MeV above the $\bar{B}B$ threshold, the branching ratio $\Gamma(\Upsilon(4S) \to B^+B^-)/\Gamma(\Upsilon(4S) \to B^0\bar{B}^0)$ is very sensitive to the B mass. This ratio is equal to 1.2 for $B^0 - B^+ = 2.3$ MeV in comparison to 1.5 for $B^0 - B^+ = 4.4$ MeV.

The crucial difference between my analysis and that of Refs. 9 and 12 is the induced dynamical isospin-breaking effects due to the difference in the up and down quark masses. I have presented a method of quark-mass interpolation in accordance with some general QCD assumptions to estimate this effect phenomenologically. The interpolation is carried out with the aid of a simple model. It is clear that the spin-spin interaction term must be present in order to achieve internal consistency in fitting the mass differences.[5] The prediction of EMD in D mesons has been confirmed to within 0.5 MeV. My prediction of $B^0 - B^+ = 2.3$ MeV should be accurate to 0.6 MeV, including possible error for the interpolation. If a precise measurement finds $B^0 - B^+$ outside the acceptable range, some of the general assumptions used here should be reexamined carefully.

[1]S. Behrends et al., Phys. Rev. Lett. 50, 881 (1983).

[2]Particle names stand for particle masses.

[3]I follow the traditional usage of the term EMD to include other isospin violations. A partial list of recent works is as follows: A. D. Sakharov, Pis'ma Zh. Eksp. Teor. Fiz. 21, 554 (1975) [JETP Lett. 21, 258 (1975)]; J. Franklin, Phys. Rev. D 12, 2077 (1975); A. W. Hendry and D. B. Lichtenberg, Phys. Rev. D 12, 2756 (1975); D. B. Lichtenberg, Phys. Rev. D 16, 231 (1977); S. Ono, Phys. Rev. D 17, 888 (1978); H. J. Lipkin, Phys. Rev. Lett. 41, 1629 (1978); I. Cohen and H. J. Lipkin, Phys. Lett. 93B, 56 (1980); D. C. Peaslee, Phys. Rev. D 14, 3495 (1977); N. Isgur, Phys. Rev. D 12, 779 (1980); C. Itoh et al., Prog. Theor. Phys. 54, 908 (1975), and 61, 548 (1979).

[4]T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975); R. L. Jaffe and J. Kiskis, Phys. Rev. D 13, 1355 (1976); N. G. Deshpande, D. A. Dicus, K. Johnson, and V. L. Teplitz, Phys. Rev. D 15, 1885 (1977).

[5]Lai-Him Chan, Phys. Rev. D 15, 2478 (1977), and "Hadron EMD and a Prediction of $B^+ - B^-$" (unpublished).

[6]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[7]The source of data in this paper is from M. Roos et al. (Particle Data Group), Phys. Lett. 111B, 1 (1982).

[8]An exception is $K^{*0} - K^{*+} = 1.34$ MeV compared to the observed value 6.7 ± 1.2 MeV. Since K^* is a wide resonance the significance of this discrepancy is not clear.

[9]K. Lane and S. Weinberg, Phys. Rev. Lett. 37, 717 (1976).

[10]R. Dashen, Phys. Rev. 183, 1245 (1969).

[11]I. Peruzzi et al., Phys. Rev. Lett. 39, 1301 (1977).

[12]E. Eichten, Phys. Rev. D 22, 1819 (1980).