Observation of a High-Frequency Cutoff for Phonon Propagation in Liquid ^{4}He

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A new high-frequency cutoff for phonon propagation which lies beyond the well-known low-frequency cutoff due to anomalous dispersion is observed in phonon propagation experiments in liquid ^{4}He using superconducting Sn and Al tunnel-junction generators and detectors. The cutoff depends strongly on pressure and is interpreted as arising from a strong increase in phase space for phonon-roton scattering as normal dispersion in the phonon branch sets in.

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The role of critical points in the shape of the excitation spectrum of He II in determining the lifetime of the elementary excitations has been the subject of intense experimental and theoretical study for many years. The propagation of high-frequency phonons ($\hbar\omega \gg kT$), for example, in He II is strongly affected by the laws of conservation of momentum and energy. The dispersion relation up to ~20 bars in anomalous, i.e., the phase velocity v_p increases above the velocity of sound c for small wave vectors $[v_p = c(1+|\gamma|q^2+\ldots)]$ as shown experimentally.[1] This anomalous dispersion allows a spontaneous decay of a phonon below a certain threshold energy $E_c = \hbar\omega_c$ into two or more phonons in accordance with the conservation laws. Dynes and Narayanamurti[2] have measured E_c as a function of pressure by using superconducting Al-oxide-Al tunnel-junction phonon generators and detectors.[3] Various consequences of anomalous dispersion have been studied theoretically[1,4-6] and Aldrich, Pethick, and Pines,[6] using a polarization potential model, have reconciled the differences between experiments at very long (ultrasonic) wavelengths with the decay-threshold measurements.

In this Letter we report an observation of a new high-frequency cutoff energy E_c'' in phonon propagation at low temperatures ($T < 200$ mK) in addition to the low-frequency cutoff due to anomalous dispersion. The occurrence of E_c'' is shown to depend markedly on pressure and is interpreted as arising from phonon-roton scattering reducing the phonon lifetime for $\hbar\omega > E_c''$.

The experiments were done with use of Sn-oxide-Sn tunnel-junction generators and either bulk or granular Al-oxide-Al detectors. The tunnel junctions were evaporated on a glass substrate and these were placed parallel within a distance of 1.10 ± 0.05 mm from each other. The maximum power dissipated at the generator was typically a few microwatts. The lowest attainable temperature of ~50 mK was reached by dilution refrigeration.

The phonon propagation was studied by the quasimonochromatic modulation technique.[3] The generator bias current was modulated typically at a frequency of ~500 Hz and the transmitted phonon signal was followed at the detector by a lock-in amplifier. This type of detection technique produces an output which is proportional to the derivative of the total signal with respect to the generator current, dS/dI_G.

Previous work[2] using such modulation techniques had been confined to the use of Al junctions both as generator and detector. The use of a Sn generator allows the probing of the region in the elementary excitation spectrum between the energy gap of Al (0.36 meV) and that of Sn (1.2 meV) in great detail, which had not been done heretofore. These differences are discussed in greater detail below.

In Fig. 1 we have plotted dS/dI_G versus the gen-

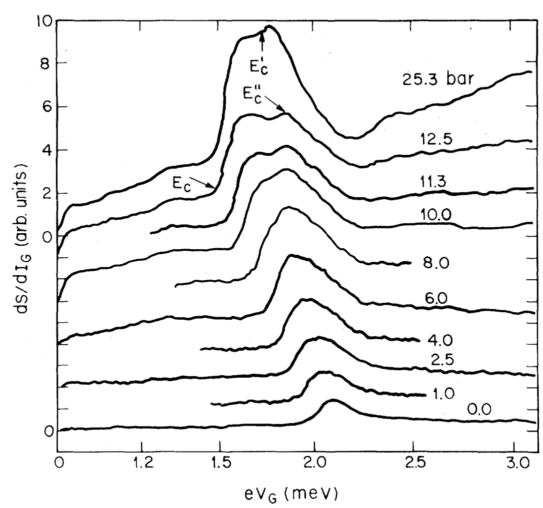

FIG. 1. The signal derivative, dS/dI_G, with respect to the generator current I_G as a function of the generator voltage V_G at various pressures. An Al detector ($2\Delta_{Al} \simeq 0.36$ meV) and a Sn generator ($2\Delta_{Sn} \simeq 1.15$ meV) were used.

erator voltage V_G at various pressures from 0 to 25.3 bars measured with a Sn-oxide-Sn generator. For $P \leq 12.5$ bars we see a sharp rise of the signal when the generator voltage reaches E_c, i.e., the low-frequency cutoff due to anomalous dispersion. At higher pressures we cannot follow E_c because the Al detector is not sensitive to phonons $\hbar\omega < 2\Delta_{\rm Al} \approx 0.36$ meV. Within the pressure range from 8 to 25.3 bars we detect another but smaller increase in the signal marked E_c'. At somewhat higher generator voltages the signal starts to decrease and finally reaches a level which is close to that just before E_c. The onset of this decrease is marked E_c''. As a function of pressure the signal amplitude increases monotonically by a factor of roughly 5 from 0 to 25.3 bars.

We show the pressure dependence of the events E_c, E_c', and E_c'' in Fig. 2. As mentioned earlier the event E_c corresponds to the threshold of spontaneous decay due to anomalous dispersion and agrees with earlier measurements[2] as well as with our own measurements when using Al-oxide-Al detectors and generators. The small bump in dS/dI_G at E_c' seems to follow the roton minimum Δ_r within our experimental accuracy; the solid line is an average value of $\Delta_r(P)$ determined by various groups and taken from Greywall.[7] This feature has been observed earlier as well and is to be associated with propagation of rotons.[2]

The new feature with a threshold at E_c'' is detectable only with Sn generators. We see at least two reasons why this effect may become unobservable with Al generators. First of all $E_c'' \gtrsim 4\Delta_{\rm Al}$ and thus we must rely on relaxation phonons at energies $\hbar\omega_r \gg 2\Delta_{\rm Al}$. Although the electron-phonon coupling in Al is not very strong, still the modulated phonon distribution at generator voltages $eV_G - 2\Delta_{\rm Al} > 2\Delta_{\rm Al}$ becomes highly nonmonochromatic. Thus only a small fraction (few percent) of the lock-in detected signal is due to phonons of energy $\hbar\omega = eV_G - 2\Delta_{\rm Al} > 2\Delta_{\rm Al}$. This problem is avoided with Sn generators since we do not have to work at frequencies $\hbar\omega_r > 2\Delta_{\rm Sn} \sim 13$ K. Furthermore, the $2\Delta_{\rm Sn}$ phonons themselves do not propagate, thus relatively enhancing the detection of the event at E_c''. The second reason may be due to the fact that with Sn generators the relaxation phonons detected must have traveled through the cloud of highly scattered phonons or rotons originating from the $2\Delta_{\rm Sn}$ phonons. This 2Δ background distribution can enhance the scattering observed at E_c''.

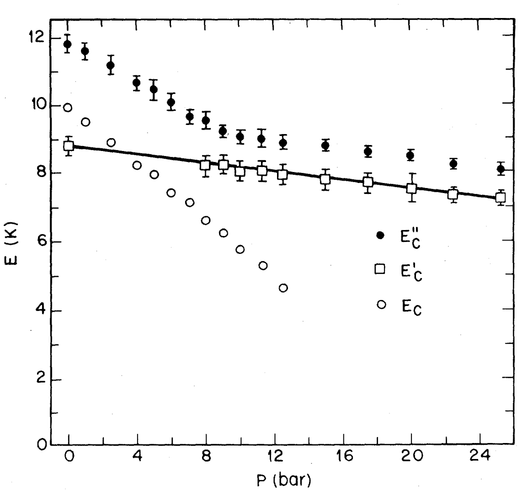

FIG. 2. The threshold energies E_c, E_c', and E_c'' plotted as a function of pressure. E_c corresponds to anomalous decay, E_c' to the roton minimum, and E_c'' is the new threshold energy. The solid line follows the roton minimum taken from Ref. 7.

We have evidence that although the density of states has a singularity at the roton minimum, most of the signal detected at $\hbar\omega \gtrsim E_c''$ is due to phonon propagation and only a small fraction is caused by rotons. Firstly, at zero pressure we do not see much, if any, signal between Δ_r and E_c. Secondly, we could not observe any appreciable expected change of phase of the signal when passing through the roton minimum at high pressures. Finally, in time-of-flight measurements, the propagation velocity was that expected for phonons.

As to the nature of the scattering process which causes the reduction of the signal at $\hbar\omega > E_c''$, we can conclude that the three-particle processes we can think of, i.e., spontaneous decay, collinear Pitaevskii scattering,[4,8] and phonon decay into two rotons, do not satisfy the conservation laws at phonon energies $\hbar\omega \approx E_c''$. We thus have to consider four-particle processes involving large-momenta phonons and rotons in detail. As is well known,[9-12] both rotons and phonons thermalize among themselves on a short time scale, but the relaxation process involving equilibrium between the phonon gas and the roton gas is rather slow. In the current experiment with a constant influx of rotons, we expect, therefore, that there will be an equilibrium density of rotons, mostly at the roton minimum Δ (since $T \ll \Delta$), determined by the long roton lifetime. Consequently, the four-particle scattering process of interest is one which involves the scattering of a phonon with a roton at Δ.

The phonon-roton scattering cross section has been calculated by Khalatnikov and Chernikova[9,11] in the limit of small phonon momenta to be

$$\sigma_{\mathrm{pr}}(p) = \frac{1}{4\pi}\left(\frac{p_0 p^2}{\hbar^2 \rho c}\right)^2\left[\frac{2}{9} + \frac{1}{25}\left(\frac{p_0}{\mu c}\right)^2 + \frac{2A}{9}\left(\frac{p_0}{\mu c}\right) + A^2\right] \quad (1)$$

where

$$A = \frac{\rho^2}{p_0 c}\left[\frac{\partial^2 \Delta}{\partial p^2} + \frac{1}{\mu}\left(\frac{\partial p_0}{\partial \rho}\right)^2\right],$$

p and p_0 are the phonon and roton momenta, c the phonon velocity, μ and Δ the roton mass and gap, respectively, and ρ the density of helium.[11] Equation (1), which shows that σ_{pr} for the four-particle process shown in Fig. 3(b) increases rapidly ($\sim p^4$) with the phonon momentum, is based on a linear dispersion curve for the phonons. However, in ^{4}He the phonon curve deviates markedly from linear behavior abruptly [see Fig. 3(a)]. This results in a better "matching" of the energy dispersion curves of the phonon and of the rotons around Δ. The consequent "opening up" of the phase space for scattering subject to the energy-conserving delta function causes an even stronger rise in the phonon-roton cross section than given by Eq. (1). Using the interpolation formula for the dispersion curve at saturated vapor pressure and at 25 bars by Donnelly, Donnelly, and Hills,[13] we have calculated the phase space with the actual dispersion, and the enhancement[14] over that with the linear dispersion curve with the sound velocity is shown in Fig. 3. The sudden increase in this cross section as normal dispersion sets in correlates very well with the position of the second "threshold" seen in the experimental signals. Besides the position of the threshold, the shape of the curves, including the softer onset (larger width) at high pressures, is consistent with the data.

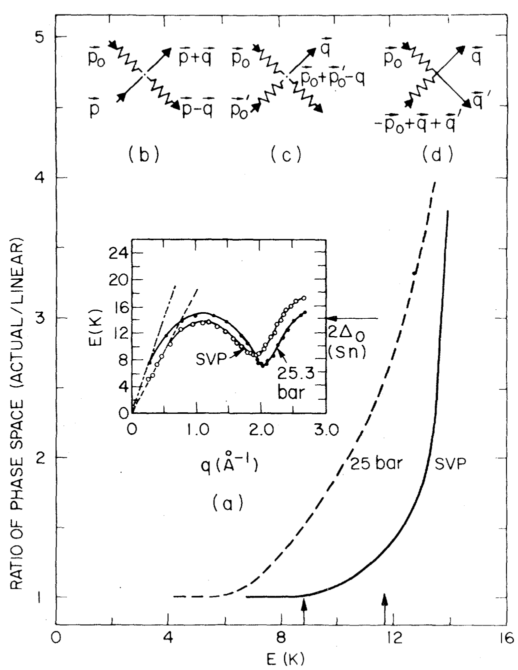

FIG. 3. Ratio of phase space for phonon-roton scattering for the actual (measured) dispersion curve to that with linear dispersion as a function of phonon energy. The arrows at 9 and 11.8 K correspond to the measured values of E_c'' at 25 bars and saturated vapor pressure. Insets: (a) the measured He dispersion curve; (b)–(d) various phonon-roton scattering processes. See text.

If we use Eq. (1) to estimate the number of rotons necessary to cause the phonon mean free path to drop below 1 mm at our observed second threshold, using parameters given by Donnelly, Donnelly, and Hills,[13] we obtain a roton density of $\sim 5\times10^{14}$ cm^{-3} at saturated vapor pressure, while at 25 bars the number must be $\sim 10^{16}$ per cm^3 because of the lower value of p and the larger value of c. This difference may seem unlikely at first sight. However, if we calculate the equilibrium roton density assuming that the decay is primarily due to the two processes shown in Figs. 3(c) and 3(d), which are essentially the inverse of the process shown in Fig. 3(b), then it becomes clear that a similar reduction is expected for the roton decay cross section as for the phonon-roton scattering process. Thus the larger equilibrium density at the higher pressure is quite reasonable. The experimental data were found to be a strong function of temperature ($T > 0.2$ K) and injection density in qualitative accord with phonon-roton scattering. Clearly for a quantitative identification, we would need to go beyond the hydrodynamic result, Eq. (1), and also beyond its recent modification.[11] Finally, we would like to mention that for roton momenta beyond p_0 the group velocity approaches the sound velocity and the scattering could be further enhanced if the colliding excitations move in a parallel direction.[10] We would like to emphasize, however, that this would simply act to enhance the low-pressure scattering cross section over and above the dominant four-particle process considered in this paper.

Finally, we would like to mention that the data presented here were repeated with a granular Al detector with a gap of 0.62 meV and we obtained similar results. In addition, when the experimental cell was filled with solid ^{4}He, the signal remained flat[15] through energies $\sim E_c''$. These results, combined with the observed pressure dependence of E_c'', rule out interface and/or detector artifacts.

In summary, we have observed a new high-frequency cutoff in phonon propagation in liquid He II which lies beyond the low-frequency cutoff due to anomalous dispersion. This cutoff depends strongly on pressure and is believed to arise from phonon-roton scattering in agreement with the pressure dependence of the dispersion curve. If so, this technique is a sensitive measure of phonon-roton scattering in the region of phonon momenta 0.4–0.8 Å^{-1} at different pressures, and indicates many avenues for further research. On the experimental side, clearly more measurements as a function of cell size, scattering angle, and input power are called for. It would simultaneously be of great interest to have precise inelastic neutron scattering measurements of the phonon dispersion curve as a function of pressure. Further theoretical investigation of the four-particle scattering process with large momenta of the phonon involved (so that the matrix elements may have to be determined from a microscopic Hamiltonian rather than the hydrodynamic Landau-Khalatnikov theory) is also indicated.

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