Very High-Resolution Heat-Capacity Measurements near the Lambda Point of Helium

J. A. Lipa and T. C. P. Chui
Physics Department, Stanford University, Stanford, California 94305
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New measurements of the heat capacity of a sample of helium 3 mm high are reported which extend to within 5×10^{-8} deg of the lambda transition at the vapor pressure. From an analysis of the results allowing for the effect of gravity, the values -0.0127 ± 0.0026 (2σ) for the exponent α ($=\alpha'$) and 1.058 ± 0.004 for the leading singularity ratio A/A' are obtained. These values are in closer agreement with the theoretical predictions than those reported previously.

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Experimental tests of the renormalization-group (RG) predictions[1] for cooperative transitions involve parameter estimation by curve fitting to data. The accuracy of the parameter values depends on the degree of distortion of the data due to sample defects, the measurement uncertainties, and the dynamic range of the independent variable. Near the λ transition of ^{4}He at the vapor pressure, an exceptionally wide range of temperature is available extending from $t=|1-T/T_\lambda|\sim10^{-3}$ to $\sim10^{-8}$, limited ultimately by the tradeoff between finite-size effects and gravitational rounding. This range is the maximum for any known cooperative transition and experiments on this system should therefore provide the best tests of theoretical predictions. Many experiments have been performed near the λ transition, but none have spanned the complete range of t available. In this paper we report heat-capacity measurements on a sample of helium 3 mm high which extend from $t=10^{-3}$ to well inside the gravity-affected region. Previously we studied[2] the heat-capacity curve of a sample of height 0.3 mm. The present experiment was undertaken both to improve the quality of the measurements and to minimize those distortions of the heat-capacity curve which are less well understood, and which might perturb values obtained for quantities of theoretical interest. By increasing the sample height from 0.3 to 3 mm, finite-size effects and nonlocal contributions to the gravity corrections are reduced, but at the expense of a large increase in the gravity effect due to the finite slope of the λ line. In addition to causing a distortion of the heat-capacity curve, which can be easily calculated,[3] this effect induces a two-phase region within the sample where both He I and He II coexist. For the present experiment the width of this region, δT_2, is about 4×10^{-7} K. We corrected our data for the effect of gravity and then performed curve fitting to extract parameters for comparison with the RG predictions. The exponent value we obtained agrees with theory better than the commonly quoted value,[4] $\alpha=-0.026\pm0.004$, obtained along the λ line,[5] and agrees well with previous measurements at the vapor pressure,[3,6] made further from the transition.

A 270-mg helium sample was sealed inside an annular copper calorimeter with a low-temperature valve. The sample filled about 80% of the volume of the calorimeter to a uniform depth of 3 mm, the minimum dimension of the sample. The calorimeter was attached to a paramagnetic-salt thermometer described elsewhere.[7] This assembly was installed in a multistage thermal enclosure shown in Fig. 1 and placed in a helium bath at 4.2 K. The first stage of thermal isolation was held at 1.6 K by a continuously pumped cold plate.[8] The second stage consisted of a com-

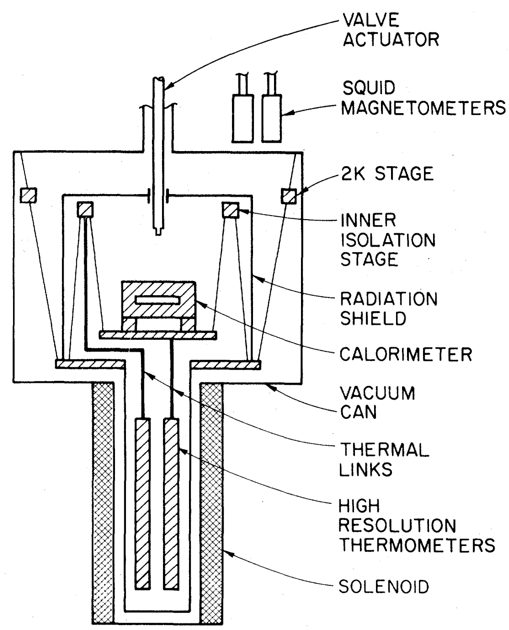

FIG. 1. Schematic view of thermal control system.

plete copper shield thermally controlled with a germanium thermometer and an integrating servo loop. The final stage was a copper ring which intercepted all connections to the calorimeter and was thermally controlled with a second paramagnetic salt thermometer.

Heat-capacity measurements were performed with use of a continuous heating (or cooling) method with constant power input. The thermometer signal as a function of time was digitally differentiated over periods from 5 to 80 sec to give heat-capacity information. Scanning rates were varied from 4×10^{-10} to 10^{-7} K/sec to search for nonequilibrium phenomena within the sample. In addition, during a measurement cycle the heater power was regularly stepped by a known amount to determine the background heat leak. At such times we observed the thermal response of the system by monitoring the associated transients. Within 0.5 μK of the transition the thermal response time was less than 8 sec, in agreement with values calculated from the thermal conductivity of the helium.[9] For scanning rates below 1.5×10^{-9} K/sec we found no difference between data obtained while heating and while cooling. For rates up to 8×10^{-9} K/sec, the data in the two-phase region were displaced a few nanodegrees, depending on scanning rate and temperature. Data affected in this way were averaged with similar data obtained while cooling at a comparable rate. For higher rates, some smearing of the transition was observed, and all affected data were rejected. The main scale factor uncertainty in the measurements ($\pm$ 1.5%) was due to the determination of the sample mass. Temperature-dependent uncertainties in other scale factors were estimated to be less than 0.1% over the range of measurements. In Fig. 2 we show the data for rates below 3×10^{-9} K/sec on a linear scale in the vicinity of the two-phase region. Each data point is an average of about twenty independent measurements which fall in bins 5×10^{-8} K wide. The temperature scale has been plotted relative to the location of the λ transition at the gas-liquid interface, $T_{\lambda 0}$, which was determined in the curve-fitting procedure described below.

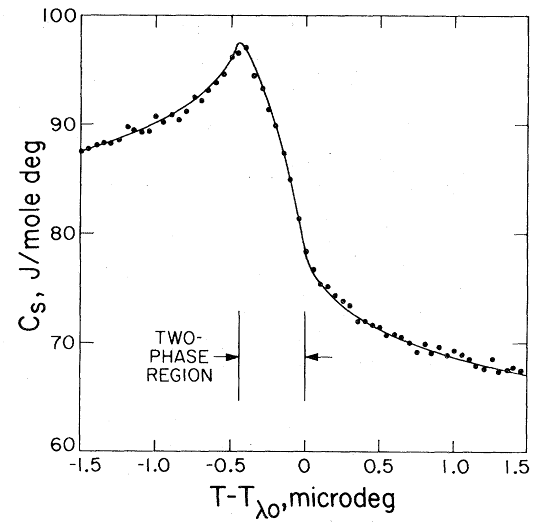

FIG. 2. Comparison of heat-capacity measurements with the predicted behavior (solid line) in the vicinity of the two-phase region.

To estimate the exponents α and α' (below T_λ), and other quantities of theoretical interest, we fitted the data corrected for the effect of gravity by functions of the form

$$C_s=(A/\alpha)t^{-\alpha}(1+Dt^x)+B \qquad (1)$$

where the coefficients may assume different values on each side of the transition. The effect of gravity was calculated in three different ways, all based on the assumption[3] that the heat capacity $C(T,h)$ of a given element of the sample at depth h below the surface is given by (1) but with t replaced by $t_h=|1-T/T_{\lambda h}|$. Here $T_{\lambda h}$ is the local value of the transition temperature given by $T_{\lambda h}=T_{\lambda 0}+\rho g h/S_\lambda$ where ρ is the density of the liquid, g is the acceleration due to gravity, and S_λ is the slope of the λ line. Initially, the heat capacity of the sample was calculated simply by integrating $C(T,h)$ over the height H of the sample. This function was then fitted to the raw data outside the two-phase region by a standard least-squares procedure. Using a second approach, we first calculated $C(T,0)$, the heat capacity at the gas-liquid interface, from the observations and then fitted (1) to the corrected data. Unfortunately, because of the negative slope of the λ line, both these methods yield no information in the temperature interval δT_2 below $T_{\lambda 0}$, which degrades the accuracy of the determination of α'. To improve this situation we modified the second method below T_λ to calculate the heat capacity at the *bottom* of the sample, and then incremented the temperature of each datum point by an amount δT_2. This approach makes use of the relationship $C(T,H)=C(T+\delta T_2,0)$ which is a direct consequence of the assumption described above, and generates estimates of $C(T,0)$ with the desired distribution about $T_{\lambda 0}$. The three methods used to correct for the gravity distortion were found to give similar results for the optimum parameter values: Here we describe the results obtained with the last method, for which the uncertainties were least.

In the curve-fitting analysis we rejected the data in the two-phase region because of larger uncertainties in the gravity corrections arising from the rapid variation of the correlation length with height near the phase boundary.[10] Outside this region, we estimate such contributions to be less than ± 0.03 J/mole K, which is negligible for our purposes. Initially, gravity corrections corresponding to a logarithmic singularity in C_s were calculated; they were then updated with use of the first estimate of the best-fit parameters to our data. Finally, the best-fit parameters were redetermined, and found to be consistent with the previous set. From this information we calculated the heat capacity of the sample through the two-phase region and compared it with the observations. This curve is shown in Fig. 2. It can be seen that we obtain a very good fit even when the uncertainties in the estimated gravity effect are relatively large. This reinforces our belief that we have observed the equilibrium heat capacity of the sample throughout this region. To our knowledge this is the first time such data have been reported. We then used the data within and near the two-phase region to decrease the uncertainties in the estimate of $T_{\lambda 0}$, which was previously a free parameter. This point was determined to within ± 10 nK by comparing the data with the calculated functional form, and was then used as a constraint in the remainder of the curve-fitting analysis. In Fig. 3 we show the data, corrected for the effect of gravity, on a logarithmic temperature scale plotted relative to $T_{\lambda 0}$. The $C(T,0)$ data below T_λ extend closer to $T_{\lambda 0}$ than the boundary of the two-phase region because of the temperature increment δT_2 which is part of the gravity correction.

A simultaneous estimate of all five free parameters in (1) on each side of T_λ leads to unacceptably large uncertainties in α, α', and A/A', the quantities of primary interest. Instead, we set D and D' equal to the values determined by Ahlers[4] (-0.022, -0.020), and $x = x' = 0.5$, close to the RG estimate, and evaluated the remaining parameters. This approach gave the results $\alpha = -0.0129 \pm 0.0033$, $\alpha' = -0.0125 \pm 0.0036$, and $B - B' = 0 \pm 4.4$, where the uncertainties correspond to the 2σ statistical confidence limit and the analysis covers the range $3 \times 10^{-8} \leq t \leq 10^{-3}$. It is clear that these results support the scaling pre-

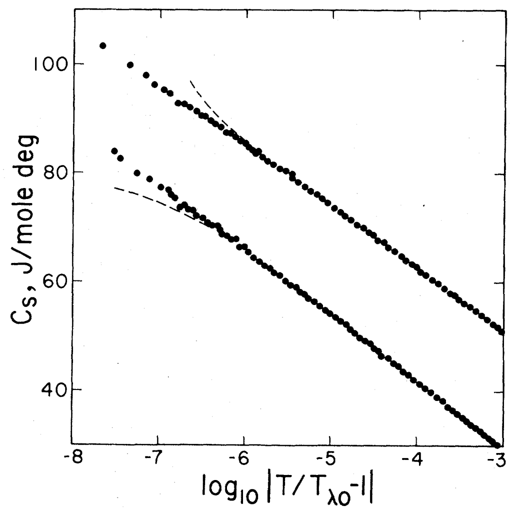

FIG. 3. Heat-capacity measurements over a wide range of temperature (upper curve, $T < T_\lambda$; lower curve, $T > T_\lambda$) corrected for the effect of gravity as described in the text. Below T_λ this correction includes a displacement of the data closer to the transition by an amount δT_2. The broken lines show the location of the data before the gravity corrections were applied.

diction $\alpha = \alpha'$ and that the heat capacity is continuous through T_λ. With the additional constraint $\alpha = \alpha'$ we obtained $\alpha = -0.0127 \pm 0.0026$, $A/A' = 1.058 \pm 0.004$, and $B - B' = 0.1 \pm 3.1$. Since improved estimates for D and D' may become available, we determined the sensitivity of our results to changes in these parameters. For the optimum values of α and A/A' we obtained sensitivities $d\alpha/dD = -0.105 \pm 0.01$ and $d(A/A')/dD = (-2.7 \pm 0.3) \times 10^{-3}$ for a simultaneous change of the same magnitude in both D and D'. Thus a 50% change in the values of D and D' would not change our results significantly.

Our experiment can be compared with two classes of previous measurements made further from the λ point. Three sets of directly comparable heat-capacity measurements[3,6] give[11] $-0.016 \lesssim \alpha \lesssim -0.020$ and $1.068 \lesssim A/A' \lesssim 1.081$ in the range $t \gtrsim 10^{-6}$. With the additional assumption of universality our results can also be compared with experiments along the λ line[5] and as a function of ^{3}He concentration,[6] which give $-0.022 \lesssim \alpha \lesssim -0.026$ and $1.088 \lesssim A/A' \lesssim 1.11$ for $t \gtrsim 5 \times 10^{-6}$. Our results, obtained from data extending significantly closer to the transition, give somewhat better agreement with the RG predictions[1] ($\alpha = -0.007 \pm 0.006$ and $A/A' \approx 1.03$) than those obtained previously. Further improvements in resolution will require either a more careful treatment of the distortions near T_λ or a reduction in gravity, for example by performing the measurements in space.

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