Atmospheric Neutrinos, Astrophysical Neutrons, and Proton-Decay Experiments

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Simple analytical estimates of the fluxes of atmospheric muons and neutrinos are presented. It is shown that the geomagnetic field strongly suppresses the fluxes of atmospheric neutrinos below 1 GeV from directions of low magnetic latitudes and may open a window for neutrino astronomy. The validity of these estimates is demonstrated through indirect tests.

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If proton decay[1] is ever discovered it will have important and profound implications for the physical understanding of the universe and its evolution. An experimental proof of proton decay requires the detection of a signal that is clearly different from that of background events.[2] These background events in the massive tracking calorimeters used deep underground to detect proton decay are believed[3] to be due to atmospheric neutrinos (mainly ν_μ's and ν_e's from the decays of π's, K's, and μ's produced in cosmic-ray–induced air showers). However, the same cosmic rays that produce atmospheric neutrinos also produce neutrinos when they propagate through matter that may be near the sources of the cosmic rays (e.g., ejected or falling matter around supernovae, pulsars, galactic nuclei, quasars, and black holes) and through interstellar matter.[4] If there are significant fluxes of such astronomical neutrinos they should be seen by the proton-decay detectors, in particular in directions of low magnetic latitude where the Earth's magnetic field strongly suppresses the primary cosmic-ray flux that reaches the atmosphere and produces atmospheric neutrinos.

In this paper I derive simple analytical formulas for the fluxes of atmospheric muons and neutrinos which display explicitly their dependence on energy, zenith angle, and geomagnetic cutoff (which depends on geographic location, zenith, and azimuth). If the ν background in proton-decay experiments is due to atmospheric ν's, it should exhibit these dependences and in particular a significant asymmetry between the total fluxes of up-going and down-going ν's at all the sites of the major proton-decay experiments, which results from the strong geomagnetic suppression of the ν fluxes from directions of low magnetic latitudes. The reliability of the predictions is demonstrated by showing that (i) the predicted spectra of atmospheric π's and μ's, which are the main source of low-energy atmospheric ν's, agree well with the measured spectra, and (ii) the predicted ν fluxes agree very well with previous numerical estimates of these fluxes in the absence of geomagnetic effects.[5]

The differential fluxes of leptons, $l=\mu, \nu$, from $M\to\mu\nu$ decays are related to the differential fluxes of their parent mesons, $M=\pi^\pm, K^\pm$, through

$$\frac{dF_\nu}{dE} = \sum_M \alpha_M \int_0^{\alpha_M^{-1}} dx \int_{\alpha_M E}^{\infty} \delta(E-xE_M)\frac{dF_M}{dE_M}\, dE_M, \quad (1)$$

$$\frac{dF_\mu}{dE} = \sum_M \alpha_M \int_{\beta_M^{-1}}^{1} dx \int_E^{\beta_M E} \delta(E-xE_M)\frac{dF_M}{dE_M}\, dE_M, \quad (2)$$

where $\alpha_M \equiv m_M^2/(m_M^2-m_\mu^2)$ and $\beta_M \equiv M_M^2/m_\mu^2$. Equations (1) and (2) are valid for energies that satisfy $m_M^2/E^2 \ll 1$. The differential fluxes of the parent mesons (i.e., the mesons that decayed into the respective leptons and photons), which are produced by a primary flux of $dF_p/dE=cE^{-p}$ nucleons/cm^2 sec sr GeV at the top of the atmosphere, can be easily estimated if one assumes that the cross sections for inclusive production of hadrons in nucleon–air-nucleus collisions obey Feynman scaling: $dn_h/dx=f_h(x)$; $h=N, M$, where N denotes either a proton or a neutron and f_h depends on x, the fraction of the momentum of the projectile carried by the produced hadron h, and not on energy. They are given by[6]

$$\frac{dF_M}{dE} = \frac{B_M}{1+\gamma_M E}\, g_M^{\,\mathrm{at}} \frac{dF_p}{dE}. \quad (3)$$

In Eq. (3) B_M is the branching ratio for the decay $M\to\mu\nu$ ($B_\pi=1$, $B_K=0.632$) and $\gamma_M^{-1} \equiv m_M c h_0 \sec\theta/\tau_M$, where τ_M is the proper lifetime of M, $h_0 \cong 6.3$ km is the scale parameter of the upper atmosphere, and θ is the zenith angle of the incident cosmic-ray particle. The production coefficients $g_M^{\,\mathrm{at}} \equiv g_M/(1-g_N)$, where $g_h \equiv \int_0^1 x^{p-1} f_h(x)\, dx$, were calculated by Liland[7] from accelerator data on inclusive production of hadrons in p-

hydrogen collisions. (They display explicitly the approach to scaling with increasing energy.) Because of the nuclear enhancement of the multiplicity they have to be multiplied by an effective factor of approximately 1.15 when applied to p-air-nuclei collisions.

When Eq. (3) is substituted into Eqs. (1) and (2) we find that

$$dF_\nu/dE \cong \sum_M g_M{}^{\rm at} B_M\, \alpha_M K_M(\alpha_M E), \qquad (4)$$

$$dF_\mu/dE \cong \sum_M g_M{}^{\rm at} B_M \alpha_M [K_M(E) - K_M(\beta_M E)], \qquad (5)$$

where $K_M(E) \equiv cE^{-p}/[p+(p+1)\gamma_M E]$. At low energies Eq. (5) has to be modified to include the effects of μ decay and energy losses in the atmosphere. The energy losses are given approximately by $\rho^{-1}\,dE/dx = \alpha \sim 2.06$ MeV cm^2/gm; consequently muons at zenith angle θ with energy E at sea level (final atmospheric depth $\lambda_F \cong 1030 \times \sec\theta$ gm/cm^2) are born with initial energy E_0 which is given approximately by $E_0 \cong E + \alpha(\lambda_F - \lambda_0)$. Their average energy $\bar{E}$ in the atmosphere is $\bar{E} \cong E + \alpha(\lambda_F - \lambda_0)/2$ and their probability to reach sea level before decay is given approximately by[6] $S_\mu(\bar{E}) = (\lambda_0/\lambda_F)^{1/\gamma_\mu \bar{E}}$, where $\lambda_0 \cong 120$ gm/cm^2 and $\gamma_\mu{}^{-1} \equiv m_\mu c h_0 \sec\theta/\tau_\mu \simeq 1.04$ GeV. Equation (5) corrected for μ decay and energy losses in the atmosphere yields for sea-level muons

$$dF_\mu/dE \cong \sum g_M{}^{\rm at} B_M\, \alpha_M S_\mu(\bar{E})[K_M(E_0) - K_M(\beta_M E_0)], \qquad (6)$$

where $g_M{}^{\rm at}$ are evaluated at energy E_0. In Fig. 1, Eq. (6) is compared with measured fluxes of sea level μ's at zenith angles $\theta = 0$[8] and $\theta = 75°$[9] under the assumption of a primary cosmic-ray flux[10] of $1.6E^{-2.67}$ nucleons/cm^2 sec sr GeV. Figure 1 demonstrates excellent agreement between theory and experiment.

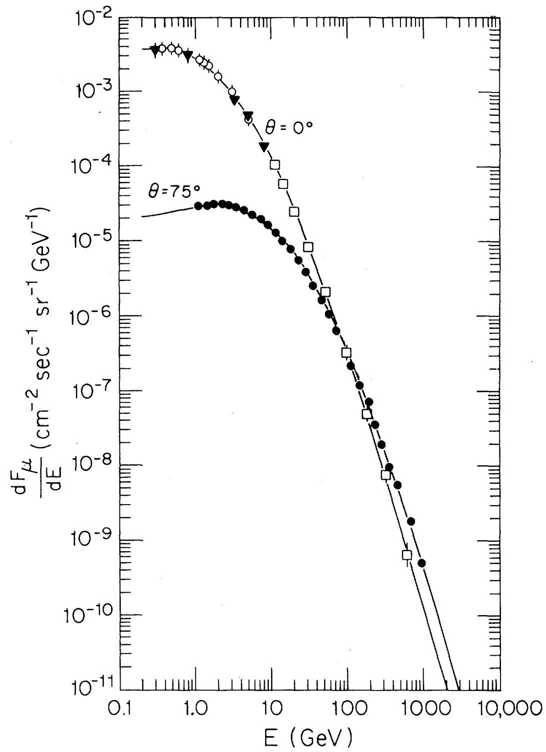

FIG. 1. Comparison between the present theoretical predictions and measured fluxes of sea-level muons at zenith angles $\theta = 0°$ and $\theta = 75°$.

The ν spectra from $\mu \to e\nu_e\nu_\mu$ are given by standard weak-interaction theory (to distinguish this ν flux from F_ν the ν flux due to $M \to \mu\nu$ decays I denote it by I_ν):

$$\frac{dI_\nu}{dE} = \int_0^1 dx \int_E^\infty \sum_{i=0}^{3} A_i x^i\, \delta(E - xE_\mu)\frac{dF_\mu}{dE_\mu}\, dE_\mu, \qquad (7)$$

where $A = (\frac{5}{3}, 0, -3, \frac{4}{3})$ for ν_μ's and $A = (2, 0, -6, 4)$ for ν_e's and where I have neglected small corrections due to the μ polarization. The probability that a muon will decay in flight is given approximately by $D_\mu(E) = 1 - S_\mu(E) \cong \delta/(\delta + \gamma_\mu E)$, where $\delta \cong \ln(\lambda_F/\lambda_0)$. Equations (6) and (7) yield then the following ν fluxes produced by μ decays in the atmosphere:

$$\frac{dI_\nu}{dE} = \sum \alpha_M B_M g_M{}^{\rm at}(1 - \beta_M{}^{-p}) \sum_{i=0}^{3} \frac{\delta A_i}{(p_i\,\delta + p_{i+1}\gamma_\mu E)}\, \frac{cE^{-p}}{p}, \qquad (8)$$

where $p_i \equiv p + i$. Note that the ν fluxes from μ decays depend on zenith angle through the dependence of δ and γ_μ on zenith angle. Figures 2 and 3 compare the analytical estimates as given by Eqs. (4) and (8) for the total fluxes of atmospheric ν_μ's + $\bar{\nu}_\mu$'s and ν_e's + $\bar{\nu}_e$'s in the absence of geomagnetic effects (full lines) and the numerical estimates of these fluxes by Volkova.[5] The figures demonstrate excellent agreement between these simple analytical formulas and the numerical estimates of Volkova.[5]

The Earth's magnetic field prevents primary cosmic rays with energy below the geomagnetic

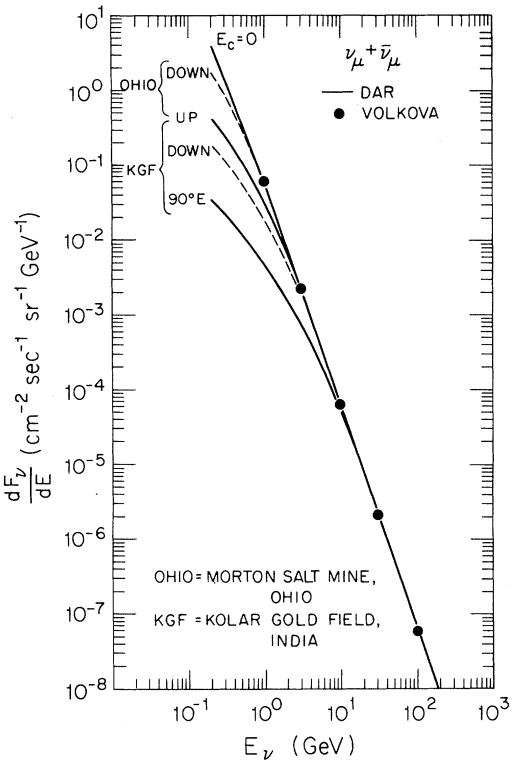

FIG. 2. Predicted fluxes of atmospheric ν_μ's $+\bar{\nu}_\mu$'s from meson and muon decays at the sites of the major proton-decay experiments.

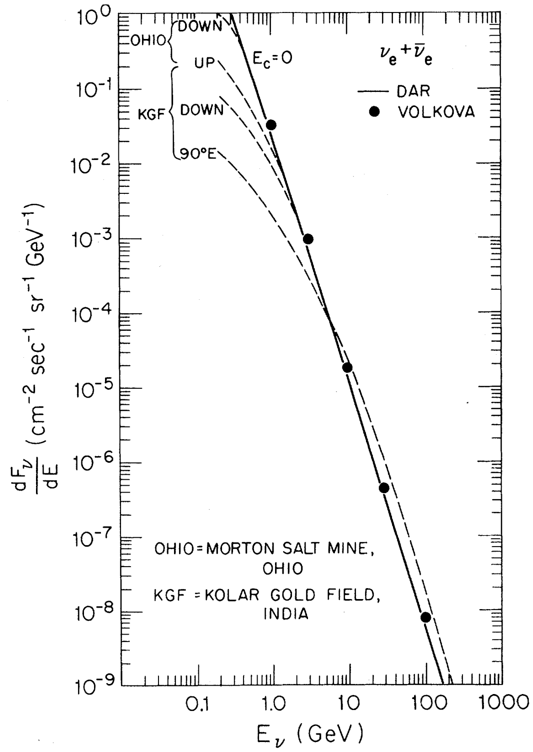

FIG. 3. Predicted fluxes of atmospheric ν_e's $+\bar{\nu}_e$'s from muon decays at the sites of the major proton-decay experiments.

cutoff energy E_c from reaching the atmosphere. For primary protons, the cutoff energy $E_c = (p_c^2 + m_p^2)^{1/2}$ is given approximately by the Stormer formula:

$$p_c = 59.4 \cos^4(\lambda/R^2)[1 + (1 - \cos^3\lambda \sin\theta \sin\varphi)^{1/2}]^2 \text{ GeV}/c,$$

where λ is the magnetic latitude, θ is the zenith angle, φ is the azimuth measured clockwise from the magnetic north, and R is the distance from the dipole center of Earth in units of Earth radii.

One can easily show that for a thick atmosphere and $dn_N/dx \sim$ const the effective flux of nucleons with energies below E_c that are produced in air showers is given by $dF_N/dE \cong g_N c E_c^{-(p-1)}/E$. For the choice $dn_M/dx \sim (1-x)^3/x$, I find that the atmospheric fluxes of mesons with energy below E_c (i.e., $x_c = E/E_c < 1$) are given by

$$\frac{dF_M}{dE} = \frac{g_M^{\text{at}}(E_c) B_M c E_c^{-p}}{(1+\gamma_\mu E) B(p-1,4)} [G_1(x_c, p) + g_N G_2(x_c)],$$

$$G_1(x,p) \equiv 1/(p-1)x - 3/p + 3x/(p+1) - x^2/(p+2), \quad G_2(x) \equiv (\tfrac{11}{6} - \ln x + 3x - \tfrac{3}{2}x^2 + \tfrac{1}{3}x^3)/x. \tag{9}$$

In Eq. (9), G_1 is due to nulceons with energies above E_c while G_2 is due to nucleons with energies below E_c. (Except for extremely low values of x_c, $G_1 \gg g_N G_2$.) When we substitute Eq. (9) into Eq. (1) we find that the atmospheric fluxes of ν's obtained directly from $M \to \mu\nu$ decays have the following form at ν energies below E_c/α_M:

$$\frac{dF_\nu}{dE} \cong g_M^{\text{at}}(E_c) B_M \alpha_M \frac{cE_c^{-p}}{p}\left\{1 + \frac{p}{B(p-1,4)}[W_1(p, z_M) + g_N W_2(z_M)]\right\},$$

$$W_1(p,z) = (1/z - 1)/(p-1) + 3\ln z/p + 3(1-z)/(p+1) - (1-z^2)/2(p+2), \tag{10}$$

$$W_2(z) = \tfrac{3}{2} - \ln z/z - 17/6z - 3\ln z + 3z/2 - z^2/6,$$

where $z_M = \alpha_M E/E_c < 1$. Similar analytical expressions can be derived for the fluxes of atmospheric μ's from $M \to \mu\nu$ decays and for atmospheric ν_μ's and ν_e's from subsequent $\mu \to e\nu_e\nu_\mu$ decays.

Using these analytical formulas and the geomagnetic cutoffs that were calculated by Cooke[11] I calculated the atmospheric ν fluxes at the sites of the major proton-decay experiments. I found that the geomagnetic field strongly suppresses the fluxes of atmospheric ν's below 1 GeV from directions of low magnetic latitudes (large geomagnetic cutoffs) and produces a strong asymmetry between up-going ($\theta > 90°$) and down-going ($\theta < 90°$) neutrinos: At the northern sites the geomagnetic field has only a little effect on down-going ν's while it strongly suppresses the total fluxes of up-going ν's; the latter are approximately the same ($\langle E_c \rangle \cong 8.5$ GeV) at all the sites including the Kolar gold field in India, where the geomagnetic field strongly suppresses also the total flux of down-going ν's ($\langle E_c \rangle \sim 15.45$ GeV). In Figs. 2 and 3 the present results are shown for the Morton salt mine site in Ohio and the Kolar gold field site in India. For reference I also plot the ν fluxes in the absence of a geomagnetic field and the predicted flux of horizontal ν's ($\theta = 90°$) from the east at the Kolar gold field ($E_c = 54.5$ GeV). Since $\sigma_\nu \sim E_\nu$, the ratio of ν interactions by up-going and down-going ν's is given by the ratio of $\int E(dF_\nu/dE)dE\,d\Omega$ for $\theta > 90°$ and $\theta < 90°$. For a threshold visible ν energy between 0.2 to 0.3 GeV this ratio for the northern sites is between 0.54 ± 0.07 and 0.64 ± 0.06, while it is 1.56 ± 0.22 at the Kolar gold field in India.

In conclusion, if the ν background in proton-decay experiments is mainly due to atmospheric neutrinos, it should exhibit the anticipated dependences on zenith, azimuth, and geographic location for both up-going and down-going ν's. For directions of low magnetic latitudes the geomagnetic cutoff strongly suppresses the fluxes of atmospheric neutrinos and may open a window for ν astronomy with proton-decay detectors. Although I believe that the present analytic model predicts reasonably well the fluxes of atmospheric ν's, neither the analytic model nor Monte Carlo codes (one-dimensional codes in particular) can be sufficiently reliable to determine whether or not neutrinos oscillate[12] by comparison of calculated and measured fluxes of atmospheric ν's, because both calculations are only approximate, because the uncertainties in the experimental input data are large, and because the results at low E_ν are very sensitive to the input and to the approximations.

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