Comments on Newtonian Particlelike Behavior of Sine-Gordon Solitons

The pioneering linear perturbational analysis[1] of Fogel, Trullinger, Bishop, and Krumhansl (hereafter I) along with the works of Nakajima, Sawada, and Onodera[2] creates a general belief that sine-Gordon solitons behave like Newtonian particles in the presence of small perturbations. This type of behavior is also very familiar in applications such as Josephson transmission lines and magnetic domain walls. But very recently Fernandez *et al.*[3] (hereafter II) raised a controversy when they said that sine-Gordon solitons do not at all behave like Newtonian particles. Here I examine the behavior of a sine-Gordon soliton in the presence of two different types of inhomogeneities using the mathematical technique of II and show the limiting Newtonian behavior. Again I also reexamine the case considered in II and find that contrary to the belief of Fernandez *et al.* their work does not contradict that of I.

(a) When we consider the sine-Gordon equation in the presence of a harmonic driving force $\chi e^{i\Omega t}$, velocity $V(t)$ becomes

$$V(t)=\frac{\pi}{4\gamma(t)}\left[\frac{\chi e^{i\Omega t}}{i\Omega}-\left\{\frac{i\Omega\chi e^{i\Omega t}}{1-\Omega^2}-\frac{i\chi e^{it}}{2(1-\Omega)}+\frac{i\chi e^{-it}}{2(1+\Omega)}\right\}\right]. \tag{1}$$

The last three terms within the brackets of Eq. (1) are the contributions due to du_∞/dt. It can be seen that for small χ and t (neglecting terms χt and higher powers of χt) the contribution of du_∞/dt is zero. Hence in the limit of small $V(t)$, when $\gamma(t)\simeq 1$, $8\,dV/dt \simeq 2\pi\chi\exp(i\Omega t)$, in agreement with the works of Fogel, Trullinger, and Bishop.[4]

(b) Again when we consider a local impurity [as in Eq. (7) of I],

$$8\gamma(t)V(t)=(2\alpha/\gamma_0)\int_0^t dt'\,\gamma(t')\{\operatorname{sech}\gamma[x_0/\gamma_0+\beta t+\int_0^t V(t')dt']-\operatorname{sech}\gamma[\beta t+\int_0^t V(t')dt'-x_0/\gamma_0]\}$$

so that for $V(t)\ll 1$ and for small t, it exhibits Newtonian particlelike behavior.

(c) In the presence of a constant external field χ Fernandez *et al.* (as in II) obtained to first approximation $u_\infty\simeq 1-\cos t$, and for small t one of the contributing terms of u_∞ cancels the term responsible for Newtonian behavior. When the external field is not time dependent the exact solution $u_\infty=\text{const}$, independent of t, is to be considered so that for small t, $8\,dV/dt=2\pi\chi/(1+V^2-V^4)\gamma(t)$. For small V, neglecting V^4 and higher terms, $8\,dV/dt=2\pi\chi(1-V^2)^{3/2}$, in nice agreement with general perturbation calculations.[5] If terms containing V^2 can be neglected $8\,dV/dt=2\pi\chi$ (Newtonian behavior as discussed in I).

(d) The linear perturbation analysis leads to the correct result when the contributions of zero mode and scattering states to the perturbation are considered separately. It will definitely lead to complications[6] if one tries to obtain the total perturbation effect by integrating the scattering states in the complex plane which involves the pole at $k=\pm i$ (as in II). As the adiabatic perturbation analysis shows, the numerical results of II are only indicative of the explicit time dependence of $u_\infty(t)$, i.e., the effect of noise, etc.

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Received 23 September 1983
PACS numbers: 03.20.+i

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