Damping of Second Sound near the Superfluid Transition of ^{4}He under Pressure

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New measurements of the damping D_2 of second sound in ^{4}He near T_λ which are a factor of 5 more precise than previous results are reported. They span a wide pressure range, whereas previous data existed only at vapor pressure. The pressure dependence of D_2 agrees well with renormalization-group theory predictions, but the detailed temperature dependence differs slightly from that predicted on the basis of the presently available theory.

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Recent comparisons[1-4] of the renormalization-group theory for the dynamics of the superfluid transition[5-8] in ^{4}He with experimental measurements of the thermal conductivity[1,9] above the transition temperature T_λ have shown that this dynamics is remarkably rich in interesting features.[10] It involves a nonuniversal weak- to strong-coupling crossover at a small reduced temperature associated with an anomalously small bare dynamic coupling constant.[1] It also involves a universal near instability[7] of the dynamic scaling fixed point at the physical dimensionality $d = 3$ which is associated with slow transients[7,10,11] that prevent direct experimental observation of the asymptotic critical behavior. These features seem to be contained accurately in both theory and experiment and it is in part because of the richness of the problem that we may regard the level of agreement with experiment that exists so far as a major success in the field of critical phenomena. There are two other reasons to view this system as a very stringent testing ground for the theory, both having to do with technical aspects of the problem. The first is the well-known particularly suitable nature of the superfluid transition for high-resolution experimental study.[12] The second is related to the existence of the weak-coupling regime mentioned above. Whereas the basic model given by the theory may well contain the dynamics exactly, specific predictions must be obtained through a perturbation expansion. In the weak-coupling regime, this expansion is a controlled approximation, and, in principle, an unambiguous test of the theory should thus be possible in a range where the reduced temperature is still a small parameter. These circumstances, and the unique opportunities which they provide, in our view warrant considerable additional experimental and theoretical effort to complete the picture. In the present paper, we add to the experimental information about this transition by reporting on new measurements of the damping D_2 of second sound for $T < T_\lambda$ which are roughly a factor of 5 more precise than the best previous data. They cover the entire pressure range of the transition, whereas previous results[13,14] were only at vapor pressure.

We used cylindrical copper cavities with second-sound generators (heaters) at one end and bolometers at the other. The amplitude of a sound burst of frequency f up to 100 kHz was measured after each of several traversals back and forth through the cavity and its decay yielded the attenuation α. Typical data in the range far from T_λ are shown in Fig. 1 as a function of f^2. The finite intercept of the straight line through the data is due in part to thermal wall losses.[15]

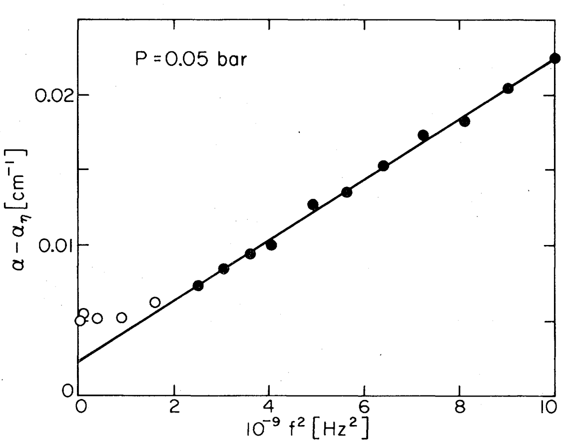

FIG. 1. The total measured attentuation, corrected for the viscous wall-loss contribution α_η, as a function of the square of the frequency f for $t = 0.10$. The data at small f, shown as open circles, were omitted in the analysis because they are influenced significantly by diffraction effects and frequency-dependent thermal wall-loss contributions. The slope of the solid line through the data is proportional to D_2.

Even at these temperatures where the bulk attenuation α_2 is relatively small, α is seen to be a linear function of f^2, and the slope α_2/f^2, which is proportional to D_2, can be determined with meaningful accuracy. The damping D_2 is derived from

$$D_2 = 2u_2^3(\alpha_2/\omega^2), \tag{1}$$

where $\omega = 2\pi f$ and u_2 is the second-sound velocity.

The results of our measurements near vapor pressure are shown as solid circles in Fig. 2. Whenever the error is larger than the circle, an appropriate error bar is given. For $t \equiv 1 - T/T_\lambda \gtrsim 0.1$, our data agree well with those of Hanson and Pellam.[16] For smaller t, the difference of up to 20% is easily attributed to temperature-scale uncertainties in the older data. For $t < 10^{-3}$, our data agree well with those of Crooks and Robinson[13] and of Ahlers.[14] However, our experimental errors are only about 2 to 4% and thus approximately a factor of 5 smaller than those of Ref. 13 and a factor of 15 smaller than those of Ref. 14. Near $t \simeq 10^{-1.5}$, the systematic difference between our data and those of Ref. 13 is no larger than the estimated errors in the older data.[13]

A comparison of the predictions of the renormalization-group theory with the experimental thermal conductivity *above* T_λ[1-4] fixes a small number of nonuniversal parameters which the theory cannot give independently. Thereafter, values of the dynamic variables f and w of the theory are known as a function of t and can be used to calculate D_2 *below* T_λ, in principle without any further adjustable parameters. The dependence of D_2 upon f and w was derived by Dohm and Folk[3,17] from the symmetric planar spin model (model E) of Halperin, Hohenberg, and Siggia.[5] Model E is expected to represent the dynamics of liquid helium only approximately and does not contain accurately the coupling of the specific heat to the order parameter. The asymmetric planar spin model (model F),[5] on the other hand, is expected to reproduce the dynamics of this transition exactly, but the corresponding calculation for D_2 has not yet been carried out for this model. The model-E formulas for D_2 when used with the dynamic variables f and w derived from a fit[1,4] of the experimental thermal conductivity above T_λ[1,9] by a model-F perturbation expansion[7,8] yielded values[1,4] of D_2 in remarkably good agreement with the older data.[13,14,16] Predictions[1] for D_2 based on two sets of thermal-conductivity data obtained in different cells (A and D) are shown as solid (cell D) and dashed (cell A) lines in Fig. 2. The difference between them may be regarded as an indication of the uncertainty in the prediction due to uncertainties in the conductivity measurement. Either line is consistent with the old data but both differ systematically by over 20% from our new results near $t \simeq 10^{-2}$. It is in this range of relatively large reduced temperatures that we expect the theory to yield numerically accurate predictions because the dynamic coupling constant f is a small parameter and the truncated perturbation expansion in f of model F should be quite accurate.

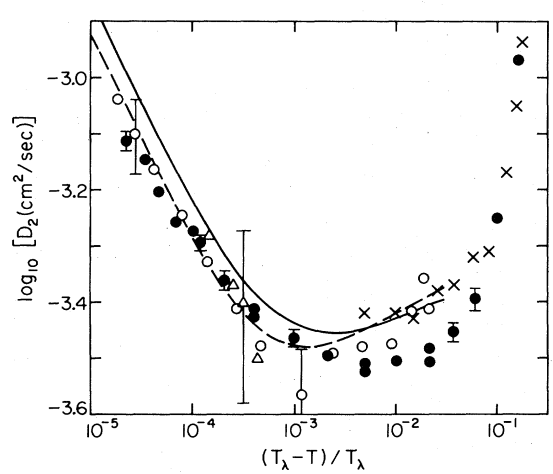

FIG. 2. The damping D_2 as a function of $(T_\lambda - T)/T_\lambda$ on logarithmic scales. Open circles: Ref. 13. Open triangles: Ref. 14. Crosses: Ref. 16. Solid circles: this work. For Refs. 13 and 14, only a few typical error bars are shown. For the present work, error bars are shown whenever they extend beyond the symbol. The prediction of Ref. 1 discussed in the text is represented by the solid (cell-D based) and dashed (cell-A based) lines.

In principle, all adjustable parameters in the theory were determined from the conductivity measurements. In practice, the prediction for D_2 involves the universal ratio of the longitudinal to the transverse correlation length ξ^L/ξ^T and the static coupling constant u. Both of these *static* parameters are given by the theory but at present are not known with high numerical accuracy. Although u enters D_2 in a complicated way, adjustment of either u or ξ^L/ξ^T resulted in a nearly parallel vertical displacement of the curves in Fig. 2. We therefore introduce a *single* new adjustable parameter by changing u from its second-order ϵ-expansion fixed-point value $u^* = 0.040$ to 0.030. This yields agreement with our new data at $t = 5 \times 10^{-3}$ where cell A and cell D give the same D_2. The result based on cell D, together with our data for D_2, is shown in Fig. 3(a) as a solid line. It now agrees with the measurements also for $t \leq 10^{-4}$, but at intermediate values of t, where the truncated perturbation expansion should still be valid, the data and the theory differ by as much as 15%. The disagreement with the cell-A-based prediction is somewhat larger.

In Fig. 3(a), we also show the more recent prediction by Ferrell and Bhattacharjee[18] which is based on their own treatment of the dynamics[10] and on the same cell-D conductivity data[9] for $T > T_\lambda$. The dashed line corresponds to the solid curve in Fig. 1 of Ref. 18. This theory also differs from the data by about 15% near $t \simeq 10^{-3}$. For larger t, the predicted D_2 rises rather too rapidly with increasing t.

The value of u^* (and of ξ^L/ξ^T) is expected to be independent of pressure. We thus use $u = 0.03$ obtained at vapor pressure also at the higher pressures and *without any further adjustments* compare the predictions based on the dynamic variables f and w derived[1] from cell-D thermal-conductivity data under pressure[9] with our D_2 measurements at 14.7 and 22.3 bars in Figs. 3(b) and 3(c), respectively. At $t = 5 \times 10^{-3}$, the agreement persists at the level of a few percent, even though the value of D_2 increases by 30% over that pressure range. We regard the correct prediction of the pressure dependence in this temperature range as a significant accomplishment of the theory. At sufficiently small t, there is also consistency between theory and measurements, even though here the pressure dependence has become negative. At intermediate temperatures, the discrepancy revealed at vapor pressure persists systematically at all pressures.

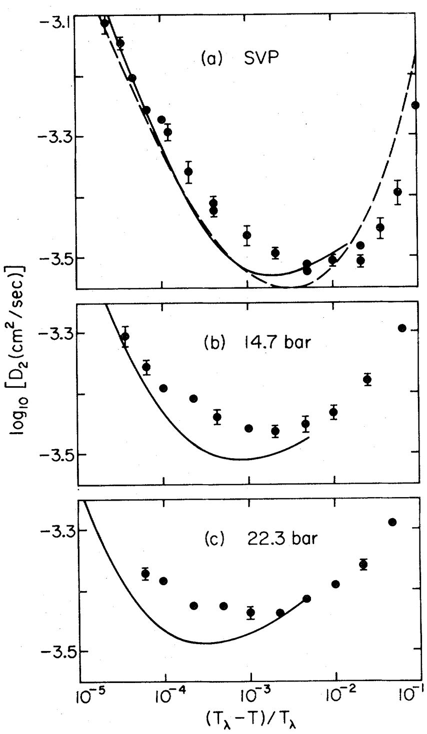

FIG. 3. The damping D_2 as a function of $(T_\lambda - T)/T_\lambda$ on logarithmic scales at several pressures. Solid lines: cell-D–based prediction with the static coupling constant u adjusted to 0.03 (instead of 0.04 as in Fig. 2 and Ref. 1). For the highest pressures, the lines terminate at $t = 0.005$ because the cell-D thermal-conductivity data were only for small t. The dashed line in (a) is the prediction given in Ref. 18.

In conclusion, the new measurements of D_2 agree in their pressure dependence but differ in their detailed temperature dependence with the theoretical prediction based on the symmetric planar spin model. The temperature dependence differs also from the "high-temperature expansion" of Ferrell and Bhattacharjee.[18] It would be highly desirable to have a calculation of D_2 based entirely on the asymmetric model (model F) as well as more accurate theoretical values of the static parameters u^* and ξ^L/ξ^T to see if the remaining differences between experiment and the renormalization-group theory will be eliminated.

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[1]G. Ahlers, P. C. Hohenberg, and A. Kornblit, Phys. Rev. Lett. 46, 493 (1981), and Phys. Rev. B 25, 3136 (1982).

[2]V. Dohm and R. Folk, Z. Phys. B 40, 79 (1980).

[3]V. Dohm and R. Folk, Phys. Rev. Lett. 46, 349 (1981).

[4]V. Dohm and R. Folk, Z. Phys. B 45, 129 (1981).

[5]B. I. Halperin, P. C. Hohenberg, and E. D. Siggia, Phys. Rev. Lett. 32, 1289 (1974), and Phys. Rev. B 13, 1299 (1976); E. D. Siggia, Phys. Rev. B 13, 3218 (1976).

[6]P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. 49, 435 (1977).

[7]C. DeDominicis and L. Peliti, Phys. Rev. Lett. 38, 505 (1977), and Phys. Rev. B 18, 353 (1978).

[8]V. Dohm, Z. Phys. B 31, 327 (1978).

[9]G. Ahlers, Phys. Rev. Lett. 21, 1159 (1968), and in

Proceedings of the Twelfth International Conference on Low Temperature Physics, edited by E. Kanda (Academic, Japan, 1971), p. 21.

[10]A somewhat different theoretical approach, which to us appears to be less systematic than the perturbation expansion for the renormalization-group models of Halperin, Hohenberg, and Siggia (Ref. 5), has been developed by R. A. Ferrell and J. K. Bhattacharjee, Phys. Rev. Lett. 42, 1638 (1979), and J. Low Temp. Phys. 36, 165 (1979). This work also predicts some of the features derived from the renormalization-group theory.

[11]P. C. Hohenberg, B. I. Halperin, and D. R. Nelson, Phys. Rev. B 22, 2373 (1980).

[12]See, for instance, G. Ahlers, Rev. Mod. Phys. 52, 489 (1980).

[13]M. J. Crooks and B. J. Robinson, Phys. Rev. B 27, 5433 (1983), and Physica (Utrecht) 107B, 339 (1981).

[14]G. Ahlers, Phys. Rev. Lett. 43, 1417 (1979).

[15]V. Steinberg, private communication.

[16]W. B. Hanson and R. J. Pellam, Phys. Rev. 95, 321 (1954).

[17]V. Dohm and R. Folk, Z. Phys. B 41, 251 (1981).

[18]R. A. Ferrell and J. K. Bhattacharjee, Phys. Rev. Lett. 51, 487 (1983), and references therein. The background correction introduced in this paper was recently criticized by G. Ahlers and P. C. Hohenberg, to be published.