Quantum Shot Noise in Tunnel Junctions

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The current and voltage fluctuations in a normal tunnel junction are calculated from microscopic theory. The power spectrum can deviate from the familiar Johnson-Nyquist form when the self-capacitance of the junction is small, at low temperatures ($C \lesssim 0.1$ pF, $T \lesssim 10$ mK), permitting experimental verification. The deviation reflects the discrete nature of the charge transfer across the junction and should be present in a wide class of similar systems.

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Recent advances in junction fabrication techniques and cryogenics have made it possible to construct devices in which quantum sensitivity limits are realized.[1] Such devices present the challenge of understanding the fundamental mechanisms of dissipation and noise generation, as well as being of considerable practical importance in their own right.

The standard noise analyses are generally restricted in one of two ways: Either the fluctuations are considered in the classical limit or they are taken to be relatively small, i.e., the fluctuating variable is treated in the Gaussian approximation. In the following, we calculate the power spectrum of current and voltage fluctuations of a tunnel junction. In a regime where both quantum effects and the discreteness of the underlying charge transfer process are important, the spectrum differs from the familiar Johnson-Nyquist form. Accordingly, consistent with the fluctuation-dissipation theorem, the response to a biasing voltage is not simply Ohmic in that case. The regime of interest is characterized by the charging energy per electron, $e^2/2C$, being of the order of kT or larger. For example, for $C = 0.1$ pF and $T = 10$ mK the ratio is $e^2/2CkT \cong 1$. This parameter range is accessible which allows experimental verification of our conclusions.

It is well known that the current through a metal-insulator-metal tunnel junction, when driven by a voltage source, is purely Ohmic: $\langle I \rangle = V/R$. Accordingly, the power spectrum of the current noise in a tunnel junction measured in a circuit closed by an ammeter is given by the familiar Johnson-Nyquist[2] result, $S_I(\omega) = (1/\pi R)\hbar\omega \coth(\hbar\omega/2kT)$. It is also known that in the case where the junction is driven by an external constant voltage source,[3] the noise spectrum is

$$S_I(\omega) = \frac{1}{2\pi R} \sum_{\pm} (\hbar\omega \pm eV) \coth\left(\frac{\hbar\omega \pm eV}{2kT}\right). \quad (1)$$

This result exhibits the typical feature of classical shot noise, viz., the linear increase of $S_I(\omega)$ with (large) average current, which reflects the discreteness of the charge transfer process. Both an ideal voltage source as well as an ideal ammeter have zero internal resistance. Hence the self-capacitance of the junction plays no role in the configuration considered above.

In contrast, if we measure the voltage fluctuations of an *open, undriven* junction, a fluctuating current causes a charging of the electrodes, which in turn act as a voltage source. Thus, we expect results reminiscent of the voltage-driven case, Eq. (1), even though the system is in thermal equilibrium. Indeed we find

$$S_V(\omega) = \frac{1}{\pi} \mathrm{Re}\left(\frac{1}{1/R(\omega) - i\omega C}\right) \hbar\omega \coth\left(\frac{\hbar\omega}{2kT}\right), \quad (2)$$

where $1/R(\omega)$ is an effective (non-Ohmic) conductivity of the junction, which is evaluated below.[4] Similarly, the response of an *open* junction to an applied bias voltage $V_b(\omega)$ (realized, for example, by placing the junction in an electric field) is non-Ohmic:

$$\langle I(\omega)\rangle|_{V_b} = \frac{1}{R(\omega) + i/\omega C} V_b(\omega) = \frac{1}{R(\omega)} \langle V(\omega)\rangle|_{V_b}. \quad (3)$$

The conductivity $1/R(\omega)$ differs from the Ohmic $1/R$ appearing in Eq. (1) unless $e^2/2CkT$ is small. Experimentally, this deviation could be verified by measuring the voltage fluctuation spectrum, Eq. (2), or by measuring the average voltage across the junction induced by a radiation field $V_b(\omega)$.[5] Inasmuch as an ideal current source has an infinite internal resistance, we expect that the response to an external current source is governed by this same $1/R(\omega)$.

The microscopic model of the tunnel junction

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that we assume is the Hamiltonian $H = H_L + H_R + H_T + H_Q$. Here $H_{L(R)}$ describes the noninteracting electrons in the left-hand (right-hand) electrode, and the tunneling across the junction is described by the standard form:

$$H_T = \int_{x\subset L} d^3x \int_{x'\subset R} d^3x'\, T(x,x')\psi_{L\sigma}^{\dagger}(x)\psi_{R\sigma}(x') + \text{H.c.} \tag{4}$$

The Coulomb interaction between the charges on the two sides of the junction is accounted for by an effective self-capacitance C such that $H_Q = (Q_L - Q_R)^2/8C$. Furthermore, the current across the junction is given by $\dot{Q}_L \equiv I = -i[Q_L, H]/\hbar$.

The tunneling process shifts the energy of all the electrons in one junction side relative to the other side. As a result, a large number of electrons—proportional to the density of states and the size of the junction—can tunnel back. By comparison, the distortion of the electrons' energy distribution has a negligible effect on the tunneling current, unless the system is of truly atomic dimensions. Thus, we do not explicitly model the inelastic interactions in the metal which thermalize the system but proceed directly to consider a canonical ensemble at temperature β^{-1}.

For a systematic analysis of current correlations we consider the generating functional

$$Z[\xi] = \mathrm{tr}\left(T_\tau \exp\{-\int_0^\beta d\tau [H - I\xi(\tau)]/\hbar\}\right). \tag{5}$$

The single macroscopically observable degree of freedom is the charge Q on one of the electrodes, or equivalently the potential drop across the junction, V. We perform the trace in Eq. (5) over all other microscopic electronic degrees of freedom and write[6] $Z[\xi] = \int DV e^{A[V,\xi]}$. Here we introduced $V(\tau)$, replacing the quartic interaction H_Q by $\frac{1}{2}CV^2(\tau) + (i/2)V(\tau)[Q_L - Q_R]$, and integrated over the (now) quadratic electron degrees of freedom. The action is given by

$$A[V,\xi] = \int_0^\beta d\tau\, CV^2/2\hbar - \mathrm{tr}\ln \hat{G}^{-1}[V,\xi], \tag{6}$$

where $\hat{G}[V,\xi]$ is the Green's function, a 2×2 matrix in the space spanned by the left- and right-hand electrodes, which have opposite potential shifts. The diagonal elements of $\hat{G}^{-1}$ are

$$G_{L(R)0}^{-1} = [-\hbar\partial/\partial\tau - \epsilon_{L(R)}(\nabla) - (+)(ie/2)V(\tau)]\delta(x-x')\delta(\tau-\tau')$$

and the off-diagonal (1,2) element is

$$T[\xi] = T(x,x')[1 + ie\xi(\tau)]\delta(\tau-\tau').$$

We assume that the tunneling matrix element T is independent of momenta and expand in T. We also ignore spatial modulations of the charge density within each electrode, which is justified for frequencies below the plasma frequency. Furthermore, it is natural to perform a gauge transformation to remove the potential shifts. This introduces a phase variable $\dot{\Theta} \equiv eV/\hbar$, such that $Z[\xi] = \int D\Theta\, e^{-A[\Theta,\xi]}$. The action becomes

$$A[\Theta] = \int_0^\beta d\tau \frac{C\hbar}{2e^2}\dot{\Theta}^2(\tau) + 2\int_0^\beta d\tau \int_0^\beta d\tau'\, \alpha(\tau-\tau')\sin^2\left(\frac{\Theta(\tau)-\Theta(\tau')}{2}\right), \tag{7}$$

and $A[\Theta,\xi]$ is obtained by replacing in this form $\cos(\Theta-\Theta')$ by $\exp[i(\Theta-\Theta')](1-ie\xi)(1+ie\xi')$. The β periodicity requires that we allow only for paths which return to $\Theta(0)$ up to integer multiples of 2π, i.e., $\Theta(\beta) = \Theta(0) + 2\pi n$. The kernel α is

$$\alpha(\tau) = \frac{2|T|^2}{h^2}\int \frac{d^3p_L}{(2\pi h)^3}\int \frac{d^3p_R}{(2\pi\hbar)^3} G_L(\tau,\vec{p}_L)G_R(-\tau,\vec{p}_R). \tag{8}$$

To proceed we assume further that $G_{L(R)}$ are the equilibrium Green's functions of the electrons in the left (right) electrodes. Thus

$$\alpha(\tau) = \frac{1}{2\pi}\frac{\hbar}{e^2R}\frac{(\pi kT/\hbar)^2}{\sin^2(\pi kT\tau/\hbar)}, \tag{9}$$

where $R = \hbar/4\pi e^2|T|^2 N_L(0)N_R(0)$ is the constant Ohmic resistance of the junction.

Notice the non-Gaussian form of the effective action $A[\Theta]$, which is an anharmonic 2π-periodic function of Θ. This cyclicity reflects the discrete nature of the charge transfer of electrons across the junction.

We can now calculate the average value of the tunneling current, $\langle I(\tau)\rangle = (1/Z[0])\int D\Theta\, e^{-A[\Theta]} I_T[\Theta,\tau]$, where

$$I_T[\Theta,\tau] = -2e\int_0^\beta d\tau'\,\alpha(\tau-\tau')\sin[\Theta(\tau)-\Theta(\tau')], \tag{10}$$

and the current fluctuations,

$$\langle I(\tau)I(\tau')\rangle = (1/Z[0])\int D\Theta\, e^{-A[\Theta]}\{I_T[\Theta,\tau]I_T[\Theta,\tau'] + 2e^2\alpha(\tau-\tau')\cos[\Theta(\tau)-\Theta(\tau')]\}. \tag{11}$$

The first term in (11) is just the square of the tunneling current I_T, while the second term represents the intrinsic noise current: $\langle I_N I_N\rangle$.

We first observe that if we fix the voltage as a constraint (as is appropriate for an ideal voltage source with zero internal resistance) then we find a simple Ohmic linear response: $\langle I_T(t)\rangle = V(t)/R$. This result obtains in the normal junction because $\alpha^>(t) - \alpha^<(t) = (i\hbar/e^2R)d\delta(t)/dt$, which makes I_T a *local*[7] function of $V(t)$. Here $\alpha^{>(<)}(t)$ are the real-time analytic continuations of Eq. (9). Under the same conditions, the noise current correlations are

$$\tfrac{1}{2}\langle[I_N(t), I_N(t')]_+\rangle = e^2\cos[\Theta(t)-\Theta(t')][\alpha^>(t-t') + \alpha^<(t-t')], \tag{12}$$

where $[\,,]_+$ denotes the anticommutator, and $e^2(\alpha^> + \alpha^<)_\omega = (\hbar\omega/R)\coth(\hbar\omega/2kT)$. For fixed *constant* voltage, the cosine shifts the $\hbar\omega$ of the Johnson-Nyquist form by $\pm eV$ and we recover the result Eq. (1).

In the undriven junction, i.e., in an open circuit, or in a circuit closed by a voltmeter, the noise spectrum follows from (11). We find results in a closed form if we restrict our attention to junctions with large resistance, viz., $R \gg (\hbar/e^2)e^2/CkT$. In this limit the second term in the action becomes negligible and the energy is determined by the charging energy of the capacitor. We then obtain the noise correlations

$$\tfrac{1}{2}\langle[I_N, I_N]_+\rangle_\omega = \frac{1}{2R}\left[\sum_{q=-\infty}^{\infty}\exp\left(-\frac{q^2e^2}{2CkT}\right)\right]^{-1} \times \sum_{q=-\infty}^{\infty}\exp\left(-\frac{q^2e^2}{2CkT}\right)\left\{\sum_{\pm}\left[\hbar\omega \pm \frac{e^2}{C}(q+\tfrac{1}{2})\right]\coth\left[\frac{\hbar\omega \pm (e^2/C)(q+\frac{1}{2})}{2kT}\right] - \frac{e^2}{C}\right\}. \tag{13}$$

This result is quite reminiscent of the voltage-driven form of Dahm *et al.*[3] in Eq. (1). Replacing the external voltage V is $(q+\frac{1}{2})e/C$, which shows the capacitor acting as an effective voltage source, with a weighting appropriate to the equilibrium Boltzmann distribution of charging energies, $E_q = q^2e^2/2C$.

For most macroscopic junctions, the parameter CkT/e^2 is very large, the discreteness of individual charge transfers becomes irrelevant, and Eq. (13) reduces to the Johnson-Nyquist result. However, for smaller values we observe interesting deviations (Fig. 1). Our result is smaller than the Johnson-Nyquist spectrum. The fluctuations which occur via a transfer of discrete charges require a finite energy E_q and are thus suppressed. Thus, for low frequencies ($\hbar\omega \ll hT, e^2/2C$) at low temperatures ($CkT/e^2 \to 0$) the result in (13) vanishes exponentially: $\frac{1}{2}\langle[I_N, I_N]_+\rangle_\omega \to (2e^2/RC)\exp(-e^2/2CkT)$. If CkT/e^2 is large it approaches the classical result as $\frac{1}{2}\langle[I_N, I_N]_+\rangle_\omega \to (2kT/R)(1-e^2/6CkT)$.

Equation (13) is our principal result. It is re-

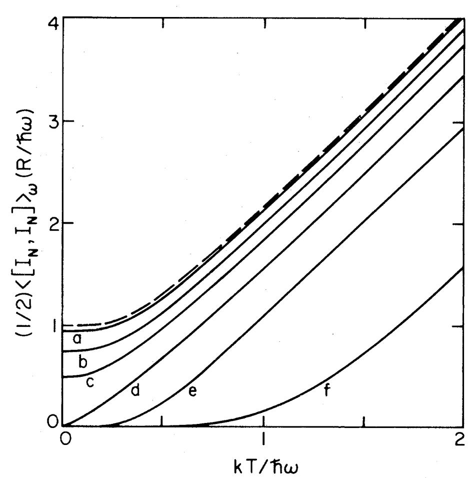

FIG. 1. The noise current correlation function as a function of temperature at fixed ω. The parameter is $\hbar\omega/(e^2/2C)$: (*a*) = 20, (*b*) 4, (*c*) 2, (*d*) 1, (*e*) 0.5, and (*f*) 0.2. The dashed line is the Johnson-Nyquist result.

lated to a response function $1/R(\omega)$ according to

$$\tfrac{1}{2}\langle[I_N, I_N]_+\rangle_\omega = [1/R(\omega)]\hbar\omega \coth(\hbar\omega/2kT). \quad (14)$$

In terms of $R(\omega)$ the *total* current fluctuations across the junction, i.e., including the contribution from the first term in (11), are given by[4]

$$\tfrac{1}{2}\langle[I,I]_+\rangle_\omega = \mathrm{Re}\left(\frac{1}{R(\omega)+i/\omega C}\right)\hbar\omega \coth\left(\frac{\hbar\omega}{2kT}\right). \quad (15)$$

For zero frequency this result vanishes, reflecting the conservation of total charge in the junction. Identifying $V = Q/C$, we readily obtain the voltage noise spectrum (2). Furthermore, the linear response of I and V to a bias voltage V_b, which is modeled by adding to the Hamiltonian the term $\Delta H = V_b(Q_L - Q_R)/2$, is summarized in Eq. (3). The same $R(\omega)$ as defined above enters again. The fluctuation-dissipation theorem connects this response to V_b and the fluctuations considered above.

We have considered the specific case of a normal tunnel junction for definiteness. However, any system in which the discreteness of the states and the underlying noise generating process is essential will show similar deviations. We presented here results for junctions with large resistance, which we can express in closed form. We expect that for stronger damped junctions $[R \lesssim (\hbar/e^2)e^2/CkT]$ the discrete level structure apparent in the result (13) is smeared out, and the power spectrum looks more like the Johnson-Nyquist result. The path integral (11) provides the prescription of how to evaluate the noise spectrum in general.

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[4]Strictly, we have verified the relations (2), (3), and (15) with $1/R(\omega)$ as defined by (14) only to first order in $1/RC\omega$. The relations also reproduce the $\omega = 0$ limit and, of course, are correct for the Gaussian limit where $R(\omega) = R$. We conjecture they are correct in general. In any case, they provide an adequate interpolation.

[5]After the main part of this paper was completed we learned that Tin-Lun Ho, preceding Letter [Phys. Rev. Lett. 51, 2060 (1983)], had derived a similar response function. He comments further on the experimental relevance.

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[7]In a superconducting junction where α depends on the energy gap, a new time scale is introduced and the current-voltage relation remains nonlocal in time and nonlinear.