Vortices with Ferromagnetic Superfluid Core in ^{3}He-*B*

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(Received 16 September 1983)

It is shown that there may exist five axisymmetric vortices in ^{3}He-B, different in their internal symmetries. The Landau theory for phase transitions between them is constructed. Numerical calculations show that one of the observed vortices possesses a novel new structure: a superfluid core, which is ferromagnetic. This explains the measured large magnetic moment of the vortex.

PACS numbers: 67.50.Fi

Two unexpected physical phenomena were found in recent NMR measurements with rotating superfluid ^{3}He-B: (i) the first-order transition[1,2] of the vortex core at $T=0.6\,T_c$ (for $p=29.3$ bar), and (ii) the intrinsic magnetic moment, concentrated in the core of the vortices.[3] Therefore, in spite of its small size—of the order of the superfluid coherence length $\xi_0 \cong 200$ Å—the core structure of vortices in ^{3}He-B proves nontrivial.

B-phase vortex-core structures have been investigated with the use of numerical[4,5] calculations in the Ginzburg-Landau (GL) regime. These approaches assumed structures for the vortex with a normal core. However, topological considerations[6] first showed that unlike the ^{4}He vortex or vortices in superconductors, superfluidity need not be broken in the ^{3}He-B vortex core, which may contain other superfluid phases, such as the A phase.

Here we introduce a new approach, based on a symmetry classification of the vortices. This is vital for understanding the vortex-core transition because, as a rule, phase transitions are accompanied with a change in the symmetry of the system (the gas-liquid transition is, however, one of the few exceptions to this general behavior). Therefore, we expect that core structures of the vortices at $T > 0.6T_c$ and at $T < 0.6T_c$, which we denote by N1 and N2, possess different symmetries. To investigate the nature of the vortex transition we analyze all the possible symmetries of the vortices. We find that in ^{3}He-B, there are five types of axially symmetric vortices with different internal symmetries. (Similar classification can also be employed for axially nonsymmetric vortices, such as continuous vortices[7] in ^{3}He-A.) To identify the observed vortices we calculate the core structure in the GL regime at low pressures, where experiments suggest[8] that only the vortex N2 is stable. We find that vortex N2 has superfluid core: a mixture of the A phase and the ferromagnetic superfluid β phase. The β phase is never stable in bulk liquid; it may only exist in the ^{3}He-B vortex core.

To illustrate the symmetries of vortices, let us first consider ^{4}He. The order parameter ψ for the vortex with m quanta of circulation is $\psi = C(r)e^{im\varphi}$, where r, φ, and z denote cylindrical coordinates, with $\hat{z}$ along the vortex axis. The total symmetry group of this state includes both continuous and discrete subgroups. The continuous symmetries, besides translations along the vortex axis, contain rotations around z combined with a gauge transformation. The generator of this combined symmetry group is $\hat{Q}=\hat{L}_z - m\hat{I}$, where $\hat{L}_z = -i\,\partial/\partial\varphi$ and $\hat{I}$ (with $\hat{I}\psi=\psi$) is the generator of the gauge transformation. The invariance of ψ under this transformation, $\hat{Q}\psi=0$, means axial symmetry.

The subgroup of the discrete vortex symmetries contains the four elements 1, $P_1 = Pe^{im\pi}$, $P_2 = TL_{y,\pi}$, and $P_3 = P_1P_2$. Here P is the parity transformation $\vec{r} \to -\vec{r}$, $e^{im\pi}$ is a gauge transformation by the phase $m\pi$, T denotes time inversion including complex conjugation, and $L_{y,\pi}$ means rotation by π around a perpendicular axis y; clearly, $P_1\psi = P_2\psi = \psi$.

Note that $\psi = C(r)e^{im\varphi}$ is the only solution of $\hat{Q}\psi = 0$. Consequently, one can expect no phase transitions for the ^{4}He vortex, unless axial symmetry is broken. However, in superfluid ^{3}He the axially symmetric structures are much richer, and one may in fact expect several phase transitions due to broken discrete symmetries without broken axial symmetry.

The order parameter in superfluid ^{3}He is a 3

$\times$ 3 complex matrix $A_{\alpha i}$, with spin (α) and orbital (i) indices. In ^{3}He-B, this reduces to the orthogonal matrix $R_{\alpha i}$: $A_{\alpha i} = CR_{\alpha i}e^{i\Phi}$, where Φ is a phase factor and C is the amplitude of the order parameter. Vortices in ^{3}He-B are quantized, such as those in He-II, but with the circulation quantum $\hbar/2m_3$. Far from the vortex axis the order parameter is $A_{\alpha i}(\infty) = CR_{\alpha i}e^{im\varphi}$. This asymptotic form defines the maximal symmetry group of the ^{3}He-B vortex. Now axial symmetry is described by the modified generator $\hat{Q} = \hat{L}_z + \hat{S}_\alpha R_{\alpha z} - m\hat{I}$: $\hat{Q}A_{\alpha i} = 0$. Here $\hat{S}$ is the operator of spin rotations $\hat{S}_\beta A_{\alpha i} = ie_{\beta\alpha\gamma}A_{\gamma i}$ and $\hat{L}_z = -i\partial/\partial\varphi + \hat{L}_z^{\text{int}}$ is the total operator of orbital rotations, including internal rotation of the order parameter: $\hat{L}_i^{\text{int}}A_{\alpha k} = ie_{ikl}A_{kl}$, and $\hat{I}A_{\alpha i} = A_{\alpha i}$. One may use simpler coordinates for the spin system: rotated by the matrix $R_{\alpha i}$ with respect to the laboratory frame. In these coordinates $R_{\alpha i} = \delta_{\alpha i}$, asymptotics are $A_{\alpha i}(\infty) = C\delta_{\alpha i}e^{im\varphi}$, and the operator $\hat{Q}$ and the modified discrete symmetry elements become

$$\hat{Q} = \hat{L}_z + \hat{S}_z - m\hat{I},$$

$$P_1 = Pe^{im\pi}, \; P_2 = TL_{y,\pi}S_{y,\pi}, \; P_3 = P_1P_2,$$

where $S_{y,\pi}$ is a spin rotation through π around y.

To find the general representation for the axially symmetric vortex, one must solve the equation $\hat{Q}A_{\alpha i} = 0$; the solution is

$$A_{\alpha i} = \sum_{\mu\nu} C_{\mu\nu}(r)\lambda_\alpha^{\mu}\lambda_i^{\nu}e^{i(m-\mu-\nu)\varphi}, \tag{1}$$

where λ_i^ν and λ_α^μ are eigenfunctions of the operators $\hat{L}_z^{\text{int}}$ and $\hat{S}_z$ with eigenvalues ν and μ: $\lambda_{i,\alpha}^{\pm} = (\hat{x}_{i,\alpha} \pm i\hat{y}_{i,\alpha})/\sqrt{2}$, and $\lambda_{i,\alpha}^0 = \hat{z}_{i,\alpha}$. This solution contains nine complex functions $C_{\mu\nu}(r)$, describing amplitudes of Cooper pairing into states with projections μ and ν of pair spin and pair orbital momenta. For example, C_{0+} represents A-phase Cooper pairing.

There are important constraints on the $C_{\mu\nu}$ imposed by the discrete symmetries, under which they transform as

$$\hat{P}_1C_{\mu\nu} = (-1)^{\mu+\nu}C_{\mu\nu}, \quad \hat{P}_2C_{\mu\nu} = C_{\mu\nu}^*,$$
$$\hat{P}_3C_{\mu\nu} = (-1)^{\mu+\nu}C_{\mu\nu}^*.$$

Here we consider only singly quantized ($m=1$) vortices. The vortex which possesses the maximal discrete symmetry, $\hat{P}_1C_{\mu\nu} = \hat{P}_2C_{\mu\nu} = \hat{P}_3C_{\mu\nu} = C_{\mu\nu}$, we denote as the o vortex. It has five real amplitudes, C_{++}, C_{+-}, C_{00}, C_{-+}, and C_{--}. All of them must enter the expressions because of their mutual nonlinear coupling in the Gorkov equations (or GL equations near T_c); hence all the possible vortex solutions in ^{3}He-B are nonunitary by necessity. Thus the one-parameter *Ansatz* $A_{\alpha i} = C(r)\delta_{\alpha i}e^{im\varphi}$, imitating a ^{4}He vortex, is not a solution of the Gorkov equations; neither is the three-parameter vortex *Ansatz* of Passvogel, Schopohl, and Tewordt.[4] The o vortex, which was first discussed by Ohmi, Tsuneto, and Fujita,[5] possesses a normal core: All the five amplitudes vanish on the vortex axis, where their phases $\Phi_{\mu\nu} = (1-\mu-\nu)\varphi$ have a discontinuity.

The discrete vortex symmetry may be broken in three inequivalent ways, depending on which symmetry is retained: P_1, P_2, or P_3. We denote the corresponding vortices as u, v, and w; see Table I. The u vortex contains five complex amplitudes $C_{\mu\nu} = (-1)^{\mu+\nu}C_{\mu\nu}$, which vanish on the vortex axis. The v vortex has nine real amplitudes $C_{\mu\nu} = C_{\mu\nu}^*$ (C_{+0}, C_{0+}, C_{0-}, and C_{-0} in addition to those in the o vortex), of which C_{0+} and C_{+0} need not vanish at $r=0$, since their phases $\Phi_{0+} = \Phi_{+0} = 0$ display no discontinuity on the axis. These describe the superfluid A phase (C_{0+}) and the ferromagnetic superfluid β-phase (C_{+0}), with its magnetic moment directed along $\hat{z}_\alpha = R_{\alpha i}\hat{z}_i$. The w vortex also contains nine amplitudes $C_{\mu\nu} = (-1)^{\mu+\nu}C_{\mu\nu}^*$, describing the same pairing states as in the v vortex.

For the construction of a Landau theory of phase transitions one must introduce three real order parameters u, v, and w, associated with the three possible broken symmetries. The GL free-energy functional, invariant under the P_1, P_2, and P_3 transformations, is

$$F = -au^2 - bv^2 - cw^2 + \tfrac{1}{2}(u^4 + v^4 + w^4) + euvw + fu^2v^2 + gu^2w^2 + hv^2w^2. \tag{2}$$

Nonzero u, v, or w describes the appearance in Eq. (1) of the five (imaginary), four (real), or four (imaginary) amplitudes, in addition to those of the o vortex; see Table I. Depending on the phenomenological coefficients $a,\ldots,h$, which are functions of temperature and pressure, Eq. (2) may exhibit different minima, corresponding to the o, u, v, w, and the least symmetric, though axial, uvw vortex, with all the nine complex amplitudes nonzero: (i) $u=v=w=0$; (ii) $u \neq 0$, $v=w=0$, (iii) $v \neq 0$, $u=w=0$; (iv) $w \neq 0$, $u=v=0$, and (v) $u \neq 0$, $v \neq 0$, $w \neq 0$.

The (p,T) plane may be divided by lines of

TABLE I. Classification of the axisymmetric ^{3}He-B vortices.

Vortex	Discrete symmetry	$C_{\mu\nu}$ with $\mu+\nu$ even	$C_{\mu\nu}$ with $\mu+\nu$ odd	Core fluid
o	P_1, P_2, P_3	Real	···	Normal
u	P_1	Complex	···	Normal
v	P_2	Real	Real	Super
w	P_3	Real	Imaginary	Super
uvw	···	Complex	Complex	Super

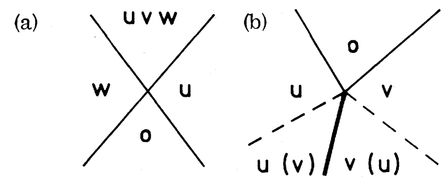

FIG. 1. Two of many possible phase diagrams for the five vortices according to the Landau theory of Eq. (2). In (a) only second-order transitions (thin lines) occur, while in (b) there also is a line of first-order transitions (solid curve). On dashed catastrophe lines metastability of one of the phases (in brackets) terminates.

first- and second-order phase transitions in parts, each corresponding to a given minimum. Figure 1 illustrates several of the possible transition lines, critical points, and catastrophe lines. Transition from an o vortex to u, v, and w vortices (and from u, v, w to the uvw vortex) is accompanied with decreasing symmetry and is therefore of second order. First-order transitions occur between u, v, and w vortices, because these possess different broken symmetries. Landau theory also allows a possibility of various asymmetric uvw vortices corresponding to different roots of Eq. (2). Liquid-gas type first-order transitions between them are also possible.

We can identify the vortex N2, known to be stable at low pressures for any temperature,[8] including the GL regime near T_c. Here we may use the well-known GL bulk free energy functional

$$f_B = -\alpha A_{\alpha i}{}^* A_{\alpha i} + \beta_1 A_{\alpha i}{}^* A_{\alpha i}{}^* A_{\beta j} A_{\beta j} + \beta_2 A_{\alpha i}{}^* A_{\alpha i} A_{\beta j}{}^* A_{\beta j} + \beta_3 A_{\alpha i}{}^* A_{\beta i}{}^* A_{\beta j} A_{\alpha j} + \beta_4 A_{\alpha i}{}^* A_{\beta i} A_{\beta j}{}^* A_{\alpha j} + \beta_5 A_{\alpha i}{}^* A_{\beta i} A_{\beta j} A_{\alpha j}{}^*$$

and the gradient energy

$$f_K = K_1 \partial_i A_{\alpha i} \partial_j A_{\alpha j}{}^* + K_2 \partial_i A_{\alpha j} \partial_i A_{\alpha j}{}^* + K_3 \partial_j A_{\alpha i} \partial_i A_{\alpha j}{}^*$$

(where for weak coupling $K_1 = K_2 = K_3 = K$) for calculating the vortex core structure in terms of the order parameter $A_{\alpha i}$. Here the coefficients β_i of the fourth-order invariants are chosen in the weak-coupling approximation $[-2\beta_1 = \beta_2 = \beta_3 = \beta_4 = -\beta_5$, with $\beta_1 = 7N(0)\xi(3)/240(\pi T_c)^2]$ and $\alpha = \frac{1}{3}N(0)(1 - T/T_c)$. We have minimized the sum of the gradient and condensation energies, $\int_0^\infty (f_K + f_B) r\,dr$, for the o, u, v, and w vortices in this approximation, believed to hold at low pressure. We find that the v vortex has minimal energy. The other vortices, including the most symmetric o vortex in Fig. 2(a), are unstable with respect to the v vortex illustrated in Fig. 2(b). Hence we suggest identifying the vortex N2 with the v vortex, which possesses a superfluid core, containing both the A and β phases.

Note that there is no general symmetry constraint on the vortex magnetization: $\langle \hat{S}_z \rangle \sim \sum_{\mu\nu} \mu |C_{\mu\nu}|^2$. Therefore, the net magnetization is nonvanishing for all the vortices: The superflow motion around the vortex produces internal orbital rotation of the pairs with $\langle \hat{L}_z{}^{\rm int} \rangle \neq 0$, which in turn produces a magnetization, because of the rigidity of the Cooper pairs.

Figure 2(c) contrasts distributions of magnetization in the v vortex (N2) and the o vortex: The parameter m_0 depends on the details of particle-hole asymmetry near the Fermi surface, and we know only its order of magnitude,

$$m_0 \sim \gamma(\hbar/m_3)\rho_s(T_c/E_F)^2 \ln(E_F/T_c)$$

(where γ is the gyromagnetic ratio for ^{3}He). This makes it difficult to compare calculated values for the magnetic moment of the vortex with the values observed experimentally at temperatures far from T_c. The measured magnetization of the liquid ^{3}He due to the magnetic vortices is of the order of 10^{-11} nuclear Bohr magnetons per one atom of the liquid which contains an equilibrium density of vortices at an angular velocity of rotation $\Omega \cong 1$ rad/s. Also our estimated vortex magnetization is consistent with this within an order of magnitude.

The appearance of the ferromagnetic phase in the core of the v vortex is essential in producing the magnetization. The long extent of the β phase in the v vortex [see Fig. 2(b)] gives rise to the large range of its magnetization. According to

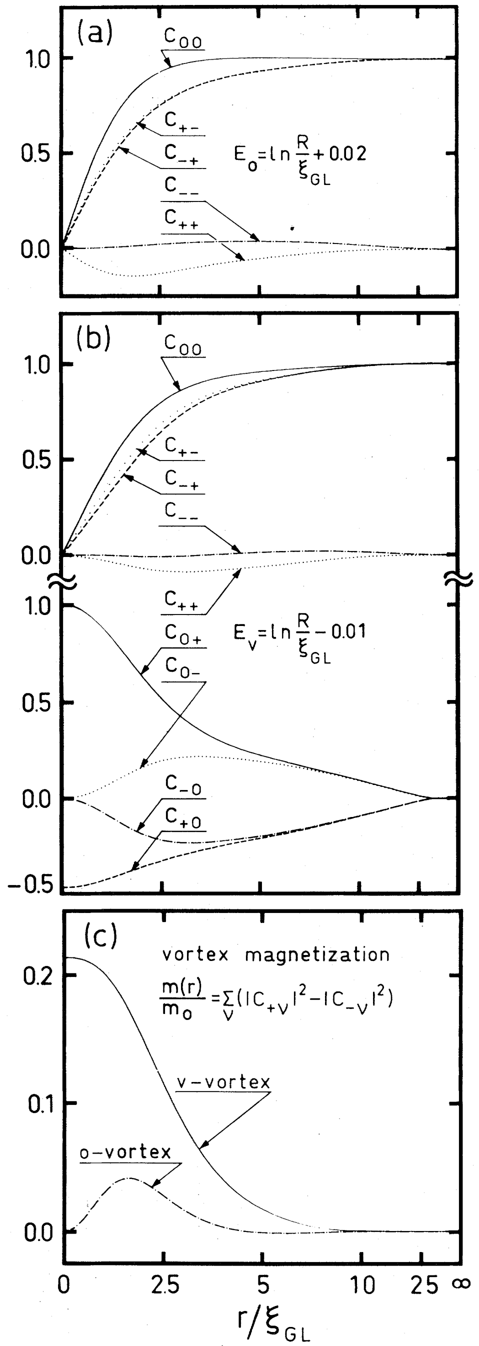

FIG. 2. Structures of (a) the most symmetric o vortex, and (b) the P_2-symmetric superfluid v vortex in the weak-coupling limit. The real parameters $C_{\mu\nu}$ are scaled such that the bulk ^{3}He-B order parameter $C_{+-}(\infty) = C_{00}(\infty) = C_{-+}(\infty) = 1$. Radial distances are in units of the GL coherence length $\xi_{GL} = (\sqrt{\frac{3}{5}})\xi_0/(1-T/T_c)^{1/2}$; for $r < 5\xi_{GL}$ the scale is linear in r, for $r > 5\xi_{GL}$ it varies as $1/r$. The energies E of the vortices are in units of $\frac{1}{4}\pi\rho_s(\hbar/m_3)^2$, where ρ_s is the superfluid density; R is a logarithmic cutoff. (c) Radial distribution of vortex magnetization $m(r) = m_0 \sum_{\mu\nu} \mu |C_{\mu\nu}|^2$, which is directed along $R_{\alpha i}\hat{z}_i$.

experiments,[8] a large magnetization is found in vortex N2 compared with vortex N1. We suggest (since we have not yet found the transition, we may only conjecture) that the observed first-order vortex transition occurs between two vortices with different internal symmetries: The v vortex with superfluid ferromagnetic core possessing a large magnetization and the u vortex with normal core.

We conclude that the vortex N2, occurring at low pressures and/or low temperatures over most of the phase diagram, possesses a superfluid core, which is ferromagnetic.[9] However, an unambiguous identification of the vortex N1 can only be done after a complete experimental investigation of the vortex phase diagram, including the catastrophe lines, which may be found from the observed metastability phenomena.

We thank P. J. Hakonen, M. Krusius, and J. T. Simola, and A. L. Fetter, T. Ohmi, J. A. Sauls, and A. Veselov for discussions on experiments and theory. This work was supported in part by the Academy of Finland.

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[9]Nonunitary vortex structures have been discussed for a hypothetical $^3P_2$ pairing of neutrons, conjectured to occur in neutron stars, e.g., by R. W. Richardson, Phys. Rev. D 5, 1883 (1972), and also by J. A. Sauls, D. L. Stein, and J. W. Serene, Phys. Rev. D 25, 967 (1982). These suggested structures are analogous to the nonunitary o vortex of Ref. 5 in that the vortex core is normal. However, the v vortex introduced in the present paper is completely different from these in that it has a superfluid core; moreover, one which is spontaneously ferromagnetic.