Stability of $n=1$ Kink Modes in Bean-Shaped Tokamaks

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Numerical studies show that by indenting the small-major-radius side of conventional finite-aspect-ratio tokamaks, significant improvements to the stability of pressure-driven ideal magnetohydrodynamic modes can be achieved. The internal $n=1$ kink mode can be stabilized completely with quite modest indentation. In the presence of a nearby conducting wall, kink-ballooning mode stability is also improved, and accessibility to a second stable region at high plasma β is possible.

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Detailed theoretical studies have established that the ideal magnetohydrodynamic (MHD) stability properties of axisymmetric tokamak equilibria can depend strongly on the shape of the poloidal cross section of the plasma. Small aspect ratio, modest elongation, and triangularity are all favorable in the sense that the critical value of $\beta \equiv 2\langle p\rangle/\langle B^2\rangle$ for the onset of internal, pressure-driven ballooning modes is larger than for the comparable circular cross-sectional shape.[1] The stabilizing effect of triangularity can be enhanced by indenting the small-major-radius side of the cross section, forming a "bean-shaped" plasma.[2,3] In fact, with certain plasma-pressure and safety-factor profiles, it has been shown recently that for sufficiently large indentations complete ballooning stability can be achieved, providing an accessible path to the second stability region at smaller indentations.[4]

While stability in the short-wavelength ballooning limit can usually be extrapolated to longer-wavelength modes, the effects of which one would expect to be most directly observable experimentally, it is necessary to carry out a separate study for the low toroidal mode numbers (such as $n=1$). This is the subject of this paper. For the internal $n=1$ kink, we find that even modest indentation can completely stabilize the mode, thus connecting the favorable effect of finite aspect ratio at low β[5] to the stabilization associated with the second stable region at very high β.[6] For the external kinks we find that in the presence of a nearby conducting shell the stability properties with large indentation are quantitatively similar to previous parameter studies at low β, but that, at larger β, a second stable region to external kinks appears in a similar way to the high-β ballooning results.

Equilibrium solutions employed for the study were prepared following now standard procedures. A flux-coordinate, Grad-Shafranov equilibrium code was used with pressure and safety-factor profiles given as

$$p(\psi)=p_0(1-\bar{\psi}^{\alpha})^{\beta}, \quad q(\psi)=\sum_{i=1}^{3} q_i \bar{\psi}^i, \tag{1}$$

with $\bar{\psi}=(\psi-\psi_a)/(\psi_l-\psi_a)$ (ψ_a and ψ_l being the flux measured at the magnetic axis and plasma surface, respectively), and p_0, α, β, q_i parameters chosen to fix β and the required profiles. In the cases reported here the q_i were fixed so that $q(0)=1.03$, $q(1)=4.2$, $q'(0)=0.84$, and $q'(1)=9.0$. The equilibria generated were obtained as fixed boundary solutions with the plasma surface determined from the formulas[4]

$$X(\theta)=\bar{X}+\rho\cos t, \quad Z(\theta)=E\rho\sin t, \tag{2}$$

with $\rho=A(1+B\cos\theta)$, $t=C\sin\theta$, and $0\leq\theta\leq 2\pi$. A typical example, showing the various parameters of interest, is given in Fig. 1. In the study reported here $R/a=4$, $b/a=1.386$, and the indentation, $i\equiv d/2a$, is varied by changing the angle C in Eq. (2). Sequences of flux-conserved equilibria at different values of the indentation were then obtained by increasing the parameter p_0, holding all other profile parameters fixed. For the external kink studies, this gives a matrix of equilibria in the $\langle\beta\rangle$-i plane, for each of which the stability must be examined. For the $n=1$ internal kink we are interested in the stability properties for equilibria with $0.5<q(0)\leq 1$. To study this, each of the equilibria described above was scaled by varying the strength of the external toroidal field.[7] This procedure generates additional sequences of equilibria at constant $\beta_p \equiv 4\int p\,dv/(\mu_0 I_\varphi^2 \bar{R})$, where $\bar{R}$ is a volume-averaged major radius. At fixed indentation, the stability can then be determined in the $\epsilon\beta_p$-vs-$q(0)$ plane.

The stability calculations were carried out with PEST2.[8] Typically, 30–50 Fourier harmonics in PEST coordinates were kept with 100 radial finite elements. Convergence studies were performed to ensure that this resolution was adequate.

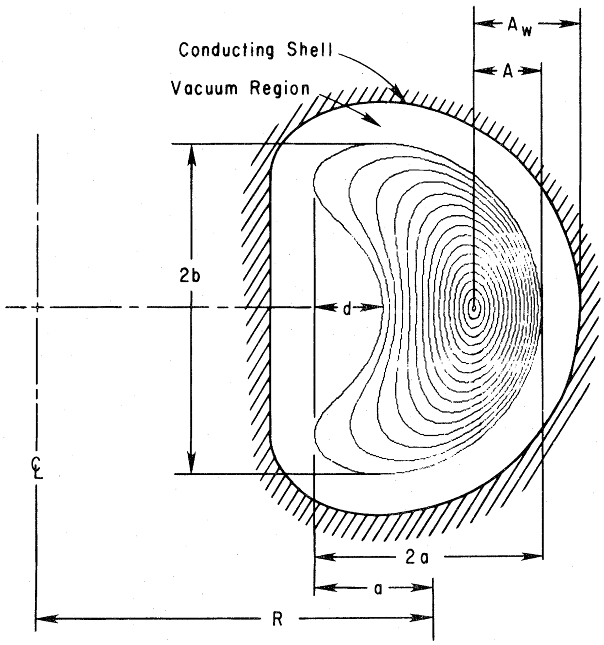

FIG. 1. Parameters used to describe the bean-shaped plasmas studied.

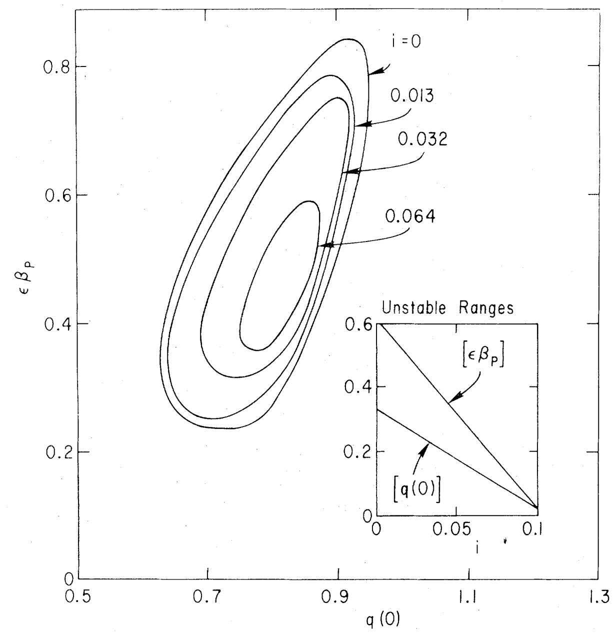

FIG. 2. Stability boundaries for the internal kink with $\alpha = \beta = 2$ and various indentations.

Stability boundaries for the $n = 1$ internal kink mode are presented in Figs. 2 and 3 for two different choices of the pressure profile. In Fig. 2 $\alpha = \beta = 2$ so that the profile is flat near the magnetic axis. This provides some stabilization when the $q = 1$ surface lies close to the magnetic axis, even without indentation. The $n = 1$ mode is unstable inside the curves, which shrink rapidly with indentation. In the inset we have plotted the maximum height and width of the unstable regions as a function of indentation. The critical indentation is seen to be approximately 0.1. Detailed calculations with $i \geq 0.11$ show complete stability to the internal kink even with $q(0)$ as low as 0.5.

In Fig. 3 we have taken $\alpha = 1$, $\beta = 2$, so that $p'(0) \neq 0$. For given indentation the instability region is larger than for the flatter pressure profile, but indentation is seen to have a similar, strongly stabilizing effect. Extrapolation of these results gives a critical indentation of approximately 0.3.

It is useful to examine the nature of the $n = 1$ eigenmodes as a function of $\epsilon\beta_p$. At low $\epsilon\beta_p$ the structure, illustrated in Fig. 4, is consistent with the conventional picture of a current-driven perturbation with driving forces localized around the $q = 1$ surface. At larger β_p, however, the mode becomes primarily pressure driven and shows a radial structure of poloidal harmonics typical of ballooning modes (similar to the internal part of Fig. 6).

External kink modes are usually difficult to avoid in MHD models of an infinitely conducting plasma surrounded by a vacuum region, and set

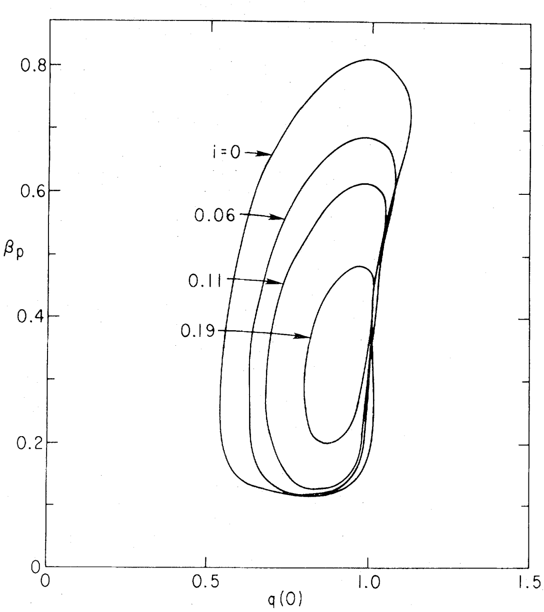

FIG. 3. Stability boundaries for the internal kink with $\alpha = 1$, $\beta = 2$, and various indentations.

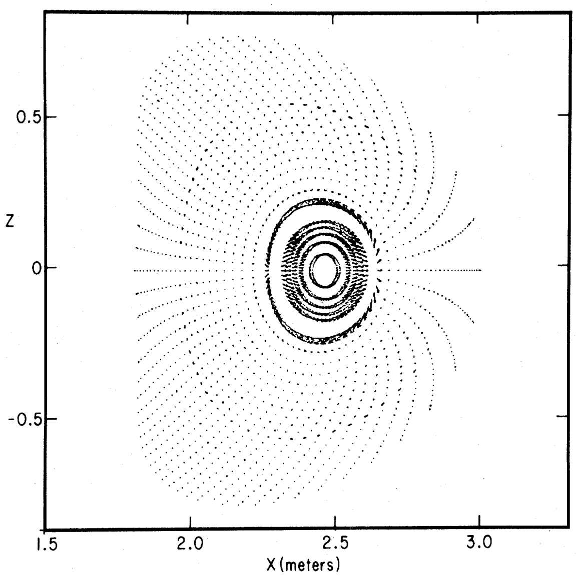

FIG. 4. Internal kink displacement vector ξ, for $\epsilon\beta_p = 0.16$, $q(0) = 0.9$.

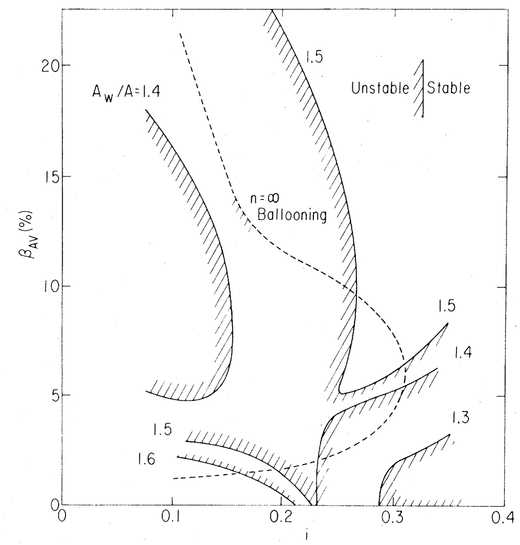

FIG. 5. Stability boundaries to the external $n = 1$ kink mode with $\alpha = \beta = 2$ for different wall positions. Shaded regions are unstable.

β limits below internal ballooning results.[1] Theoretical studies show a sensitivity to the shape of the current profile near the plasma surface. Stability depends on the presence of a nearby conducting shell and we adopt the usual approach of presenting the stability boundaries as a function of wall radius. To do this we have examined each of the equilibria in the $\langle\beta\rangle$-i plane as a function of A_w/A, and plot contours of marginal stability in Fig. 5.

At low β the external modes are localized near the plasma surface and can be described as surface kinks. Indentation is destabilizing in the sense that the wall must be closer to achieve stability as indentation is increased. At modest indentations, $i \leq 0.2$, the modes take on a strong ballooning character as β is increased (see Fig. 6). Eventually, as the second stable region to internal ballooning modes is approached, the advantages associated with favorable local shear and the magnetic well arising from the Shafranov shift appear, and the configuration becomes stable. At strong indentations, finite β effects are seen to stabilize the kink modes, an effect previously observed in a circular-cross-section, quasi-uniform-current, $\beta \sim a/R$ model.[9] When the results of Fig. 5 are considered simultaneously with the infinite-n ballooning mode boundary[4] (broken line), we see that, with a perfectly conducting wall at approximately 1.3 plasma radii, accessibility to the second stable region without encountering current-driven kinks should be achievable.

While the experimental verification of external kink modes in modern tokomaks remains an open issue, operation with $q(0) < 1$ is usual and measurements indicate significant activity associated with the $m = 1$, $n = 1$ internal mode. At low β the mode occurs as a resistive kink, whose nonlinear evolution is associated with the sawtooth phenomenon; but at higher β the instability will be more aptly described as an ideal MHD internal

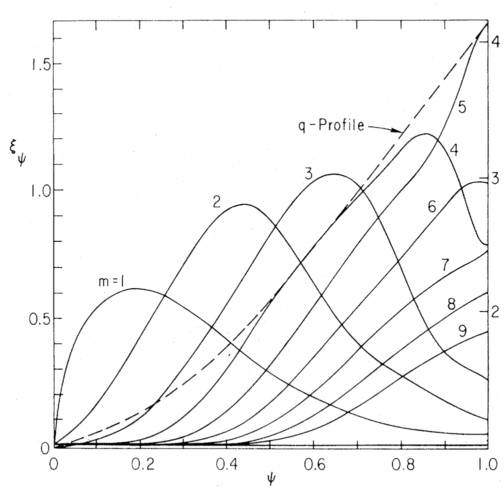

FIG. 6. Strongly ballooning driven kink mode with $i = 0.112$, $\langle\beta\rangle = 8.2\%$, $A_w/A = 1.5$.

kink. Recently this mode has been associated with the onset of fishbone MHD oscillations in PDX,[10] resulting in poor fast-ion confinement. Since, to date, this is the only clearly identifiable detrimental ideal MHD experimental effect to come with increasing β, it is obviously of considerable interest to study configurations which are stable to this mode.

Currently, preparations are underway to modify the PDX tokamak to make possible a study of the effect of indentation on the achievable $\langle\beta\rangle$ values. Considerable analysis of equilibria with profiles similar to those used to model typical high-β PDX discharges has been carried out with $i=0.27$, $b/a=1.4$, $q(0)=0.85$, and $q(1)=3.0$. Stability to the internal kink up to the maximum $\langle\beta\rangle$ studied of 20% has been observed. These equilibria do not display the second stable region to ballooning modes but they indicate a ballooning limit of ~10%, considerably above previous studies.

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