Shell-Model Study of the (p, π^-) Reactions in the $f_{7/2}$-Shell Region

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Relative cross sections for the (p, π^-) reaction for nuclei in the region $40 < A < 56$ are calculated with use of $(1f_{7/2})^n$ wave functions and a zero-range plane-wave approximation for this three-nucleon process. The results of the calculation for the low-lying discrete states are in excellent agreement with recent data and many detailed aspects of the spectra are understood for the first time.

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Recently the results of a series of (p, π^-) experiments on a number of nuclei have been reported by Vigdor *et al.*[1] They have pointed out that this reaction populates selective high-spin states which can be correlated in many instances with those identified previously in gamma decay and particle transfer experiments. In addition, they have shown evidence from analyzing-power measurements that this reaction involves a relatively simple two-nucleon pion production process.[2]

For the Ca isotopes and the $N=28$ isotones the (p, π^-) cross sections for populating discrete low-lying final states have a strong dependence on N and Z (see Fig. 2). The cross sections in units of nanobarns per steradian for 206-MeV protons and a pion emission angle of 30°, when integrated over the strong states below 5-MeV excitation energy in the final nucleus, are 10.6 ± 1.3 for ^{43}Ti, 8.8 ± 2.0 for ^{45}Ti, 53.2 ± 1.4 for ^{49}Ti, 18.0 ± 3.0 for ^{51}Cr, and 9.9 ± 1.5 for ^{53}Fe.[3]

The final states most strongly populated in the ^{48}Ca$(p, \pi^-)^{49}$Ti reaction are a doublet,[4] the higher member of which is consistent in energy with the $\frac{19}{2}^-$ state at 4.4-MeV excitation energy observed in a gamma decay experiment.[5] However, the lower peak at 4.0 MeV cannot be associated with any high-spin states known previously. In addition, in a recent experiment with 165-MeV protons,[4] the pions from these two states have been observed to have quite different angular distributions. The 4.4-MeV peak cross section falls off by about a factor of 2 over the angular range of 30–135° and the 4.0-MeV peak cross section falls off about an order of magnitude in the same angular range.

In this Letter we will show that all of the above features of the relative cross sections can be qualitatively understood within the context of the $(1f_{7/2})^n$ shell model for these nuclei together with some general assumptions about the reaction mechanism within the plane-wave Born approximation. The success of our results should encourage the development of distorted-wave calculations for this reaction which will be necessary to obtain more quantitative results as well as the absolute cross sections.

From the basic Feynman diagrams for this reaction shown in Fig. 1 it is clear that the reaction involves three nucleons. The box at the outgoing pion line in Fig. 1 can be the s-wave and/or p-wave vertex.[6] As the proton energy increases, the p-wave process with a delta-isobar intermediate-state excitation becomes dominant.[6,7] The general nuclear structure amplitude for this reaction can be expressed in terms of matrix elements of creation and annihilation operators which are calculated with shell-model wave functions. In order to separate the spatial and spin degrees of freedom it is useful to express these amplitudes in LS coupling:

$$A(i,f,L_{ph},S_{ph},L,S,J,n_p,n_h,n_3,l_p,l_h,l_3,r)$$
$$= \sum_{J_{ph},j_p,j_h,j_3} \begin{bmatrix} l_p & l_h & L_{ph} \\ \frac{1}{2} & \frac{1}{2} & S_{ph} \\ j_p & j_h & J_{ph} \end{bmatrix} \begin{bmatrix} L_{ph} & l_3 & L \\ S_{ph} & \frac{1}{2} & S \\ J_{ph} & j_3 & J \end{bmatrix} \langle f \| [[a^\dagger(\pi\alpha_p) \otimes \tilde{a}(\nu\alpha_h)]^{J_{ph}} \otimes a^\dagger(\pi\alpha_3)]^J \| i \rangle F(r),$$

where $F(r) = R(\alpha_p, r)R(\alpha_h, r)R(\alpha_3, r)$, i/f stand for the quantum numbers of the initial and final states, and α stands for the set of quantum numbers (n, l, j) of the shell-model orbits. Square brackets denote the normalized 9-j coefficient. In the cases of interest here $\alpha_p = \alpha_h = \alpha_3 = 1f_{7/2}$, $J_i = 0$, and hence $J = J_f$ for the total angular momentum transfer. These amplitudes have been calculated with the wave functions obtained with the "^{42}Sc" interaction of Kutchera, Brown, and Ogawa.[8] A harmonic-oscillator potential with $\hbar\omega = 10.5$ MeV was used for the bound-state radial wave function $R(r)$.

To go further we must make some assumptions about the reaction mechanism. We first work out the kinematics. The most important aspect of the kinematics is the large momentum mismatch. For $E_p = 200$ MeV, the momentum of the incoming proton is $|p| = 645$ MeV/c. If the outgoing pion carries off all of the energy brought in by the proton, the pion momentum is $|k| = 144$ MeV/c. Hence (with neglect of the small Q-value dependence) the momentum transfer to the nucleus for pions outgoing at 0° is $\Delta p = 501$ MeV/c. If the reaction takes place at the nuclear surface ($R_0 = 1.2A^{1/3}$ fm), the angular momentum transfer is $\Delta l = R_0 \Delta p = 11$ for ^{48}Ca (and is as large as 17 for backward angles). Hence, this reaction excites high-spin states (high orbital angular momentum states) almost exclusively.

Since the major part of this momentum has to be carried away by the interaction line in Fig. 1, the interaction is of short range and the zero-range approximation should be adequate. A plane-wave calculation for the incoming proton and outgoing pion then leads to the following expression for the cross section:

$$d\sigma/d\Omega = \sigma_0(2J_f+1) \sum_{L,S,J} \Big\{ \sum_{L_{ph},S_{ph},n_i,l_i} \epsilon(S_{ph})[(2l_p+1)(2l_h+1)(2l_3+1)/(2L+1)]^{1/2}(l_p0l_h0|L_{ph}0)$$
$$\times (L_{ph}0l_30|L0)\int A(i,f,L_{ph},S_{ph},L,S,J,n_i,l_i,r)j_L(qr)r^2\,dr\Big\}^2.$$

In our case the total spin transfer can only be $S=\frac{1}{2}$ which can be understood by first coupling the two protons to spin S_{pp}. Since the two protons are in the same orbit, in the zero-range approximation L_{pp} must be even, and hence $S_{pp}=0$. In order to conserve parity in the zero-range approximation, L must be odd, and thus each final state spin has a unique total L transfer given by $L=J\pm\frac{1}{2}$. In the above equation we have introduced the parameter $\epsilon(S_{ph})$ for the relative strength of the $S_{ph}=0$ and 1 amplitudes. The ϵ weighting will depend on the details of the reaction mechanism. For the example of s-wave rescattering process in the box in Fig. 1, process (a) provides a $S_{ph}=1$ amplitude and process (b) provides a $S_{ph}=0$ amplitude. However, for our one-orbit calculation it can be shown that $A(S_{ph}=1)=\sqrt{3}A(S_{ph}=0)$ and hence the ϵ dependence enters in our case just as an overall scale factor (independent of angle) in the cross sections. [The results presented here were obtained with $\epsilon(S_{ph}$

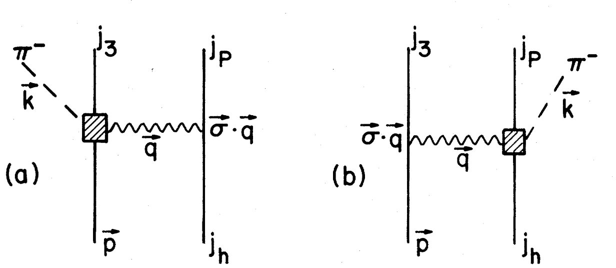

FIG. 1. Feynman diagrams for the (p,π^-) reaction. The incoming proton is represented by a solid line with momentum p and is transferred into a bound orbit in the nucleus j_3. j_p and j_h are the orbits of the particle-hole states. The dotted line denotes the outgoing pion with momentum k and the intermediate boson with momentum q is denoted by a wavy line.

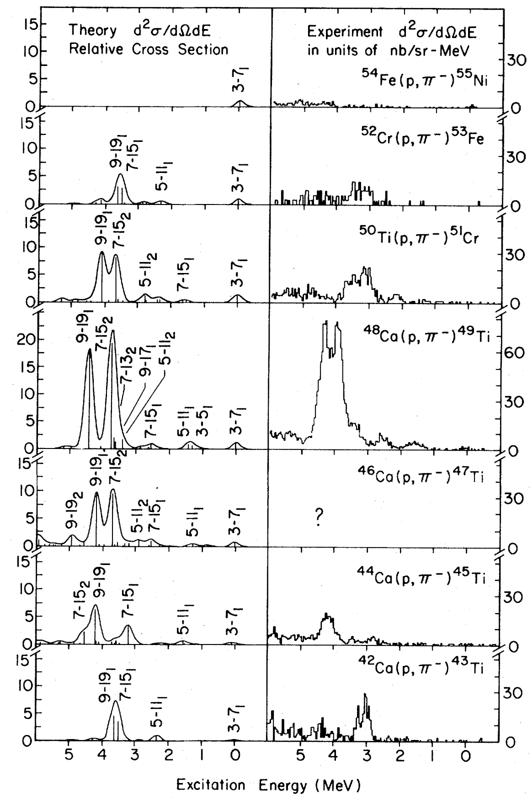

FIG. 2. Comparison of the experimental (Ref. 4) (p,π^-) cross sections with the calculated relative cross sections ($\sigma_0=1000$) for $E_p=206$ MeV and pions at 30°. In the theoretical spectra the individual states are indicated by lines labeled by $L-2J_\nu$ and the curve represents a Gaussian average with $\Gamma_{\rm FWHM}=0.3$ MeV. Above 6 MeV in excitation energy there are no strong states in the theoretical spectra and the experimental spectra show a smoothly rising background (Ref. 1).

$=1)=1$ and $\epsilon(S_{ph}=0)=0$.]

The reaction will be peaked at the nuclear surface because of the strong pion absorption and this can be taken into account in an approximate fashion by using a lower cutoff in the above radial integral. The angular distributions are in fact rather sensitive to this cutoff and we have chosen a value of 3.3 fm (about $1.1A^{1/3}$) which gives the best reproduction of the experimental ^{48}Ca(p, π^-) angular distributions. The momentum transfer as a function of angle is $q=|\vec{p}_{\text{eff}}-\vec{k}_{\text{eff}}|$ where the effective momenta take into account the Coulomb energy at the nuclear surface.

The cross sections (ignoring the relatively small Q-value dependence in the radial integral) for a given L transfer ($L\neq 3$) are proportional to the probability of stripping two protons into an $f_{7/2}$ orbit with $L_{pp}\neq 0$ multiplied by the probability of picking up a neutron from an $f_{7/2}$ orbit. For the $(f_{7/2})^n$ seniority-zero ground configurations of the Ca isotopes and $N=28$ isotones it can be shown that this spectroscopic sum rule is proportional to $(N-20)(28-Z)(26-Z)$ for each $L(\neq 3)$ for N and Z of the target. For the cases of interest these numbers are proportional to 12 for ^{43}Ti, 24 for ^{45}Ti, 48 for ^{49}Ti, 24 for ^{51}Cr, and 8 for ^{53}Fe. Except for ^{45}Ti, these values are closely proportional to the experimental cross sections noted above.

The calculated spectra for $E_p=206$ MeV and pions at 30° are shown in Fig. 2. The agreement with experiment (Refs. 1, 3, and 4) is excellent. Several details of the spectra which have not been previously understood come out of our calculations. In the theoretical ^{49}Ti spectrum there are only two strong states, one $\frac{15}{2}^-$ ($L=7$) and one $\frac{19}{2}^-$ ($L=9$). They agree in energy with the two states observed experimentally within a few hundred kiloelectronvolts. Since they have different L-transfer values, the angular distributions are very different and at 100° the $\frac{19}{2}^-$ ($L=9$) state dominates the spectra. The $\frac{19}{2}^-$ ($L=9$) state is much stronger than the $\frac{17}{2}^-$ ($L=9$) state because the latter is unfavored in the jj to LS transformation (this same difference between $\frac{19}{2}^-$ and $\frac{17}{2}^-$ also occurs, for the same reason, in three-nucleon transfer[9]).

Whereas the lowest $\frac{15}{2}^-$ ($L=7$) state is strong in ^{43}Ti, it is the second $\frac{15}{2}^-$ state which is strong in ^{49}Ti because of the change in going from the particle-particle to the particle-hole structure of the wave functions. This feature of the $\frac{15}{2}^-$ states in ^{49}Ti is confirmed by comparisons with the gamma-decay data[5] in which only the yrast $\frac{15}{2}^-$ state is seen, and by comparison with recent ^{51}V(d, α) data,[10] in which both $\frac{15}{2}^-$ states (and other high-spin states) are seen with relative strengths in agreement with $(f_{7/2})^n$ calculations. Finally we note that the fractions of the total $L=7$ and $L=9$ strength concentrated in the low-lying strong states for the five final nuclei (A) of interest are 92% (43), 62% (45), 89% (49), 83% (51), and 91% (53). This increased fragmentation in ^{45}Ti explains most of the deviation in this case from the trends expected from the sum rules noted above (the remaining strength in ^{45}Ti is fragmented over many levels but mainly into the $T=\frac{3}{2}$, $J^\pi=\frac{15}{2}^-$ and $\frac{19}{2}^-$ states around 10 MeV in excitation).

In summary, we have made plane-wave calculations using the $(f_{7/2})^n$ shell-model structure amplitudes for the (p, π^-) reaction which are able to reproduce qualitatively the experimental data for the low-lying discrete states. These spectra are dominated by an "l window" created on the lower side by the momentum mismatch in the reaction and on the higher side by the maximum transfer ($L_{\text{max}}=9$) available for three particles in the fp shell. It will thus be interesting to pursue experimentally the (p, π^-) reaction on ^{88}Sr ($L_{\text{max}}=12$) and on ^{144}Sm ($L_{\text{max}}=15$). In ^{89}Zr we expect a cluster of three closely spaced states with $L=12$, 10, and 8 around 4 MeV in excitation. (More angular momentum transfer can be achieved by putting the proton into the higher shells and this process may be part of the continuum seen above the discrete states.) The success of our simple calculations should encourage progress in the development of distorted-wave calculations. Indeed, because of the selectivity of this reaction, a great deal may be learned from such calculations about the interactions of pions in nuclei.

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