Enhanced T-Nonconserving Nuclear Moments

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T- and P-nonconserving nuclear moments induced by a parity- and CP-nonconserving interaction in the QCD Lagrangian are discussed. The mixing of nearly degenerate opposite-parity ground-state doublets in certain deformed nuclei leads to electric dipole and magnetic quadrupole moments 10^1–10^3 and 10^3–10^4 times larger, respectively, than those generated by the unpaired valence nucleon.

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The observation of CP nonconservation in the decay $K_L{}^0 \to 2\pi$,[1] coupled with the CPT theorem, suggests that elementary particles may have nonzero "odd" static moments ($C1/E1$, $M2$, $C3/E3$, ...) forbidden by time-reversal invariance. Experimental limits on the size of such moments, particularly the electric dipole moment (edm) of the neutron ($d_n \leq 6\times10^{-25}$ cm),[2] impose important constraints on theories attempting to deal with possible mechanisms for CP nonconservation.

One possible technique for determining the odd moments of charged particles is to consider their interaction energy with an external electromagnetic field when bound in a neutral, composite system. In particular, we will be concerned with nuclear moments in a neutral atom. At first such atomic experiments look unattractive because of a theorem of Schiff that, for a point nucleus, there exists no term in the interaction with an external field that is linear in the electric dipole moment.[3] However, the consequences of perfect electronic shielding are mitigated to the extent that, for a finite nucleus, the charge and electric dipole moments have different spatial distributions.[4] This effect has been exploited successfully by Harrison, Sandars, and Wright, who greatly improved the limit on the proton edm in a molecular beam resonance experiment using the polar molecule TlF.[5]

In this Letter we illustrate an attractive feature of such experiments, that their sensitivity to the underlying parameters governing the strength of CP nonconservation may be much greater than one would expect from free-particle estimates. The phenomenon we exploit is the occurrence of nearly degenerate opposite-parity ground-state doublets in certain rare-earth and actinide nuclei whose members are connected by (surprisingly) strong $E1$ matrix elements. The possibility that the mixing of doublet states could enhance T-nonconserving nuclear moments appears to have been first discussed by Feinberg.[4]

It has been known for some time that the most general form of the Lagrangian in quantum chromodynamics contains a P- and CP-nonconserving interaction of the form[6]

$$L_{CPNC} = \theta(g^2/32\pi^2)F_a{}^{\mu\nu}F_{a\mu\nu}{}^*. \tag{1}$$

where θ is a dimensionless parameter, F and F^* are the field tensor and its dual, a is the color index, and $g^2/4\pi$ is the color coupling constant. Inclusion of weak-interaction effects shifts θ from its bare value to a new angle $\bar{\theta}$.[7] A long-sought observable consequence of this term is an induced neutron edm

$$d_n \simeq 3.6\times10^{-16}\bar{\theta}\ \text{cm}$$

so that experiment demands $|\bar{\theta}| \lesssim 10^{-9}$.[8,9] Considerable effort has been expended in attempts to understand why $\bar{\theta}$, a combination of pure QCD and weak-interaction parameters, is so much smaller than any natural scale. (See Ref. 7 for background). Our calculation of T-odd nuclear moments takes such a small but nonzero value for $\bar{\theta}$ as the source of CP nonconservation.

If the CP-nonconserving term in Eq. (1) is added to the usual QCD Lagrangian one obtains, in addition to the familiar pion-nucleon coupling $g_{\pi NN}$, an effective CP-nonconserving coupling $\bar{g}_{\pi NN}$:

$$L_{\pi NN} = \vec{\pi}\cdot\bar{N}\vec{\tau}(i\gamma_5 g_{\pi NN} + \bar{g}_{\pi NN})N. \tag{2}$$

The strength, $|\bar{g}_{\pi NN}| \cong 0.027|\bar{\theta}|$, is determined by SU(3) and known quark mass ratios.[8] The corresponding induced coupling, proportional to $\bar{g}_{\pi NN}$, of the electromagnetic current to nucleons can be evaluated rigorously in the chiral limit $m_\pi \to 0$, as shown by Crewther *et al.*[8] The leading order $[M^{-1}\ln(M/m_\pi)]$ terms are given entirely by the one-pion intermediate-state diagrams of Fig. 1(a):

$$\langle p' | J_{\mu(1)}^{\text{em odd}} | p\rangle = e\overline{U}(p') d_n \sigma_{\mu\nu} q^\nu \gamma_5 \tau_3 U(p) \tag{3}$$

where $q^\nu = (p'-p)^\nu$ and $d_n = (g_{\pi NN}\bar{g}_{\pi NN}/4\pi^2 M) \times \ln(M/m_\pi)$ is the neutron electric dipole moment. $J_\mu{}^{\text{em odd}}$ has the opposite behavior under P and CP from the ordinary electromagnetic current.

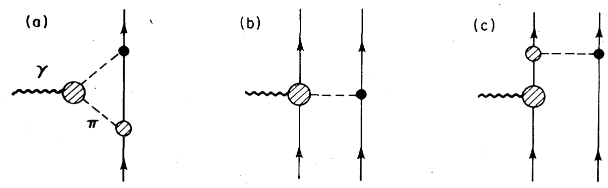

FIG. 1. Representative contributions to nuclear electric dipole moment: (a) One-body mechanism, (b) exchange-current contribution, (c) contribution from CP- and P-nonconserving potential via nuclear wave-function mixing. The solid blobs represent the CP-nonconserving πNN coupling $\bar{g}_{\pi NN}$. Only positive (negative) energy nucleon intermediate states are included in Fig. 1(c) [in the photoproduction vertex of Fig. 1(b)].

In a nucleus the effective Lagrangian of Eq. (2) induces, in addition to single-nucleon odd moments, an exchange-current contribution to the electromagnetic current and a CP-nonconserving nucleon-nucleon potential, as illustrated in Figs. 1(b) and 1(c). The former, to leading order in M^{-1}, adds a two-body component $\vec{J}_{(2)}^{\text{em odd}}$ to the three-current that satisfies

$$\nabla \cdot \vec{J}_{(2)}^{\text{em odd}} = -i[V^{\text{odd}}, \rho_{(1)}{}^{\text{em}}] \tag{4}$$

where $\rho_{(1)}{}^{\text{em}}$ is the usual charge operator and V^{odd} is the CP-nonconserving one-pion-exchange potential discussed below. Equation (4) is required to satisfy current conservation to order $\bar{g}_{\pi NN}$. As a result, contributions to T-nonconserving static nuclear moments from electric projections of the three-current $\vec{J}^{\text{ em odd}}$ vanish.

The CP-nonconserving nucleon-nucleon potential of Fig. 1(c) is

$$v^{\text{odd}}(1,2) = -\frac{1}{8\pi}\frac{m_\pi^2}{M} g_{\pi NN}\bar{g}_{\pi NN}\vec{\tau}(1)\cdot\vec{\tau}(2)[\vec{\sigma}(1)-\vec{\sigma}(2)]\cdot\hat{r}\,\frac{\exp(-m_\pi r)}{m_\pi r}\left[1+\frac{1}{m_\pi r}\right], \tag{5}$$

where $\vec{r}$ is the separation between nucleons. This interaction will induce P-odd components in nuclear states as a result of mixing with nearby levels of the same spin and opposite parity. Expectation values of odd multipole projections of the ordinary electromagnetic charge operator and even magnetic projections of the ordinary three-current are proportional to the strength of this component.

The interaction linear in $\bar{g}_{\pi NN}$ of an atomic electron with the nucleus can be calculated in the long-wavelength limit. Because of current conservation, the surviving nuclear operators involve only the $\Delta J = 1$ matrix elements of the electromagnetic charge. We find

$$V(r) = -\frac{\alpha d_n \vec{r}}{r^3}\cdot\left[\langle 0|\sum_{i=1}^{A}\tau_3(i)\vec{\sigma}(i)|0\rangle + 3.09\,\mathrm{Re}\left(\left\langle 0\left|\sum_{i=1}^{A}\frac{1}{2}+\frac{\tau_3(i)}{2}\,M\vec{r}(i)\right|1\right\rangle\frac{\langle 1|V^{\text{odd}}|0\rangle/\bar{g}_{\pi NN}}{\Delta E}\right)\right]$$
$$\equiv -\frac{\alpha D_N \vec{r}}{r^3}\cdot\langle 0|\sum_{i=1}^{A}\tau_3(i)\vec{\sigma}(i)|0\rangle, \tag{6}$$

where we use $8\pi^2/g_{\pi NN}\ln(M/m_\pi) = 3.09$ and assume that the unperturbed nuclear ground state $|0\rangle$ mixes principally with a single nearby excited state $|1\rangle$ of the same spin and opposite parity. Equation (6) defines the total nuclear electric dipole moment so that $D_N \to d_n$ in the absence of such mixing [i.e., without contributions from Fig. 1(c)]. In this limit $V(r)$ would be comparable to, or smaller than, the potential exerted by a single nucleon, depending on the extent to which the ground-state Gamow-Teller (GT) matrix element in Eq. (6) differs from the single-particle value. The interaction energy of a neutral atom in an electric field should scale roughly like D_N; it also depends in detail on the dipole moment *density*, specifically the second moment of the difference between the normalized nuclear charge and electric-dipole distributions.[4] Calculations of this quantity and of the atomic wave functions needed to estimate the interaction energy will be presented elsewhere.

Our present concern is the enhancement of D_N due to wave-function mixing. According to Eq. (6) such enhancement is likely to occur for a ground-state opposite-parity doublet whose members are separated in energy by a small ΔE and are connected by a relatively strong $E1$ matrix element. In addition, as one can demonstrate by reducing $V^{\text{odd}} = \frac{1}{2}\sum_{ij} v_{ij}{}^{\text{odd}}$ to an effective one-body potential $\sum_i U_i$, $\langle V\rangle$ depends linearly on $(N-Z)/A$. These criteria comprise a figure of merit that we employed in selecting nuclei that might have enhanced nuclear dipole moments. The best candidates, as shown in Table I, are found in the rare-earth and actinide regions. The Nilsson wave functions for the relevant levels in these deformed nuclei are given in terms of their asymptotic quantum numbers $[Nn_Z\Lambda, K^\pi]$. In all cases the transitions between doublet levels involve $\Delta K=0$, so that mixing by a scalar potential is allowed. The $E1$ matrix elements were determined from the experimental lifetimes of the excited-state members of the doublet and from calculated internal-conversion coefficients.[10] These estimates should be quite reliable in all cases except ^{229}Pa, where the very small energy release of $\Delta E = 220$ eV introduces considerable uncertainty in the internal-conversion calculation.[11] However, the surprisingly large $E1$ matrix element found in ^{229}Pa is consistent with the value determined for the analogous $\frac{5}{2}^+$-$\frac{5}{2}^-$ doublet in ^{225}Ac.[12] The ratios of the experimental $E1$ matrix elements to those predicted by the Nilsson model generally exceed unity and in two cases do so significantly (^{153}Sm and ^{229}Pa, where the ratios are >19.2 and 13.8, respectively). Such enhancements contradict the general trend in nuclei that the collective giant-dipole resonance results in a substantial depletion of low-lying $E1$ strength.

We evaluated the Nilsson-model matrix elements of the two-body potential and of the Gamow-Teller operator that appear in Eq. (6). As the matrix elements of $\vec{\sigma}\tau$ and $\vec{\sigma}\cdot\vec{r}$, the operator generated by reducing V to an effective one-body form U, are generally strong [$O(1)$ and $O(b)$, respectively, where b is the oscillator parameter], we expect these estimates to be reliable. The resulting predictions for D_N/d_n are shown in Table I.

The atomic shielding problem discussed earlier for the electric dipole moment does not arise for the inhomogeneous external electric and magnetic fields required to interact with higher odd nuclear electromagnetic moments. However, the magnitude of such interactions are expected to be suppressed by $[(\text{nuclear size})/(\text{atomic size})]^{L-1}$, where L is the multipolarity. We consider enhancements due to wave-function admixing for the first such moment, $M2$. The quantity analogous to D_N/d_n in Eq. (6) is

$$\frac{M2}{m2} = 1 + \frac{2i\,\mathrm{Im}(\epsilon\langle 0\|T_2^{\mathrm{mag}}\|1\rangle}{\langle 0\|T_2^{\mathrm{mag}\,\bar{g}}\|0\rangle}$$

where $T_2^{\mathrm{mag}\,\bar{g}}$, proportional to $\bar{g}_{\pi NN}$, is the $M2$ projection of $J_\mu^{\mathrm{em\ odd}}$,

$$T_2^{\mathrm{mag}\,\bar{g}} = \frac{id_n}{3M}\left(\frac{2}{5}\right)^{1/2}\sum_{i=1}^{A} r(i)[[Y_1(\Omega_i)\otimes\vec{\sigma}(i)]_2\otimes\vec{\nabla}(i)]_2,$$

and T_2^{mag} is the ordinary $M2$ operator

$$T_2^{\mathrm{mag}} = -\frac{i}{30M}\sum_{i=1}^{A}\{[1+\tau_3(i)]r(i)^2[Y_2(\Omega_i)\otimes\vec{\nabla}(i)]_2 - \tfrac{1}{2}[\mu^s+\mu^v\tau_3(i)](15)^{1/2}r(i)[Y_1(\Omega_i)\otimes\vec{\sigma}]_2\}. \qquad (7)$$

The scalar and vector magnetic moments are $\mu^s = 0.88$ and $\mu^v = 4.706$, $\|$ and $\otimes$ denote reduced matrix elements and tensor products, and $\epsilon \equiv \langle 1|V^{CP\mathrm{NC}}|0\rangle/\Delta E$. Terms arising in the nonrelativistic reduction of Eq. (3) that vanish on shell[13] as well as exchange currents (which do affect magnetic multipoles) have been ignored in Eq. (7). The enhancements shown in Table I, of order 10^2-10^4, are considerable

TABLE I. Nuclear electric dipole and magnetic quadrupole moments.

Nucleus	$[Nn_Z\Lambda,K^\pi]_{\mathrm{g.s.}}$ [a]	$[Nn_Z\Lambda,K^\pi]_{\mathrm{e.s.}}$ [a]	ΔE (keV)	$\langle 1\lvert V\rvert 0\rangle/\bar{g}$ (keV) [b]	$\langle 0\|\mathrm{GT}\|0\rangle$ [b]	$\langle 0\|E1\|1\rangle$ [c]	D_N/d_n	$M2/m2$
^{153}Sm	[651, $\frac{3}{2}^+$]	[521, $\frac{3}{2}^-$]	35.8	−170	−0.65	>3.74	>86.1	>10.1
^{161}Dy	[642, $\frac{5}{2}^+$]	[523, $\frac{5}{2}^-$]	25.7	−237	−1.21	0.39	10.3	−541
^{165}Er	[523, $\frac{5}{2}^-$]	[642, $\frac{5}{2}^+$]	47.2	213	1.03	0.64	9.6	664
^{225}Ac	[532, $\frac{3}{2}^-$]	[651, $\frac{3}{2}^+$]	40.0	180	−0.56	<−0.74	>19.3	<−610
^{227}Ac	[532, $\frac{3}{2}^-$]	[651, $\frac{3}{2}^+$]	27.4	187	−0.56	−0.21	8.7	−926
^{229}Pa	[642, $\frac{5}{2}^+$]	[523, $\frac{5}{2}^-$]	0.22	39	1.05	−4.58	2390	12400

[a] Nilsson model parameters taken from A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, London, 1975), Vol. II, p. 220. Wave functions for first three nuclei computed for $\delta=0.3$, last three $\delta=0.2$. All states have $J=K$.

[b] Theoretical value.

[c] Magnitudes calculated from lifetimes given in Nuclear Data Sheets and internal conversion coefficients of Ref. 10; sign from theory.

but reflect in part the weakness of the one-body contribution to the $M2$ nuclear moment.

To set the scale of the effects under discussion, limits of $(-1.4\pm 6)\times 10^{-21}$ and $<5.5\times 10^{-19}$ $e\cdot$cm have been placed on the proton edm by experiments on the molecule TlF and on atomic cesium, respectively.[5] It is believed that the sensitivities of such experiments can be improved by several orders of magnitude.[14] Our results suggest that, for certain T-nonconservation mechanisms, additional leverage can be gained by exploiting the mixing of nearly degenerate opposite-parity ground-state doublets found in some deformed nuclei. Thus it is possible that the ultimate sensitivity of atomic and molecular experiments to the underlying parameters governing T nonconservation may match or exceed that achieved for the neutron.

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