# *CP* Nonconservation in Hyperon Decays

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Upper bounds on rate differences between particle and antiparticle decays of hyperons are estimated in the standard six-quark model. For each of $\Lambda^0 \to p\pi^-$, $\Lambda^0 \to n\pi^0$, $\Sigma^+ \to n\pi^+$, $\Sigma^+ \to p\pi^0$ fractional rate differences of order $10^{-6}$ are found whereas for $\Sigma^- \to n\pi^-$, $\Xi^- \to \Lambda\pi^-$, and $\Xi^0 \to \Lambda\pi^0$ *none is expected.* Some other tests of *CP* conservation in hyperon decays relevant to left-right symmetric models are pointed out.

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It is well known that while the *CPT* theorem requires equal lifetimes for particles and antiparticles, it allows different partial widths for particular decay modes. Such particle/antiparticle rate asymmetries due to *CP* nonconservation in the standard six-quark (Kobayashi-Maskawa) model[1] have been estimated[2] for several processes recently. We know that any *CP*-nonconserving effect below the threshold for $b$-quark production in this scheme is expected[3] to be proportional to $s_2 s_3 s_\delta$ [$s_i = \sin\theta_i$ and $s_\delta = \sin\delta$ where the angles are defined as usual in the Kobayashi-Maskawa (KM) matrix]. Here we estimate the rate asymmetries for hyperon decays and find that while they are proportional to this factor, they are reduced further by at least 3 orders of magnitude as a result of dynamical effects, viz. the dominance of a single final state and the smallness of final-state interactions.

We define the *CP*-nonconserving rate asymmetry in a given decay mode as

$$\Delta = (\Gamma - \bar{\Gamma})/(\Gamma + \bar{\Gamma}), \tag{1}$$

where $\Gamma$ is the particle decay rate for this channel and $\bar{\Gamma}$ is the antiparticle rate for the charge-conjugate channel. Let the amplitude for the particle decay be $A$ given by

$$A = A_1 \exp(i\delta_1) + A_2 \exp(i\delta_2), \tag{2}$$

where $A_1$ and $A_2$ are the weak amplitudes for two different final eigenstates which differ in one or more of the (strongly) conserved quantum numbers, e.g., isospin. $\delta_1$ and $\delta_2$ are the final-state scattering eigenphase shifts. The corresponding antiparticle decay amplitude is then

$$\bar{A} = A_1^* \exp(i\delta_1) + A_2^* \exp(i\delta_2). \tag{3}$$

Then $\Delta$ is found to be given by

$$\Delta = \frac{-2|A_1||A_2|\sin(\varphi_1 - \varphi_2)\sin(\delta_1 - \delta_2)}{|A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\varphi_1 - \varphi_2)\cos(\delta_1 - \delta_2)}, \tag{4}$$

where $A_1 = |A_1|\exp(i\varphi_1)$ and $A_2 = |A_2|\exp(i\varphi_2)$. From Eq. (4) it is clear that for $\Delta$ not to vanish we must have (i) at least two channels in the final state, (ii) differing weak-interaction phases, and (iii) unequal final-state strong-interaction phases. This treatment is useful for the problem at hand, viz. hyperon decays where the energy is below threshold and the phase shifts are known. In $B$ decays, on the other hand, many channels are open and the techniques of Ref. 2 are more useful.

We now apply the above analysis to hyperon decays. First, we consider $\Xi^- \to \Lambda\pi^-$, $\Xi^0 \to \Lambda\pi^0$, and $\Sigma^- \to n\pi^-$ decays. In each of these there is only one final state, viz. $I=1$, $I=1$, and $I=\frac{3}{2}$, respectively. Hence by our reasoning above there can be *no* *CP*-nonconserving rate asymmetry for these three decay modes. Note that the fact that both $P$- and $S$-wave amplitudes can be present is irrelevant since they do *not* interfere in the total decay rate.

Next we turn to the modes $\Lambda \to p\pi^-$ and $\Lambda \to n\pi^0$. Since experimentally[4] $\Gamma(\Lambda \to n\pi^0) \approx \frac{1}{2}\Gamma(\Lambda \to p\pi^-)$ and by *CPT* we have $\Gamma(\Lambda \to n\pi^0) - \bar{\Gamma}(\bar{\Lambda} \to \bar{n}\pi^0) = -[\Gamma(\Lambda \to p\pi^-) - \bar{\Gamma}(\bar{\Lambda} \to \bar{p}\pi^+)]$, it follows that $\Delta(\Lambda \to n\pi^0) = -2\Delta(\Lambda \to p\pi^-)$. Hence we need only estimate $\Delta(\Lambda \to p\pi^-)$. The $S$-wave contribution to the total decay rate for $\Lambda \to p\pi^-$ is almost[4] 90% of the total rate and so the contribution from the $P$-

wave amplitudes to the rate asymmetry can safely be ignored. The $S$-wave amplitude can be written as

$$S=-(\tfrac{2}{3})^{1/2}S_1\exp(i\delta_1)+(\tfrac{1}{3})^{1/2}S_3\exp(i\delta_3), \qquad (5)$$

where $S_1$ and $S_3$ are amplitudes for $I=\frac{1}{2}$ and $\frac{3}{2}$ final states, respectively, and $\delta_1$ and $\delta_3$ are $\pi N$ $S$-wave scattering phase shifts for $I=\frac{1}{2}$ and $\frac{3}{2}$ at $E=M_\Lambda$. In the KM model for $CP$ nonconservation (with the usual phase convention), a $CP$-nonconserving phase for $\Delta S=1$ decays can only arise[3] from $c\bar{c}$ and $t\bar{t}$ intermediate states. Since these can give rise to $\Delta I=\frac{1}{2}$ only, $S_3$ is real. To *maximize* the possible $CP$-nonconservation we assume that penguin diagrams[5] are the dominant source for $\Delta I=\frac{1}{2}$ amplitude and hence for $S_1$. Then $S_3$ is real and the phase of $S_1$ is given by $\varphi$ where

$$\tan\varphi\cong\frac{(-2c_2/c_1c_3)(s_2s_3s_\delta)}{1+2s_2^{\,2}+2s_2s_3c_\delta c_2/c_1c_3} \qquad (6)$$

from the evaluation of the coefficient of operator $O_6$,

$$O_6=(\bar{s}_L\lambda_\alpha d_L)(\bar{u}_R\lambda_\alpha u_R+\bar{d}_R\lambda_\alpha u_R+\bar{s}_R\lambda_\alpha s_R), \qquad (7)$$

by Gilman and Wise.[6] The parameters they used are $m_t\approx 30$ GeV, $m_b=4.5$ GeV, and $m_c=1.5$ GeV. Now from $\mathrm{Re}\,\epsilon\approx 10^{-3}$ in the $K_L$-$K_S$ system, we have[3,7] $s_2s_3s_\delta\sim 0.3\times10^{-3}$, and hence $\varphi\sim\tan\varphi\sim-0.6\times10^{-3}$. With $\sin\varphi\sim-0.6\times10^{-3}$, $\delta_1-\delta_3\approx 9.8°$ and $S_3/S_1\approx 0.027\pm0.008$, we find[8]

$$\Delta(\Lambda\to p\pi^-)\cong-\sqrt{2}(S_3/S_1)\sin\varphi\sin(\delta_1-\delta_3)\approx 3\times10^{-6} \qquad (8)$$

and hence $\Delta(\Lambda\to n\pi^0)=-2\Delta(\Lambda\to p\pi^-)\approx-2\times10^{-6}$.

Next we turn to $\Sigma^+$ decay modes, $\Sigma^+\to n\pi^+$ and $\Sigma^+\to p\pi^0$. Again by $CPT$ we have $\Gamma(\Sigma^+\to n\pi^+)-\bar{\Gamma}(\bar{\Sigma}^+\to\bar{n}\pi^-)=-[\Gamma(\Sigma^+\to p\pi^0)-\bar{\Gamma}(\bar{\Sigma}^+\to\bar{p}\pi^0)]$ and by the approximate equality[4] of observed rates $\Gamma(\Sigma^+\to p\pi^0)\approx\Gamma(\Sigma^+\to n\pi^+)$; hence $\Delta(\Sigma^+\to n\pi^+)\cong-\Delta(\Sigma^+\to p\pi^0)$. We choose to estimate $\Delta(\Sigma^+\to n\pi^+)$. In the $\Sigma^+\to n\pi^+$ decay mode, the $P$-wave amplitude dominates,[4] while the $S$ wave (in the rate) is down by a factor of about 1000, and so the $S$-wave amplitude can be safely ignored. The $P$-wave amplitude can be written as[9]

$$P=(-\tfrac{2}{3}P_{11}+\tfrac{1}{3}P_{31})\exp(i\delta_{11})+[\tfrac{1}{3}P_{13}-\tfrac{2}{3}(\tfrac{2}{5})^{1/2}P_{33}]\exp(i\delta_{31}), \qquad (9)$$

where $P_{IJ}$ are the weak amplitudes engendered by $\Delta I=I/2$ leading to the final state of isospin $J/2$ and $\delta_{J1}$ are $P$-wave $\pi$-$N$ scattering phase shifts for isospin $J/2$ at $E=M_\Sigma$. Now we know that the dominant $\Delta I=\frac{1}{2}$ $P$-wave amplitude does *not* lead to $I=\frac{3}{2}$ in the final state (notice, e.g., that the $\Sigma^-\to n\pi^-$ $P$-wave amplitude is nearly zero). Hence, to a good approximation $P_{13}\approx 0$. Next, to maximize possible $CP$-nonconserving effects we also let $P_{31}\approx 0$. Then

$$P\approx-\tfrac{2}{3}P_{11}\exp(i\delta_{11})-\tfrac{2}{3}(\tfrac{2}{5})^{1/2}P_{33}\exp(i\delta_{31}). \qquad (10)$$

As before, the $\Delta I=\frac{1}{2}$ amplitude $P_{11}$ has a phase $\varphi$ and $\Delta I=\frac{3}{2}$ amplitude $P_{33}$ is real. For $\Delta(\Sigma^+\to n\pi^+)$ we obtain

$$\Delta(\Sigma^+\to n\pi^+)\approx-2(\tfrac{2}{5})^{1/2}(P_{33}/P_{11})\sin\varphi\sin(\delta_{11}-\delta_{31}). \qquad (11)$$

With $P_{33}/P_{11}\approx 0.050\pm0.018$, $\delta_{11}-\delta_{31}=1.7°$, and $\sin\varphi\sim-0.6\times10^{-3}$, we find $\Delta(\Sigma^+\to n\pi^+)\cong-10^{-6}$ and $\Delta(\Sigma^+\to p\pi^0)\cong-\Delta(\Sigma^+\to n\pi^+)=10^{-6}$.

In the KM model $CP$-nonconserving phases do not distinguish parity-conserving ($P$-wave) from parity-nonconserving ($S$-wave) amplitudes. But in some models[10] $S$- and $P$-wave amplitudes, in general, are expected to get different $CP$ phases. In that case the following tests of $CP$ invariance become interesting. In $\Lambda\to p\pi^-$ a violation of the equality

$$\beta^-/\alpha^-=\tan(\delta_{11}-\delta_1) \qquad (12)$$

signals $CP$ nonconservation.[9] At present $\delta_{11}-\delta_1\sim-(6.5\pm1.5)°$ and $\beta^-/\alpha^-$ is of order $\tan(-7.7\pm4.0)°$. Another test[11] for the presence of $CP$-nonconserving phase difference between $S$- and $P$-wave amplitudes is the deviation of $\bar{\alpha}/\alpha$ from $-1$, e.g.,

$$\frac{\bar{\alpha}}{\alpha}=-\frac{\cos[\delta_S-\delta_P-(\Delta_S-\Delta_P)]}{\cos[\delta_S-\delta_P+(\Delta_S-\Delta_P)]}, \qquad (13)$$

where $\delta_S$ and $\delta_P$ are the final-state interaction phases and $\Delta_S$ and $\Delta_P$ are $CP$-nonconserving phases of $S$ and $P$ amplitudes, respectively.

In $\Sigma$ decay, to the extent that the $\Delta I=\frac{1}{2}$ rule is satisfied and $\alpha(\Sigma^+\to n\pi^+)$ and $\alpha(\Sigma^-\to n\pi^-)$ are nearly 0, $CP$ invariance is tested[9] by the following equality for $\Sigma^+\to p\pi^0$ asymmetry parameters:

$$\beta^0/\alpha^0=\tfrac{1}{3}\tan(\delta_{11}-\delta_1)+\tfrac{2}{3}\tan(\delta_{11}-\delta_3)\approx 0.03\pm0.05. \qquad (14)$$

The test of $CP$ invariance in $\Xi$ decay is that[9] $\beta^-/\alpha^-=\beta^0/\alpha^0=\tan(\delta_{21}-\delta_2)$ independent of the $\Delta I=\frac{1}{2}$ rule. Here $\delta_{21}$ and $\delta_2$ are $\Lambda$-$\pi$ $S$- and $P$-wave scattering phase shifts at $E=M_\Xi$ and difficult to obtain experimentally. However, if $\beta^-/\alpha^- \neq \beta^0/\alpha^0$ both $CP$ invariance and the $\Delta I = \frac{1}{2}$ rule are violated, and furthermore $CP$ nonconservation must be unequal in $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$ amplitudes. Present data[4] are that $\beta^-/\alpha^- = \tan(5 \pm 13)^\circ$ and $\beta^0/\alpha^0 = \tan(3.6^{+1.3}_{-1.9})^\circ$.

To summarize, if the source of observed $CP$ nonconservation in $K$ decay is the phase $\delta$ in the KM matrix, then the only nonzero rate asymmetries possible in hyperon decays (in $\Lambda$ and $\Sigma^+$) are smaller than $10^{-6}$. It seems unlikely that these are experimentally detectable in the near future. To test other models, $\beta/\alpha$ for the various decay modes and the phase shifts have to be known to one part in $10^3$.

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*Note added.*—We would like to elaborate a little on the neglect of the contributions of the $P$ wave in $\Lambda \to p\pi^-$ and the $S$ wave in $\Sigma^+ \to n\pi^+$ to the rate asymmetries. In the model at hand (the Kobayashi-Maskawa model) the $CP$ nonconservation does not distinguish between parity-nonconserving and parity-conserving pieces of the Hamiltonian. Hence, unless the penguin-diagram contribution is very different for the $S$-wave and $P$-wave amplitudes, the phase difference $\varphi$ is the same for $S$ waves and $P$ waves. In that approximation the contribution to $\Delta(\Lambda \to p\pi^-)$ from $P$ waves relative to the one from $S$ waves is

$$\frac{\Delta_p}{\Delta_s} = \frac{P_3 P_1 \sin(\delta_{11} - \delta_{31})}{S_3 S_1 \sin(\delta_1 - \delta_3)} = \frac{(P_3/P_1)}{(S_3/S_1)} \frac{P_1^2}{S_1^2} \frac{\sin(\delta_{11} - \delta_{31})}{\sin(\delta_1 - \delta_3)}.$$

Now experimentally $(P_3/P_1)/(S_3/S_1)$ is of order 1, certainly less than 2; $P_1^2/S_1^2 \sim 0.115$; and $\sin(\delta_{11} - \delta_{31})/\sin(\delta_1 - \delta_3) = -\sin 0.4^\circ/\sin 9.8^\circ = -0.041$. Hence $|\Delta_p/\Delta_s| \lesssim 0.01$ and our approximation is justified. In the case of $\Sigma^+ \to n\pi^+$, the small $S$-wave amplitude is well described[12] by a small pure $\Delta T = \frac{1}{2}$ piece which has an equal mixture of final-state $T = \frac{1}{2}$ and $\frac{3}{2}$. Then the contribution to the rate asymmetry due to the $S$ wave relative to that of the $P$ wave is

$$\frac{\Delta_s}{\Delta_p} \simeq \frac{2}{9} \frac{5^{1/2}}{2^{1/2}} \left(\frac{S^+}{P^+}\right)^2 \frac{P_1}{P_3} \frac{\sin(\delta_1 - \delta_3)}{\sin(\delta_{11} - \delta_{31})}.$$

With the experimental values $(S^+/P^+)^2 \sim 10^{-3}$, $P_3/P_1 \sim 0.05$, $\delta_1 - \delta_3 \sim 19^\circ$, and $\delta_{11} - \delta_{31} \sim 1.7^\circ$, we find $(\Delta_s/\Delta_p) \lesssim 0.06$ and again our approximation is justified.

It should be emphasized that should the rate asymmetries turn out to be much larger than our estimates (say $\gtrsim 10^{-5}$) then the observed $CP$ nonconservation *must* be due to sources other than a phase $\delta$ in the KM matrix.

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