Determination of the Number of Generations of Quarks and Leptons from Flavor-Color Symmetry

O. W. Greenberg, R. N. Mohapatra, and M. Yasuè
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
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It is shown that the fact that there are six flavor-color degrees of freedom (two flavors in each generation, three colors, and lepton number) implies, under plausible assumptions, that there are three generations of light quarks and leptons. The assumptions include (a) supersymmetry for the preon theory, (b) validity of the chiral group $SU(4)_L \otimes SU(2)_L \otimes SU(4)_R \otimes SU(2)_R$ at the stage prior to the generation of the quark and lepton masses, and (c) chiral symmetry and the supersymmetric Nambu-Goldstone mechanism to protect quarks and leptons from getting large masses.

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Two of the fundamental puzzles in present elementary-particle physics are the number of generations of quarks and leptons and the mechanism which protects the quarks and leptons from acquiring masses of the order of the large mass scales which exist. Research in composite models of quarks and leptons is largely motivated by the hope that such models will shed light on these puzzles.[1] 't Hooft[2] suggested that the mass puzzle could be solved by using chiral symmetry to protect fermion masses in models in which preons are permanently confined by $SU(N)_H$ binding; however, he found that the requirement that the chiral anomalies match between the preons and the composites, together with some other conditions, was too strong to allow solutions. Since 't Hooft's work, many attempts have been made to find solutions to the anomaly-matching conditions; this work has led to the conclusion that simple solutions do not exist in models with only spin-$\frac{1}{2}$ preons.[3] There are simple solutions in models with both spin-$\frac{1}{2}$ and spin-0 preons,[4] but these models do not shed light on the number of generations; although they do have the virtue that they provide stimulus to consider supersymmetric preon models.[5]

A second mechanism to protect fermion masses was suggested recently by Buchmüller, Peccei, and Yanagida[6]: quarks and leptons are quasi-Nambu-Goldstone partners of Nambu-Goldstone bosons arising from spontaneous breaking of global symmetry in a supersymmetric preon theory where the confining interaction arises from a supersymmetric $SU(N)_H$ hypercolor gauge theory whose mass scale is Λ_H. If the original global symmetry group is G, and the spontaneous breaking leads to a subgroup H, then the Nambu-Goldstone particles have the quantum numbers of the coset space G/H. If H is an anomaly-free group, then no anomaly-matching conditions need be applied; however, among the models which have been suggested,[6-11] those in which H is anomaly free do not have the group-theory properties to give the desired chiral structure of the low-energy weak interactions. On the other hand, when H has left-right symmetry, and thus can give rise to the proper chiral structure of the low-energy weak interactions, H is not anomaly free, and the matching constraints must be applied.

We have studied the anomaly-matching constraints in a family of supersymmetric preon models in which $G = SU(6)_L \otimes SU(6)_R \otimes U(1)_V \otimes U(1)_X$, where $U(1)_X$ is an instanton-anomaly-free axial symmetry, and we take various subgroups for H.[8] Most of these models are not completely satisfactory for one of three reasons: (a) Although the weak-interaction structure is correct, anomaly matching forces fermions which do not belong to one or more generations of quarks and leptons to be massless; (b) the massless fermions belong to one or more generations, but the weak-interaction structure is not guaranteed by the unbroken group H; or (c) the weak-interaction structure is guaranteed by H, and the massless fermions can be assigned to several generations, but the multiplicity of generations has to be inserted explicitly [as in the $SU(4M+2N)$-type model[8]], thus undermining the economy of the model.

The purpose of this Letter is to describe a model in which (a) the weak-interaction structure is guaranteed by the unbroken subgroup $H = SU(4)_L \otimes SU(2)_L \otimes SU(4)_R \otimes SU(2)_R$; (b) all massless fermions have quantum numbers of quarks and leptons (and the quantum numbers correspond to the Nambu-Goldstone sector of the spontaneously broken global symmetry) except for quasi-Nambu-Goldstone fermions associated with the spontaneous breakdown of U(1)'s, which do not enter the anomaly equation considered below; and (c) the repetition of generations is forced by the anomaly-matching conditions, rather than being put in from the start, and the number of generations is required to be three.[12]

We choose as fundamental matter preons two sets of chiral superfields, $\Phi^{(s)}$ $(s=1,2)$, the first being the left-handed preons, and the second being the charge conjugates of the right-handed preons, belonging to the N and N^* representation of $\mathrm{SU}(N)_H$, the supersymmetric hypercolor binding gauge theory, respectively. Both superfields also carry N_{fc} of flavors: up and down flavors and three colors plus lepton number $(N_{fc}=6)$. At this stage, N is arbitrary. The theory also, of course, contains the vector multiplet of the supersymmetric $\mathrm{SU}(N)_H$. We call the components of the chiral multiplets $\Phi=(A,\psi,F)$, and those of the vector multiplet $V=(G_\mu,\lambda,D)$.

The global symmetry group of our preon theory is

$$G_{\mathrm{cl}}=\mathrm{U}(6)_L\otimes\mathrm{U}(6)_R\otimes\mathrm{U}(1)_X, \tag{1}$$

where $\mathrm{U}(1)_X$ is an R symmetry. Because of the instanton of $\mathrm{SU}(N)_H$, G_{cl} is broken to

$$G=\mathrm{SU}(6)_L\otimes\mathrm{SU}(6)_R\otimes\mathrm{U}(1)_V\otimes\mathrm{U}(1)_A, \tag{2}$$

where Q_V of $\mathrm{U}(1)_V$ is $+1$ for $\Phi^{(1)}$ and -1 for $\Phi^{(2)}$, and Q_A of $\mathrm{U}(1)_A$ is $N/6-1$ for $A^{(s)}$, $N/6$ for $\psi^{(s)}$, and -1 for λ. Recent extensive studies of the spontaneous breakdown of G [13-16] seem to suggest that bilinear condensates like $\langle A^{(1)}A^{(2)}\rangle$ and $\langle\psi^{(1)}\times\psi^{(2)}\rangle$ vanish in the massless limit, which is the present case. However, there is one exception: $N=N_{fc}(=6)$, which is the number of flavors plus colors including lepton number. $N=N_{fc}$ allows $\langle\det(A^{(1)}{}_iA^{(2)j})\rangle$ to be independent of the preon masses, where i and j specify flavors. In the massless limit, $\langle\det(A^{(1)}{}_iA^{(2)j})\rangle$ can acquire a nonvanishing value.[14,15] It implies the hypercolor- and $\mathrm{SU}(6)_L\otimes\mathrm{SU}(6)_R$-singlet condensates $\langle A^{(s)}A^{(s)}\times A^{(s)}A^{(s)}A^{(s)}A^{(s)}\rangle\neq0$.[14,17] The condensates break the vector $\mathrm{U}(1)_V$ symmetry but preserve the axial $\mathrm{U}(1)_A$ symmetry since by this choice of N, $Q_A=0$ for $A^{(s)}$. Therefore, we choose

$$N=N_{fc}\ (=6) \tag{3}$$

in order to support the spontaneous breakdown of $\mathrm{U}(1)_V$, the preon-number conservation. Other condensates free from the preon masses are given by $\langle A^{(s)\dagger}A^{(s)}\rangle$, which can break $\mathrm{SU}(6)_{L,R}$. At this stage, G is broken to

$$G'=\mathrm{SU}(4)_L\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_L\otimes\mathrm{SU}(4)_R\otimes\mathrm{SU}(2)_R\otimes\mathrm{U}(1)_R\otimes\mathrm{U}(1)_A$$

by condensates: $\langle A^{(s)\dagger}A^{(s)}\rangle=v^{(s)}\mathrm{diag}(1,1,1,1,-2,-2)$ and $\langle A_{[1}{}^{(s)}A_2{}^{(s)}A_3{}^{(s)}A_4{}^{(s)}A_5{}^{(s)}A_{6]}{}^{(s)}\rangle\neq0$, where subscript [ ] stands for total antisymmetrization with respect to subscripts $1,2,3,\ldots$. We further assume spontaneous breaking of three U(1)'s in G' by appropriate condensates such as $\langle\lambda\lambda\rangle$ and $\langle\Lambda_{[5}{}^{(s)}\Lambda_{6]}{}^{(s)}A_{[5}{}^{(s)}A_{6]}{}^{(s)}\Lambda_{[5}{}^{(s')*}\Lambda_{6]}{}^{(s')*}\rangle$, where $\Lambda_i{}^{(s)}=\bar{\lambda}A_i{}^{(s)}$ $(i=5,6)$ and $s\neq s'$.[18] We are thus left with

$$H=\mathrm{SU}(4)_L\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(4)_R\otimes\mathrm{SU}(2)_R. \tag{4}$$

The Nambu-Goldstone phenomenon requires that there be massless composite fermions transforming as $(4,2,1,1)$, $(1,1,4^*,2)$, and four neutral ones as $(1,1,1,1)$ under H.

Because there are no U(1) factors in H, and SU(2) is an anomaly-free group, the only anomaly-matching constraint comes from the SU(4) factors; we require left-right symmetry of the theory and need satisfy only one constraint:

$$2(l_1-l_2+l_3)=6(=N), \tag{5}$$

where l_1, l_2, and l_3 are the indices for $(4,2,1,1)$ and $(1,1,4^*,2)$, $(4^*,2,1,1)$ and $(1,1,4,2)$, and $(4,1,1,2)$ and $(1,4^*,2,1)$, respectively. The choice,

$$l_1=3 \text{ and all other indices } =0, \tag{6}$$

clearly satisfies the constraint. Since l_1 is the index for the Nambu-Goldstone sector, there are three generations of quarks and leptons with the identical quantum numbers of this sector. Only one of these is a quasi-Nambu-Goldstone fermion with double mass protection. The other two generations have single mass protection from chiral symmetry. Other massless fermions are four neutral quasi-Nambu-Goldstone fermions which have only supersymmetric mass protection.

We have not studied in detail the mass and mixing pattern which occurs when we break H to give quarks and leptons masses. For the present we close by giving a plausible mass matrix among generations which seems compatible with the present model:

$$M=\begin{pmatrix}\epsilon & \epsilon' & \epsilon'\\ \epsilon' & m_4 & m_4\\ \epsilon' & m_4 & m_4\end{pmatrix}\quad(\epsilon,\epsilon'\sim m_4M_s/\Lambda_H), \tag{7}$$

where M_s and m_4 stand for the mass scales of the supersymmetry breaking and the breaking of $SU(4)_L \otimes SU(4)_R$ down to $SU(4)_V$, respectively. This mass matrix is easily obtained by noticing that only one generation has a double mass protection ($\epsilon \to 0$ as m_4 or $M_s \to 0$),[19] while the other two have a single mass protection ($m_4 \to 0$). Furthermore, there will be no mixing between these two groups if M_s or m_4 vanishes ($\epsilon' \to 0$ as m_4 or $M_s \to 0$). Diagonalization of this matrix leads to a pattern of masses in qualitative agreement with the experimental situation. The masses of the three generations are given by 0, $\sim\epsilon$, and $\sim 2m_4$ for $\epsilon, \epsilon' \ll m_4$ ($<\Lambda_H$), which respectively turn out to be 0, O(1 GeV), and O(10 GeV) if $m_4 \sim O$(10 GeV), $M_s \sim O$(100 GeV), and $\Lambda_H \sim O$(1 TeV). The four neutral quasi-Nambu-Goldstone fermions are also expected to acquire masses of order M_s since they are not protected by the chiral symmetry. To discuss the weak mixing pattern needs further study.[20]

We summarize our conclusions as follows: Experiment shows the existence of six flavors and colors: the up and down flavors in each generation, the three colors carried by quarks, and the lepton number. The spontaneous preon-number breaking seems to suggest that the number N of hypercolors equals the number N_{fc} of flavors plus colors. Implementation of the Buchmüller-Peccei-Yanagida mechanism in a model in which the unbroken symmetry group guarantees the correct low-energy weak interactions and all massless fermions belong to the Nambu-Goldstone sector requires the choice $H = SU(4)_L \otimes SU(2)_L \otimes SU(4)_R \otimes SU(2)_R$ and leads to an index for particles in this sector (which is the number N_g of generations) which is half the number of flavors plus colors via the intermediate requirement that this number agree with the number of hypercolors. In one elementary equation, we find

$$2N_g = N = N_{fc}\,. \tag{8}$$

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[18]The condensate $\langle\lambda\lambda\rangle$ may not be developed in the massless limit (Refs. 14 and 15). Other condensates like $\langle \Lambda_5{}^{(s)}\Lambda_6{}^{(s)}A_5{}^{(s)}A_6{}^{(s)}A_5{}^{(s')*}A_6{}^{(s')*}\rangle$ may be used.

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[20]For the case $N=4$ or two generations ($N_g=N/2$), the mass matrix takes the interesting form

$$\begin{pmatrix} \epsilon & \epsilon' \\ \epsilon' & m_4 \end{pmatrix},$$

which is similar to that suggested by S. Weinberg, in *Festschrift for I. I. Rabi,* edited by L. Motz (N.Y. Academy of Sciences, New York, 1977), and by F. Wilczek and A. Zee, Phys. Lett. 70B, 418 (1977), and will imply a value for the Cabibbo angle $\theta_C \simeq (m_d/m_s)^{1/2}$.