Comment on "Diffusion in a Medium with a Random Distribution of Static Traps"

In their Letter,[1] Kayser and Hubbard consider a particle diffusing among random static traps. Let $P(t)$ denote the probability of survival of the particle up to time t, averaged over the trap distribution; they show an upper bound on $P(t)$ and note that together with a lower bound of Grassberger and Procaccia[2] it yields

$$C_1 \leq \lim_{t\to\infty} \frac{-1}{t^{d/(d+2)}} \ln P(t) \leq C_2, \tag{1}$$

where d is the space dimension and C_1, C_2 are finite, strictly positive constants. It is the purpose of the present Comment to stress the fact that such a result can be viewed as a direct consequence of a probabilistic result which was proved some years ago and which furthermore provides the *exact* asymptotic behavior of $P(t)$!

Let us suppose the traps to have radius ϵ and uniform probability distribution with density n, as in Refs. 1 and 2. The averaged survival probability $P(t)$ is easily seen to be equal to $E(\exp\{-n \times S_t^{\epsilon}\})$ where S_t^{ϵ} is the volume of the Wiener sausage (i.e., of the set of points at distance less than ϵ from a Brownian path up to time t) and E denotes averaging over all Brownian paths. Donsker and Varadhan have proven[3] the following theorem:

Theorem.—For any $\epsilon > 0$,

$$\lim_{t\to\infty} \frac{1}{t^{d/(d+2)}} \ln (E\{\exp - n S_t^{\epsilon}\}) = -k(n,d),$$

where

$$k(n,d) = n^{2/(d+2)} \left(\frac{d+2}{2}\right)\left(\frac{2\gamma}{d}\right)^{d/(d+2)}$$

and γ is the lowest eigenvalue of $-\frac{1}{2}\Delta$ in the d-dimensional sphere of unit volume with zero boundary conditions.

We add a few comments:

(i) In fact the authors of Refs. 1 and 2 consider the mean return probability $\rho(t)$ at time t, instead of $P(t)$. It is easily verified that the asymptotic behavior of $\rho(t)$ and $P(t)$ coincide within a multiplicative power of t.

(ii) The result of Ref. 3 is a difficult one, in particular the proof of the upper bound. It is remarkable that one obtains the exact asymptotic behavior of $P(t)$. After computation, $k(n,d)$ exactly coincides with the lower bound of Ref. 2; notice that in the theorem the diffusion constant has been taken implicitly as $\frac{1}{2}$, which corresponds to the normalized Brownian motion.

(iii) The result of Donsker and Varadhan extends to other diffusion processes[3,4] and to various random walks. In particular the case of random walks with long-range jumps can be treated[4]: The asymptotic behavior is then $t^{d/(d+\alpha)}$ instead of $t^{d/(d+2)}$ where α is a parameter governing the range of the jumps.

(iv) However, the method of Ref. 3 is difficult and does not extend easily to various other situations whereas the method of Ref. 1 makes explicit the basic mechanism and can be directly generalized to study for example the case of partially absorbing traps.[5]

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