Density Fluctuations from Strings and Galaxy Formation

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The spectra of density fluctuations caused by strings in a universe dominated either by baryons, neutrinos, or axions are presented. Realistic scenarios for galaxy formation seem possible in all three cases. Examples of grand unified theories which lead to strings with the desired mass scales are given.

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Most of the work on galaxy formation theory has centered on the evolution of cosmological density fluctuations, the amplitude and spectrum of the fluctuations being postulated as an initial condition. The origin of the fluctuations remains an unresolved problem. It has been recently suggested[1,2] that cosmologically interesting density fluctuations could be generated by strings produced at a phase transition in the early universe. In this Letter we shall analyze the string model of galaxy formation from both cosmological and particle-physics points of view.

Suppose the strings are formed at a phase transition when a Higgs field φ acquires a vacuum expectation value, $\langle\varphi\rangle=\eta$. Then the linear mass density of the strings is[3] $\mu\sim\eta^2$. [For global strings[4] resulting from a global symmetry breaking, $\mu=2\pi\eta^2\ln(\eta r_c)$, where r_c is a cutoff radius equal to the typical distance between the strings.] The evolution of strings in an expanding universe has been discussed previously.[2-7] Expansion of the universe conformally stretches the strings on scales greater than the horizon and straightens them out on scales smaller than the horizon,[2] so that at any time t we have a system of Brownian strings with a persistence length $\sim t$ and typical distance between nearby strings also $\sim t$. Intersecting strings intercommute and form closed loops[3] (this process also contributes to straightening of the strings). Loops can rapidly decay as a result of multiple self-intersections; then they are cosmologically unimportant, and the density fluctuations are produced by open strings.[1] On scales greater than the horizon the density fluctuations due to strings are balanced by the corresponding variations of the radiation density. Fluctuations of the order $(\delta\rho/\rho)_h\sim\rho_s/\rho$ are produced on each scale at the time t_h when that scale comes within the horizon.[1] Here $\rho_s\sim\mu t/t^3\sim\mu t^{-2}$ is the density of strings and $\rho=(AGt^2)^{-1}$ is the unperturbed density. Thus, we have

$$(\delta\rho/\rho)_h\sim AG\mu, \qquad (1)$$

where $A\sim 30$ for $t_h<t_{\rm eq}$, $A\sim 20$ for $t_h>t_{\rm eq}$; $t_{\rm eq}$ is the time of equal matter and radiation densities. The cosmological evolution of such a scale-invariant spectrum has been studied by a number of authors, and we shall have nothing new to say about it here. Instead, we shall concentrate on the alternative possibility when the lifetimes of the loops are long.

Kibble and Turok[5] have found a large class of loop trajectories which never self-intersect. This suggests that a substantial fraction of loops may avoid rapid decay by multiple self-intersections.[8] Then the dominant energy-loss mechanism for the loops is gravitational radiation, and the lifetime of a loop of size R is[2] $\tau\sim R/G\mu$. Loops formed at time t_1 have size $R\sim t_1$ and decay at $t\sim t_1/G\mu\gg t_1$. (We shall see that the symmetry-breaking scale η is well below the Planck scale.) Such loops can serve as seeds for galaxies and clusters of galaxies.[2]

In a baryon-dominated universe the density fluctuations start growing like $(1+Z)^{-1}$ at the red shift $Z\sim\min(Z_{\rm dec}, Z_{\rm eq})$, where $Z_{\rm dec}\sim 1300$ and $Z_{\rm eq}\sim 2\times10^4\Omega h^2$ are[9] the red shifts at decoupling and at $t_{\rm eq}$, respectively. The parameters Ω and h are in the range $0.1\lesssim\Omega\lesssim 1$, $0.5\lesssim h\lesssim 1$. Considerations of nucleosynthesis[10] constrain $\Omega_{\rm baryon}$ to be $\lesssim 0.1$, which leaves us with $\Omega\sim 0.1$. For such values of parameters, we have $Z_{\rm eq}\sim Z_{\rm dec}$.

Consider a comoving scale which has size $l<t_{\rm eq}$ at $t\sim t_{\rm eq}$. It came within the horizon at time $t_h\sim l^2/t_{\rm eq}$. At that time about one loop of size $\sim t_h$ was formed on this scale.[2,3] The correspond-

ing density fluctuation is[11] $(\delta\rho/\rho)_h \sim 30G\mu$, and so ($\rho_B$ is the baryon density)

$$\left(\frac{\delta\rho}{\rho_B}\right)_h \sim \left(\frac{t_{eq}}{t_h}\right)^{1/2}\left(\frac{\delta\rho}{\rho}\right)_h \sim 30G\mu t_{eq}/l. \tag{2}$$

The loops lose energy by gravitational radiation and disappear at $t' \sim t_h/G\mu$. If $t' > t_{eq}$, then the loops are still there at $t \sim t_{eq}$. The density constrast $\delta\rho/\rho_B$ stays approximately constant[12] at $t < t_{eq}$, so that at $t \sim t_{eq}$ it is still given by Eq. (2). After decoupling, baryons pick up the perturbations due to the loops, and for $t > t_{eq}$ the fluctuations are given by

$$\frac{\delta\rho}{\rho} \sim 30G\mu\left(\frac{M}{M_{eq}}\right)^{-1/3}\left(\frac{1+Z_{eq}}{1+Z}\right), \tag{3}$$

where $M = (4\pi/3)\,\rho_{eq}(l/2)^3$ is the baryonic mass associated with scale l and $M_{eq} \sim 5.5\times10^{13}\,(\Omega h^2)^{-2} \times M_\odot$ is the mass within a sphere of diameter t at $t = t_{eq}$. Equation (3) applies for $M_1 < M < M_{eq}$, where $M_1 \sim (G\mu)^{3/2}M_{eq}$.

On smaller scales, $M < M_1$, the loops decay before t_{eq}, when the baryons are still strongly coupled to radiation, and no density fluctuations are produced. (Small-scale adiabatic density fluctuations produced by the strings are erased by Silk damping.[13]) On large scales, $M > M_{eq}$, we have $(\delta\rho/\rho)_{hor} \sim 20\,G\mu$ and

$$\frac{\delta\rho}{\rho} \sim 20G\mu\left(\frac{M}{M_{eq}}\right)^{-2/3}\left(\frac{1+Z_{eq}}{1+Z}\right). \tag{4}$$

Statistical analysis of galaxy distribution suggests[12] that the maximum scale that has gone nonlinear by present is $\sim 8h^{-1}$ Mpc, which corresponds to $M_{nl} \sim 6\times10^{14}\Omega h^{-1}M_\odot$. We shall require that $\delta\rho/\rho \sim 1$ for $M \sim M_{nl}$ at $Z \sim \Omega^{-1}$. (In an open universe with $\Omega < 1$ linear perturbations stop growing at $Z \sim \Omega^{-1}$.) This determines the value of $G\mu \sim 4\times10^{-6}(\Omega h)^{-1}$. The resulting spectrum of fluctuations is shown in Fig. 1[13] for $\Omega = 0.1$, $h = 1$. This spectrum corresponds to the gravitational clustering picture: The fluctuations increase towards smaller scales. The lower cutoff of the spectrum is at $M_1 \sim 10^9 M_\odot$. It gives the mass of the first objects to be formed. Large galaxies ($M \sim 10^{12}M_\odot$) start forming at $Z \sim 40$.

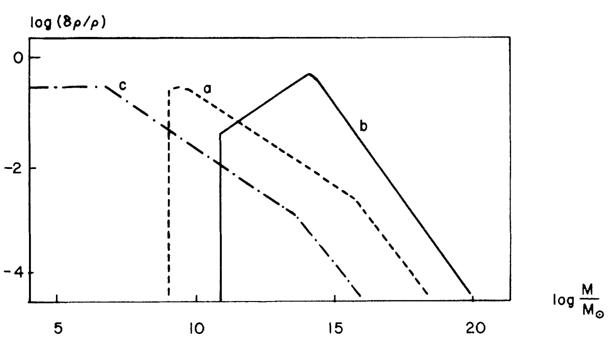

FIG. 1. The spectrum of perturbations in (curve a) baryon-, (curve b) neutrino-, and (curve c) axion-dominated universe. The scale of $\delta\rho/\rho$ is arbitrary.

It has been suggested that the dark matter in the universe can be in the form of massive neutrinos[14] or axions.[15] If at present the universe is neutrino dominated, then $\Omega h^2 = 0.3m_{30}$ and $Z_{eq} = 10^4 m_{30}$, where $m_{30} = m_\nu/(30\text{ eV})$ and we assume for simplicity that only one of the three neutrino species is massive. Neutrinos erase their density perturbations on scales smaller than their Jeans mass, M_J. For $Z < Z_{eq}$, M_J is given by[14]

$$M_J \sim M_{J,eq}[(1+Z)/(1+Z_{eq})]^{3/2}, \tag{5}$$

where $M_{J,eq} \sim 1.3\times10^{15}m_{30}^{-2}M_\odot \sim M_{eq}$. Perturbations on scales $M < M_{eq}$ start growing at $1+Z_M \sim (M/M_{eq})^{2/3}(1+Z_{eq})$, when M_J drops down to M. At $Z < Z_M$,

$$\frac{\delta\rho}{\rho} \sim 30G\mu\left(\frac{M}{M_{eq}}\right)^{-1/3}\frac{1+Z_M}{1+Z} \sim 30G\mu\left(\frac{M}{M_{eq}}\right)^{1/3}\frac{1+Z_{eq}}{1+Z}. \tag{6}$$

Equation (6) does not apply if the loops on scale M decayed at $Z > Z_M$. This condition gives a lower cutoff of the spectrum at $M_c \sim (G\mu)^{3/5}M_{eq}$. For $M > M_{eq}$, $\delta\rho/\rho$ is given by Eq. (4).

The spectrum we have obtained has a maximum at $M \sim M_{eq}$, which is the first scale to go nonlinear in this model. Requiring that at present $\delta\rho/\rho \sim 1$ for $M \sim M_{nl}$, we obtain $G\mu \sim 4.4\times10^{-6}$. (Note that Ω and h must satisfy the inequality $\Omega h \gtrsim 0.6$, which follows from $M_{nl} \gtrsim M_{eq}$.) Figure 1 shows the spectrum of density fluctuations for $\Omega = h = 1$, which corresponds to $m_\nu \sim 100$ eV. In this case $M_{eq} \sim 10^{14}M_\odot$ and $M_c \sim 6\times10^{10}M_\odot$. The mass scale M_{eq} collapsed at $Z \sim 3$. According to Zel'dovich[16] this led to pancake formation. In our model (unlike the standard pancake scenario) the pancake fragmentation is helped along by the presence of perturbations on scales $M_c < M < M_{eq}$. Note that M_c has the order of magnitude of a typical galactic mass.

The analysis of the axion-dominated universe is similar to the baryon-dominated case. The only differences are that (i) axions are not coupled to radiation, and perturbations in axions start growing at $t \sim t_{eq}$, and (ii) there is no Silk damping for axions, and so the small-scale density fluctuations are not erased.[17] On scales $M > M_1 = (G\mu)^{3/2} M_{eq}$ the spectrum is given by Eqs. (3) and (4). For $M < M_1$ the loops disappear before t_{eq}, when the universe is still radiation dominated. It can be shown that on such scales (for $Z < Z_{eq}$)

$$\delta\rho/\rho \sim 30(G\mu)^{1/2}(1+Z_{eq})/(1+Z). \qquad (7)$$

Normalizing the spectrum at $M \sim M_{nl}$, we obtain $G\mu \sim 1\times 10^{-5}$ for $\Omega h > 0.45$ and $G\mu \sim 4\times 10^{-6} \times (\Omega h)^{-1}$ for $\Omega h < 0.45$. Figure 1 shows the spectrum of perturbations for $\Omega = h = 1$. Like in the baryon-dominated case, this spectrum corresponds to the gravitational clustering picture. Baryons pick up the axion density fluctuations at $t > t_{dec}$.

Examples of grand unified theories which lead to strings with the desired superheavy mass scales are readily constructed. Consider the case of the baryon-dominated universe. One requires $G\mu \sim 4\times 10^{-6}(\Omega h)^{-1}$, which corresponds to a superheavy scale $\eta \sim (2-3)\times 10^{16}(\Omega h)^{-1/2}$ GeV [or $\eta \sim (3-4)\times 10^{14}(\Omega h)^{-1/2}$ GeV] if strings arise as a consequence of spontaneous breaking of a local (or global) symmetry. As an example of the former consider the following breaking pattern of SO(10):

$$\mathrm{SO}(10) \xrightarrow[M_G]{} \mathrm{SU}(5)\otimes Z_2 \xrightarrow[M_x]{} \mathrm{SU}(3)\otimes \mathrm{SU}(2)\otimes \mathrm{U}(1)\otimes Z_2 \xrightarrow[M_w]{} \mathrm{SU}(3)\otimes \mathrm{U}(1)_{em}\otimes Z_2.$$

Provided the first breaking is done with a $\underline{126}$ of Higgs fields, topologically stable Z_2 strings are produced.[18] The mass per unit length of the strings is characterized by $M_G[>M_x \sim (3-4)\times 10^{14}$ GeV]. The value of M_G can be adjusted to lie between 10^{16} and 10^{17} GeV, as required.

For an example of a grand unified theory (GUT) model that leads to global strings, consider the minimal SU(5) model which possesses, in addition, global $B-L$ invariance. The spontaneous breaking of this global symmetry at scale M_x leads to the desired topologically stable strings.[4]

A variant of the preceding examples can be employed to implement the scenario in which neutrinos dominate the energy density of the universe. We tacitly assumed above that the spontaneous breaking of $B-L$ invariance [which is a local symmetry in the SO(10) case] leads to neutrino masses much smaller than an electronvolt, in which case baryons comprise the dominant component of the universe. In the SO(10) example, for instance, this is indeed the case if the Yukawa couplings of the $\underline{16}$-dimensional fermions to the $\underline{126}$ of Higgs are of order 1. The neutrinos then acquire masses $m_{\nu_i} \sim M_w^2/M_G \sim 10^{-3}-10^{-4}$ eV. However, if some of the Yukawa couplings are chosen to be much less than 1 (say $\sim 10^{-5}-10^{-4}$), there will be some neutrinos which acquire masses in the 10–100-eV range. Thus, SO(10) models can be constructed which lead to Z_2 strings characterized by scales $\sim 3\times 10^{16}$ GeV, with neutrinos dominating the mass density of the universe. Analogous arguments can be made for the SU(5) $\otimes \mathrm{U}(1)_{B-L}$ model, provided SU(5) singlet fermions are also introduced.[19]

Finally, consider the case where axions dominate the mass density of the universe. Two new scales are needed here. One of them characterizes the mass scale of the strings and is larger than M_x, the SU(5) unification scale. The second scale characterizes the symmetry-breaking scale of the Peccei-Quinn $\mathrm{U}(1)_{PQ}$ symmetry and is of order[15] $10^{11}-10^{12}$ GeV. It turns out that the simplest GUT model where this scenario can be implemented is based on the gauge group E_6.[20] Consider the following breaking of E_6: $E_6 \to \mathrm{SO}(10) \otimes (Z_8/Z_4)$, where Z_4 is the center of SO(10). This breaking can be achieved with a $\underline{351'}$ of Higgs field and leads to topologically stable $Z_8/Z_4 \sim Z_2$ strings,[21] with mass scale characterized by $M_u \sim (3-4)\times 10^{16}$ GeV. The complete symmetry-breaking pattern [with SO(10) breaking via SU(4) $\otimes$SU(2)$\otimes$U(1)] requires additional Higgs fields belonging to the $\underline{27}$ and $\underline{78}$ representations of E_6. We note in passing that baryon- and neutrino-dominated scenarios can also be implemented in models based on the gauge group E_6.

To conclude, topologically stable strings with mass per unit length characterized by a superheavy mass scale appear in many realistic GUT models. Their presence in the very early universe leads to significant density fluctuations. Realistic scenarios for galaxy formation appear possible with a baryon-, a neutrino-, or an axion-dominated universe.

In the standard model of galaxy formation with a primordial spectrum of adiabatic fluctuations, all density perturbations are damped below the Silk mass ($M \sim 10^{14} M_\odot$) or neutrino free-streaming mass ($M_\nu \sim 4\times 10^{14} M_\odot$) in the baryon- and neutrino-dominated cases, respectively. An important feature of the string scenario is that it preserves fluctuations on smaller scales. Baryons and neutrinos erase their own fluctuations, but when the Jeans mass drops down to the corresponding scale, they pick up the perturbations produced by surviving loops. The presence of fluctuations on smaller scales resolves some of the difficulties (see, e.g., Primack and Blumenthal[22]) of the baryon- and neutrino-dominated models. In the axion-dominated case, the spectrum of perturbations produced by loops is qualitatively similar to that resulting from open strings (or from primordial scale-invariant fluctuations[22,23]). A discussion of residual temperature fluctuations of the microwave background and further details will be given elsewhere.

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