# Precision Muonic-Atom Measurements of Nuclear Quadrupole Moments and the Sternheimer Effect in Rare-Earth Atoms

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The ground-state quadrupole moments of $^{151}$Eu, $^{153}$Eu, $^{155}$Gd, $^{157}$Gd, $^{159}$Tb, $^{163}$Dy, $^{167}$Er, $^{177}$Hf, $^{179}$Hf, $^{191}$Ir, and $^{193}$Ir were determined with an uncertainty of less than one percent by measuring the quadrupole hyperfine-splitting energies of muonic $M$ x rays. The results are used to determine experimentally Sternheimer shielding factors for the $4f$, $5d$, and $6p$ electronic states of the respective atoms. The deduced shielding factors for the $5d$ electronic states were found to vary considerably among these elements, presumably as a result of configuration mixing.

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Values of nuclear quadrupole moments $Q_I$ deduced from electronic-atom hyperfine measurements are rather uncertain even though measurements of optical hyperfine-splitting energies are very precise and reliable. The reason lies in the difficulty of exactly calculating the electrostatic gradient at the nucleus produced by the multielectron environment. This difficulty is especially pronounced for rare-earth atoms, where the electronic valence states involve a considerable amount of configuration mixing. In general, the nonspherical shape of the nuclear quadrupole field causes a nonspherical distribution of the core electrons (the Sternheimer effect), which in turn affects the electronic valence states. For an atom with valence electrons in a pure state with quantum numbers $nl$, the influence of the deformed core on a nuclear-quadrupole measurement is taken into account by the shielding factor $R(nl)$ introduced by Sternheimer[1,2]:

$$Q_I = Q(nl)/[1 - R(nl)], \tag{1}$$

where $Q(nl)$ is the nuclear quadrupole moment deduced from the hyperfine-splitting energies of the electronic valence state $(nl)$, under the assumption that the electron core is spherical.

In a muonic atom, however, the (single) muon is overwhelmingly responsible for the field gradient at the nucleus, allowing nuclear quadrupole moments to be precisely extracted from measurements of the hyperfine-splitting energies of muonic $3d$ and $4f$ states (i.e., by observation of the muonic $M$ x rays).

We have measured the ground-state quadrupole moments of 11 odd-$A$ nuclei in the region from $Z = 63$ to 77 by observing the hyperfine structure of muonic $M$ x rays. For details of the measurements and methods of analysis, see Tanaka and co-workers.[3,4] (The present analysis is different from that used in Ref. 3. There, the fitted peak energies were compared with the theoretically predicted transition energies, while in the present work the whole hyperfine spectrum was fitted with the computed spectrum. This direct spectrum-fitting method reduces the statistical error from 0.9% to 0.3%.) Figure 1 shows, as an example, the observed spectrum of the $M$ x rays of muonic $^{177}$Hf.

The total errors of the present quadrupole moments, including the uncertainty arising from the model assumed for the radial quadrupole charge distribution within the nuclear volume, are less than 1%. The quadrupole moments are listed in column 3 of Table I. Figure 2 shows the systematic behavior of the quadrupole nuclear deformation for the region between $N = 88$ and 116. The deformation is plotted in terms of the parameter $\beta_2$, which is related to the measured quadrupole moments by the rotational-model relationship. For the even-$A$ nuclei, the deformation parameters $\beta_2$ were deduced from published results of Coulomb-excitation experiments.[10]

 

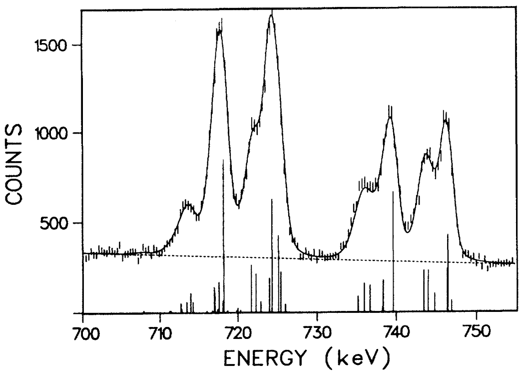


FIG. 1. The muonic $M$ x-ray spectrum of $^{177}$Hf. The solid curve through the data represents the best fit to the observed spectrum which was made by convoluting the theoretically computed hyperfine-splitting pattern indicated by the vertical lines with the known detector line shape. The intensities of the hyperfine components are computed from a cascade calculation that assumes statistical population at the muonic $n=5$ states.

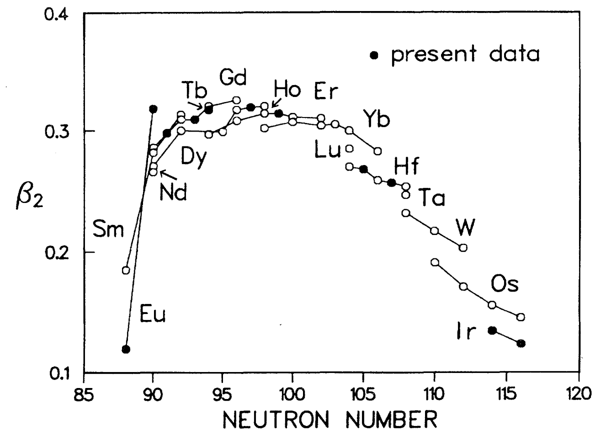


FIG. 2. Systematics of the nuclear deformation parameters $\beta_2$ in the rare-earth region. The data are taken from this work, Refs. 3, 5–9, and Table II of Ref. 10.

Our data for the odd-$A$ nuclei demonstrate that, after the abrupt onset of deformation at $N=90$, the deformation parameter $\beta_2$ varies smoothly with neutron number. Contrary to the rather scattered electronic-atom results, the present results indicate that the deformations of the odd-$A$ nuclei are quite consistent with those of the adjacent even-$A$ neighbors. This fact confirms that the valence single-particle degrees of freedom play only a minor role in nuclear deformations.

Using the electronic-atom values of $Q(nl)$, as given in the 5th column of Table I, together with the muonic quadrupole-moment values, we have computed experimental Sternheimer shielding factors $R(nl)$ from Eq. (1). These are listed in columns 6 to 8 of Table I. By comparing the values of $R(nl)$ among themselves and with the theoretical calculations, the following conclusions can be drawn. (1) The shielding factors have the same value for isotopes of the same element, even though the nuclear quadrupole moments may be quite different (by almost a factor of 3 in the case of the europium isotopes). (2) The $R(5d)$ shielding factors vary irregularly from $-0.16(13)$ to $-0.75(17)$, in serious disagreement with the theoretical estimate of $R(5d) = -0.25(5)$.[11] The $R(5d)$ values for Gd, Tb, and Ir agree reasonably well with the theoretical estimate. (3) The $R(4f)$ shielding factors for Tb, Dy, and Er, and the $R(6p)$ shielding factors for Eu and Gd are close to the theoretical values of $R(4f) = +0.10(5)$,[2] and $R(6p) = -0.18(5)$,[11] respectively. The $R(6p)$ value for Dy, however, is several times smaller than the theoretical value. It is interesting that the experimental $R(6p)$ values for the neighboring elements Eu and Gd are, within limits of error, the same, whereas their $R(5d)$ values differ by a factor of 3.

It is well known that the so-called $5d$ electronic valence states in rare-earth atoms contain a considerable amount of configuration mixing. Thus it is not surprising that the shielding factor for the pure $5d$ electronic states is not adequate to describe the effects of core polarization on an electronic valence state whose $5d$ amplitude is considerably less than unity. Clearly, the theory of the Sternheimer shielding effect in the $5d$ electronic valence states of rare-earth atoms must be reexamined.

Even though our understanding of the electronic electrostatic gradients at the nucleus in rare-earth atoms is far from complete, it seems that whatever effects contribute to the shielding phenomena, the same shielding factor is valid for all isotopes of a particular element regardless of the value of the nuclear quadrupole moment. Hence electronic-atom hyperfine measurements, which require only small amounts of target material, are well suited for determining accurate *ratios* of ground-state nuclear quadrupole moments of different isotopes of the same element. Furthermore, if the appropriate muonic-atom data are available for empirical calibration of the Sternheimer effect, the electronic-atom measurements can furnish absolute measurements of the quadrupole moments of otherwise inaccessible nuclei.

TABLE I. Measured ground-state quadrupole moments of deformed nuclei and Sternheimer shielding factors for electronic levels.

| Nucleus | $I^\pi$ | Present work $Q_I$(eb) | Electronic-atom experiment ($n\ell$) | $Q(n\ell)$(eb) | Shielding factor R(5d) | R(4f) | R(6p) | Reference |
|---|---|---|---|---|---|---|---|---|
| $^{151}$Eu | $5/2^+$ | 0.903(10) | 5d | 1.53(5) | -0.69(6) | | | a) |
| | | | 6p | 1.12(7) | | | -0.24(8) | b) |
| $^{153}$Eu | $5/2^+$ | 2.412(21) | 5d | 3.92(12) | -0.63(5) | | | a) |
| | | | 6p | 2.85(18) | | | -0.18(8) | b) |
| $^{155}$Gd | $3/2^-$ | 1.30(2)[c)] | 5d | 1.59(16) | -0.22(12) | | | d) |
| $^{157}$Gd | $3/2^-$ | 1.36(2)[c)] | 5d | 1.63(10) | -0.20(8) | | | e) |
| | | | 6p | 1.63(13) | | | -0.20(10) | e) |
| $^{159}$Tb | $3/2^+$ | 1.432(8) | 5d | 1.66(18) | -0.16(13) | | | f) |
| | | | 4f | 1.26(8) | | 0.12(6) | | f) |
| $^{163}$Dy | $5/2^-$ | 2.648(21) | 4f | 2.26(23) | | 0.15(9) | | g) |
| | | | 6p | 2.73(5) | | | -0.03(2) | h) |
| $^{167}$Er | $7/2^+$ | 3.565(29) | 4f | 2.827(12) | | 0.207(7) | | i) |
| $^{177}$Hf | $7/2^-$ | 3.365(29) | 5d | 5.85(65) | -0.74(19) | | | j) |
| $^{179}$Hf | $9/2^+$ | 3.793(33) | 5d | 6.63(65) | -0.75(17) | | | j) |
| $^{191}$Ir | $3/2^+$ | 0.816(9) | 5d | 1.10(23) | -0.35(28) | | | k) |
| $^{193}$Ir | $3/2^+$ | 0.751(9) | 5d | 0.99(21) | -0.32(28) | | | k) |

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