Supersymmetric Black Holes in $N=2$ Supergravity Theory

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An exact, asymptotically flat, stationary solution of the field equations of O(2) extended supergravity theory is presented. This solution has a mass, central electric charge as well as a supercharge, and constitutes the first exact, supersymmetric generalization of the black-hole geometries. The solution generalizes the extreme Reissner-Nordström black holes.

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A remarkable property of the extreme Reissner-Nordström black holes is their quantum-mechanical stability. These black holes have zero Hawking temperature, saturate the Bogomolny bound for general relativity, and play the role of the gravitational solitons.[1,2] When one searches for possible generalizations of black holes in the framework of O(2) extended supergravity theory, one finds that it is precisely these stable black holes which are distinguished by the local supersymmetry. This can be seen by linearizing the equations of O(2) supergravity with respect to the spin-$\frac{3}{2}$ fields and employing the resulting scheme to examine the spin-$\frac{3}{2}$ perturbations of the Kerr-Newman black holes. In this approximation the supersymmetry invariance manifests itself as a gauge freedom associated with the perturbations and dictates that Kerr-Newman black holes cannot support regular, stationary, nongauge spin-$\frac{3}{2}$ fields unless the Bogomolny bound is saturated.[3] Moreover, when the Bogomolny bound is attained, supersymmetry allows only a particular multipole of the spin-$\frac{3}{2}$ fields to possess these three qualities simultaneously. Among the static, nongauge spin-$\frac{3}{2}$ perturbations of the extreme Reissner-Nordström black holes there is a unique mode which is regular on and outside the future horizon and which dies off at spatial infinity. This mode is the $l=\frac{1}{2}$ multipole of the spin-$\frac{3}{2}$ fields and carries the conserved supercharge as a linearized "superhair" for the black holes.[4]

It is known that in classical relativity a spin-s, zero-rest-mass field can lead to new black-hole parameters only through its $l<s$ multipoles.[5] The above results are in complete agreement with this observation and indicate that the supersymmetry can extend the notion of a black hole in a well-defined manner to the quantum domain. This expectation is based on the fact that, at the linearized level, the supersymmetry singles out not only the expected multipole of the spin-$\frac{3}{2}$ fields but also the quantum-mechanically stable backgrounds. Therefore, it is of considerable interest to examine how the linearized results are generalized at the level of the exact solutions of the O(2) supergravity theory. The number of relevant exact solutions is clearly very limited. If these solutions can be found, they will constitute the only exact, fully supersymmetric generalizations of the black-hole geometries and may offer new insights into the nonperturbative structure of the O(2) supergravity theory.

In this Letter we shall present such an exact solution. This is an asymptotically flat, stationary solution of the field equations of O(2) supergravity which generalizes the extreme Reissner-Nordström black holes. The solution has a mass, and central electric charge as well as a supercharge. When one sets the supercharge parameter to zero in the solution, one switches off the spin-$\frac{3}{2}$ fields and the extreme Reissner-Nordström geometry results. On the other hand, when one retains the linear terms in the supercharge, one obtains the $l=\frac{1}{2}$ multipole of the spin-$\frac{3}{2}$ fields on the extreme Reissner-Nordström background.

Consider the O(2) extended supergravity theory in the form where the internal O(2) symmetry is not gauged and where all the auxiliary fields are set equal to zero.[6] Then, in the second-order formalism, the field variables appearing in the O(2) supergravity action may be taken to be the orthonormal tetrad one-forms[7] V^a, the graviphoton potential one-form A, and an O(2) doublet of Majorana spinor-valued one-forms ψ^j. Among these fields the fermionic variables ψ^j must be treated as odd elements of a Grassmann algebra[8] and because of the supersymmetry invariance this implies that the bosonic fields V^a and A are even elements of the same algebra. The fields V^a, A, and ψ^j which extremize the O(2) supergravity action are obtained in the usual manner by a variational principle and are governed by a set of nonlinear field equations. In order to write down these equations it is convenient to introduce first the two-forms $2K_1=\epsilon^{jk}\bar\psi^{j}\wedge\psi^k$ and $2K_2=-i\epsilon^{jk}\bar\psi^{j}\wedge\gamma_5\psi^k$, the supercovariant field strength $\hat F=dA+K_1$, and the supercovariant derivative,[9]

$$\hat D\psi^j = D\psi^j - (i/2)\epsilon^{jk}(\hat F_{ab}\sigma^{ab})\gamma\wedge\psi^k. \tag{1}$$

Here D is the covariant derivative,

$$D\psi^j = d\psi^j + \tfrac12\omega_{ab}\sigma^{ab}\wedge\psi^j, \tag{2}$$

involving the exterior derivative d, the torsion-free connection one-forms $\omega_{ab}(V^c)$, and the contorsion K_{ab}:

$$\omega_{ab}=\omega_{ab}(V^c)+K_{ab}\,;\quad K_{ca}\wedge V^c=\frac{i}{2}\bar\psi^{j}\wedge\gamma_a\psi^j. \tag{3}$$

Defining further the space-time curvature two-forms $\Omega_{ab}=d\omega_{ab}+\omega_{ac}\wedge\omega_b{}^c$ and denoting the Hodge dual by an asterisk, the field equations of the O(2) extended supergravity take the form

$$\tfrac12\epsilon_{abcd}\Omega^{ab}\wedge V^d=2T_{ca}(\hat F)*V^a+\bar\psi^{j}\wedge\gamma_5\gamma_c D\psi^j, \tag{4}$$

$$d(*\hat F-K_2)=0, \tag{5}$$

$$\gamma_5\gamma\wedge\hat D\psi^j=0, \tag{6}$$

where

$$T_{ab}(\hat F)=-\hat F_{ca}\hat F_b{}^c+\tfrac14\eta_{ab}\hat F_{cd}\hat F^{cd}. \tag{7}$$

The solution to these field equations that we wish to report is obtained with the aid of a Newman-Penrose-Debever[10,11] formalism which takes into account the torsion generated by ψ^j. Therefore, we choose the Weyl representation for the Dirac matrices and take, instead of the orthonormal tetrad fields V^a, the null-tetrad-basis one-forms $(l,n,m,\bar m)$ as the gravitational field variables. These basis one-forms may be related to V^a by

$$V^a\sigma_a=\sqrt2\begin{pmatrix} n & -\bar m\\ -m & l\end{pmatrix}, \tag{8}$$

where the spatial components of σ_a are the Pauli spin matrices and σ_0 is the two-dimensional identity matrix. We denote the entries of the Majorana fields ψ^j as

$$\psi^j=\begin{pmatrix} G_1{}^j\\ G_2{}^j\\ \bar G_2{}^j\\ -\bar G_1{}^j\end{pmatrix} \tag{9}$$

and work in a coordinate chart $(v,r,\zeta,\bar\zeta)$, where v is the advanced time, r is the usual radial, and ζ is the complex stereographic coordinate. In this coordinate chart we have verified that the null tetrad

$$l=[\Delta/2r^2+(3M^2/r^4)S\bar S]dv-dr, \tag{10a}$$

$$n=dv, \tag{10b}$$

$$m=(r/\sqrt2 P)d\bar\zeta+(2M/r^2)S\,dv, \tag{10c}$$

the graviphoton potential one-form

$$A=-(M/r)dv, \tag{11}$$

and the spin-$\frac32$ fields ψ^j, whose one-form entries are

$$G_1{}^j=(iM/\sqrt2 r^2)\epsilon^{jk}\bar S^k n, \tag{12a}$$

$$G_2{}^j=\left[\frac{M(r+M)}{2r^3}-\frac{i9M^2}{\sqrt2 r^4}S^k\bar S^k\right]S^j n+\frac{i\sqrt2M}{r^2}\epsilon^{jk}\bar S^k m, \tag{12b}$$

constitute an exact solution to Eqs. (4)–(6). Here we are using the abbreviations $\Delta=(r-M)^2$, $2P=1+\zeta\bar\zeta$, and $S=\epsilon^{jk}S^jS^k$, and

$$S^j=P^{-1/2}(c^j-i\epsilon^{jk}\bar c^k\bar\zeta) \tag{13}$$

is a particular combination of the spin-$\frac12$ spherical harmonics.[4] Moreover, M and c^j are the only free parameters appearing in the solution; M is a positive real constant and c^j are odd elements of a finite-dimensional Grassmann algebra:

$$c^jc^k+c^kc^j=0,\quad c^j\bar c^k+\bar c^kc^j=0. \tag{14}$$

By an appeal to the relevant surface integrals[12] it can be shown that M is the mass of the solution and that c^j together with $\bar c^j$ constitute the entries of the spinor supercharge. The solution also has a central electric charge e which satisfies $e=M$. [Because of the invariance of Eqs. (4)–(6) under the duality-chiral transformations,[13] the inclusion of a central magnetic charge is straightforward.] From Eqs. (10)–(12) it can easily be inferred that the terms which are nonlinear in c^j all die off very rapidly at spatial infinity and do not contribute to the surface integrals. In particular, at spatial infinity the total supercharge of the solution is anchored only by the linearized parts of the spin-$\frac32$ fields. Hence a simple way to verify the nature of the constants is to linearize the solution with respect to the parameters c^j. Then Eqs. (10) and (11) reduce to the extreme Reissner-Nordström solution and Eq. (12) reduces to the $l=\frac12$ multipole of the spin-$\frac32$ fields in the supergauge of Güven.[14]

Certain obvious properties of the solution can be exhibited by noting that any Grassmann-algebra–valued field can be decomposed into a "body" which takes values in the field of real or complex numbers and a "soul" which is nilpotent.[15] Consider for example the space-time metric

$$g = l \otimes n + n \otimes l - m \otimes \bar{m} - \bar{m} \otimes m, \qquad (15)$$

corresponding to the solution (10)–(12). This metric can be written as

$$g = g_B - g_S, \qquad (16)$$

where the body g_B is the usual Reissner-Nordström metric and the soul g_S is

$$g_S = \frac{2M^2}{r^4} S\bar{S} dv^2 + \frac{2\sqrt{2}M}{rP} dv\,(S d\zeta + \bar{S} d\bar{\zeta}). \qquad (17)$$

The body g_B is, of course, static and spherically symmetric. The full metric g itself is only stationary. This is quite understandable because, the fermionic fields ψ^j—which cannot be axially symmetric—generate, in addition to the space-time torsion, a nonzero twist for the timelike Killing vector $K^\mu = \delta_v{}^\mu$.

How can this solution be interpreted as describing a black hole? Here one may take the viewpoint that the body of a Grassmann-valued field can always be given an operational interpretation and use g_B to identify the black hole. In this approach the horizon of the black hole will be located at $r = M$ which is a null hypersurface with respect to g_B. This horizon is a degenerate horizon and intersects the v = const hypersurfaces on a two-sphere of surface area $4\pi M^2$. On this two-sphere the soul of the metric vanishes and ψ^j have purely tangential components (see Fig. 1). Within this interpretation one may evaluate the supercharge of the solution with the aid of the supercovariantly constant spinors[2] which are admitted by g_B. Using the supercovariantly constant spinors in the manner described by Aichelburg and Güven[16] one obtains that the total supercharge Q, calculated at spatial infinity, is

$$Q = Q_H + Q_{\text{ext}}, \qquad (18)$$

where Q_H is the supercharge evaluated on the two-sphere located at $r = M$. Q_{ext} is the supercharge calculated on a spacelike hypersurface which starts at the horizon and ends at spatial infinity. Then one finds that, whereas Q_H and Q_{ext} may be mixed by an infinitesimal supersymmetry transformation, the total supercharge Q is a supergauge-invariant quantity. This property follows from the sharp falloff of the fields at spatial infinity.

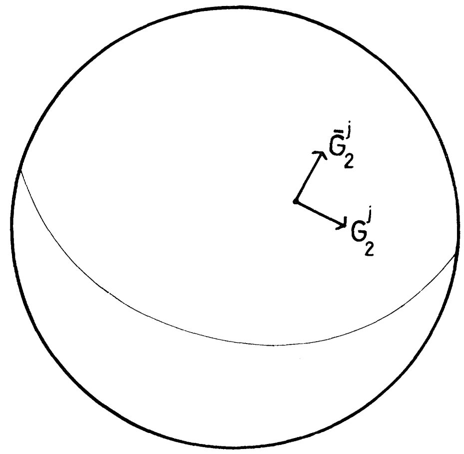

FIG. 1. In the solution the intersection of the horizon with the v = const surfaces is a two-sphere. On this two-sphere spin-$\frac{3}{2}$ fields have purely tangential components.

Within the above framework one may calculate the value of the $N = 2$ supergravity action for the solution by specifying a region between two Cauchy hypersurfaces of g_B which extend from the internal infinity B to the spatial infinity i^0. (For a discussion of these boundaries, see Hajicek[17].) Before integrating the Lagrangian over such a region, it is necessary to consider the surface terms[18] which must be added to the action in order to have well-defined variations at the boundaries. When these are taken into account one finds that all the surface terms at B vanish for Eqs. (10)–(12) and that the only nonzero contribution of the solution to the action comes from the surface terms at i^0:

$$I = \tfrac{1}{8}\int_{i^0} \epsilon_{abcd}(\omega^{ab} - \omega_F{}^{ab}) \wedge V^d \wedge V^c - \tfrac{1}{2}\int_{i^0} (*\hat{F} - K_2) \wedge A, \qquad (19)$$

where ω^{ab} is the full connection, $\omega_F{}^{ab}$ is the torsion-free flat connection and, following Ref. 18, the graviphoton gauge is chosen so that $A = 0$ at B. Moreover, one finds that Eq. (19) yields for (10)–(12) precisely the value of the action for the extreme Reissner-Nordström solution. Therefore, with respect to the above boundaries, the lowest-order contributions of the two solutions are the same in the stationary-phase approximation to the Feynman path integral. It will be interesting to compare the one-loop quantum corrections to these two solutions in the framework of $N = 2$ supergravity.

Let us finally note a more challenging approach to the solution which is linked to the anticommutativity of c^j and which points towards a quantum interpretation. A curious consequence of the anticommutativity of c^j is that the norm of the timelike Killing vector K^μ for the full metric g is a purely radial, Grassmann-valued function. If one could interpret this function in terms of the expectation values of the quantum operators in a yet undefined sense and treat the anticommuting constants as ordinary numbers after obtaining the solution, *then the full metric g could be used to describe a black hole.* As can easily be inferred from Eqs. (16) and (17), with this assumption the norm of K^μ would vanish on a *nondegenerate horizon.* This horizon would still have the topology of a two-sphere (see Fig. 1) but the theorem, implying that the stationary black holes must be axially symmetric,[19] would presumably be transcended. At the present state of quantum gravity, a better understanding of this interesting possibility awaits the determination of the back reaction of the spin-$\frac{3}{2}$ fields which are quantized on an extreme Reissner-Nordström background.

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[1]P. Hajicek, Nucl. Phys. B185, 254 (1981).

[2]G. W. Gibbons, in *Unified Theories of Elementary Particles*, edited by P. Breitenlohner and H. P. Dürr, Lecture Notes in Physics Vol. 160 (Springer, New York, 1982), p. 145; G. W. Gibbons and C. M. Hull, Phys. Lett. 109B, 190 (1982).

[3]P. C. Aichelburg and R. Güven, Phys. Rev. D 24, 2066 (1981).

[4]R. Güven, Phys. Rev. D 25, 3117 (1982).

[5]C. Misner, K. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), p. 866.

[6]S. Ferrara and P. van Nieuwenhuizen, Phys. Rev. Lett. 37, 1669 (1976).

[7]Our conventions are as follows: The letters $a, b, \ldots$ are reserved for the orthonormal tetrad indices and take the values 0, 1, 2, 3. The tangent space metric is $\eta_{ab} = \mathrm{diag}\,(1, -1, -1, -1)$. The space-time orientation is specified by the choice $\epsilon_{0123} = 1$, where ϵ_{abcd} is the four-dimensional completely antisymmetric tensor. The units are chosen so that $4\pi G c^{-4} = 1$, where G is Netwon's constant and c is the speed of light. The O(2) group indices are denoted by j and k and take the values 1, 2. The symbol ϵ^{jk} is the completely antisymmetric tensor with two indices and $\epsilon^{12} = 1$. Throughout the text flat-space Dirac matrices γ_a are utilized in the form $\gamma = \gamma_a V^a$ and $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$, $\gamma_5 = (i/4!) \times \epsilon_{abcd}\gamma^a\gamma^b\gamma^c\gamma^d$. The wedge $\wedge$ denotes the exterior product, $\otimes$ is the tensor product, and a bar over a quantity signifies the adjoint operation in the Grassman algebra (see F. A. Berezin, *The Method of Second Quantization* (Academic, New York, 1966), p. 49. For the usual complex-valued functions, the adjoint operation reduces to complex conjugation.

[8]Berezin, Ref. 7.

[9]E. Cremmer and J. Scherk, Nucl. Phys. B127, 259 (1977).

[10]E. Newman and R. Penrose, J. Math. Phys. 3, 566 (1962).

[11]R. Debever, Bull. Cl. Sci. Acad. Roy. Belg. LXII, 662 (1976).

[12]C. Teitelboim, Phys. Lett. 69B, 240 (1977).

[13]S. Ferrara, J. Scherk, and B. Zumino, Nucl. Phys. B121, 393 (1977).

[14]R. Güven, in Proceedings of the Eighth Ettore Majorana School of Cosmology and Gravitation, edited by V. de Sabbata and E. Schmutzer (to be published). See also P. C. Aichelburg and R. Güven, Phys. Rev. D 27, 456 (1983).

[15]B. S. DeWitt, P. van Nieuwenhuizen, and P. West, "Supermanifolds and Supersymmetry" (to be published).

[16]Aichelburg and Güven, Ref. 14.

[17]P. Hajicek, Phys. Rev. D 26, 3384 (1982).

[18]G. W. Gibbons and S. W. Hawking, Phys. Rev. D 15, 2752 (1977).

[19]S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge Univ. Press, Cambridge, 1973), p. 323.