Magnetohydrodynamic Effects of a First-Order Cosmological Phase Transition

Craig J. Hogan
Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125
(Received 8 June 1983)

The author analyzes the spectrum, amplitude, and evolution of a tangled primeval magnetic field generated by macroscopic dynamo activity arising during a first-order phase transition in the early universe. The effects of the field on the matter distribution at late times could be much larger than that of the adiabatic or isothermal density fluctuations generated by the transition. With optimistic assumptions, magnetic fields generated during a transition at $T \sim 0.2$ GeV could lead to the collapse of gas clouds of $\simeq 10^{1\text{-}5} M_\odot$ after recombination, and to the formation of pregalactic stars or quasars.

PACS numbers: 98.80.Bp, 52.30.+r, 98.50.Eb

Recent work in the theory of fundamental interactions has shown that hot expanding matter probably undergoes various instabilities during one or more first-order phase transitions at temperatures of order $kT \simeq 0.2-100$ GeV.[1] In a cosmological context these instabilities lead to macroscopic structure, and it is tempting to speculate that they are ultimately responsible for the large-scale inhomogeneity of matter that we observe. Thus it is interesting to explore the various ways in which these instabilities could affect the motion and distribution of matter on larger scales than the domains formed in the transition (which will be called "bubbles"), and at temperatures much lower than the transition temperature. Here I estimate the possible effect of a tangled macroscopic magnetic field which may be generated during such a transition. I find that in principle these magnetic fields could have nonnegligible consequences at late times; with optimistic assumptions, they could trigger the gravitational collapse of matter after recombination.

The role of primordial magnetic fields in cosmology has been studied previously, particularly in relation to the origin of galactic spin and magnetic field, and in relation to the formation of galaxies themselves.[2] In most of this work, the existence of the primordial field on galactic scales was either postulated *ad hoc* or attributed to large-scale primordial helical perturbations or turbulence, which are themselves implausible for other reasons.[3] Here, I calculate the properties the field would have if it were produced by an astrophysical dynamo powered by the ordered release of free energy during a phase transition at high temperature. Since the dynamos which generate and amplify astrophysical magnetic fields are not well understood even in more accessible environments, this paper deals primarily with the separable problem of large-scale properties of the field and its behavior at late times. The statistical spatial distribution and amplitude of the field on scales much larger than the maximum bubble size is derived from scaling and equipartition arguments without reference to the precise dynamo mechanism which creates the field. A simplified description is then given of the evolution of the field in the standard "big band" model up to recombination and its effect on the matter distribution. As it turns out the residual field at the present epoch is insignificant on galactic scales compared with the observed fields. Nevertheless, the field at recombination may be strong enough to have significant dynamical effects. The field stores some of the free energy released by the transition and converts it later into ordered macroscopic motion on very much larger scales than the original bubbles, with an amplitude which may be sufficient to trigger the formation of population III stars or other "seeds" for galaxy formation.[4]

Several different types of dynamo mechanism[5] could operate during the period when the kinetic energy of bubble walls is being dissipated. The most direct way to channel energy into the field would be to exploit macroscopically correlated forces which act differently on positive and negative charge carriers in an environment where the resulting motion of the current-carrying particles serves to amplify an existing field. (A "seed" field is probably provided because bubbles nucleate on a microscopic scale.) A laboratory precedent for this type of field amplification occurs in laser-irradiated plasmas,[6] and a similar process may occur in the crusts of neutron stars at the interface between two phases associated with a large heat flux.[7] An analogous thermoelectric effect could occur in cosmological bubble walls separating two phases, in which the steep gradients in radiation temperature and Higgs field expectation value (or other order parameter) could power currents in electrons and quarks (and possibly charged bosons) to amplify an existing seed field. In what follows we simply assume that bubbles act as independent dynamos.

The maximum amount of power which can be injected initially into large-l magnetic field structure may be estimated as follows. For simplicity suppose that each bubble's magnetic field lines form loops confined to the bubble walls. When bubble walls collide the fields from each bubble are "stitched" to those of its neighbors by local magnetic reconnection. Because the dynamo activity is uncorrelated on scales larger than the bubble radius l_b, the loop structure in different bubbles is uncorrelated, and an individual field line, instead of turning back in a closed loop on the scale l_b, soon loses any memory of where it started and executes an infinite Brownian walk in space with a step length l_b. These lines traverse regions of space even on scales where no causal connection has yet occurred. Define B_l to be the flux of B field lines remaining after tangles below a scale l are smoothed out by viewing the system with spatial resolution l; therefore B_l^2 is the kinetic energy density ultimately available from straightening tangles on scale l. To estimate the flux B_l we need to calculate the rms net flux through "smeared" surfaces of radius l and effective half-widths of order l (it is necessary to use such smeared window functions because sharp surfaces would sample high-wave-number, small-l tangles.) Because the orientation of the lines is random the rms net magnetic flux through such a fuzzy surface is $B_l \simeq \sqrt{\eta}/l^2$, where η is the total number of lines which traverse the surface in any direction (as seen with resolution l). If the lines were smooth on scale l or if the surfaces were sharp we would have $\eta \propto l^2$, but for Brownian walks the total effective length of each of the lines decreases with coarser resolution, like l^{-1}, so that the total number intersecting a fuzzy surface must only increase like $\eta \propto l$. (As the surface gets less fuzzy we pick up more flux because some of the lines which turn back on themselves get resolved.) Therefore this model leads to a spectrum $B_l \propto l^{-3/2}$.

It is interesting that this acausal propagation is capable of producing large-scale fields as strong as the causal superposition of dipole fields in a vacuum. An easy way to derive this scaling is to imagine space filled with randomly placed, randomly oriented current loops of radius l_b and local field strength B_{l_b}; the magnetic dipole moments of these loops add linearly and so the mean field strength at a distance l due to $N=(l/l_b)^3$ randomly oriented loops is about

$$B_l \cong \sqrt{N}\times B_{l_b}(l_b/l)^3 \cong B_{l_b}(l/l_b)^{-3/2}, \tag{1}$$

where $B_{l_b}(l_b/l)^3$ is the field strength contributed by each loop at a distance l. It would be misleading to derive the scaling this way for the cosmological model because field lines do not propagate freely through the plasma; however, this example is instructive because it demonstrates that one does not need to generate currents with correlations extending to infinity in order to create a field of infinite random walks. Note that the available kinetic energy density $B_l^2 \propto l^{-3}$ and is not the same as the dispersion in total magnetic energy density between regions of size l, which goes like $N^{-1/2} \propto l^{-3/2}$. Most of this dispersion is contained in small-scale tangles.

The amplitude of B_{l_b}, if it is not entirely negligible, may be estimated from conventional "equipartition" arguments. For the turbulent dynamo, this would suggest that $B_{l_b}^2 \simeq v^2\epsilon$, where $\epsilon \simeq gT^4$ is the total energy density of matter and radiation, $2g$ is the number of effective degrees of freedom, and v is the characteristic velocity of the motion. For a relativistic phase transition, the total free energy available is often of order ϵ, so that v may not be much less than unity—in other words, a significant fraction f of ϵ may be shared with the magnetic field, at least for a short time, on the scale l_b, and we may write $B_{l_b}^2 = f\epsilon$ regardless of the specific dynamo at work. At the time the field is generated we then have $B_l^2=(l/l_b)^{-3}f\epsilon$. Also, the scale of the bubbles l_b is characteristically of the order of the Hubble length scale $t \cong \epsilon^{-1/2}G^{-1/2}$, which we parametrize by writing $l_b=\lambda t$ (for typical transitions,[8] λ lies in the range $10^{-3}<\lambda<10^{-1}$). It is convenient to write in terms of the comoving rest mass M of baryons in a volume l^3:

$$B_M^2=(M/M_b)^{-1}f\epsilon, \tag{2}$$

where

$$M_b \simeq \lambda^3 S^{-1} g^{-1/2} T_{\mathrm{GeV}}^{-3} M_\odot \quad (T \gtrsim 1\ \mathrm{GeV}) \tag{3}$$

($\propto T^{-2}$ for $T \lesssim 1$ GeV) is the rest mass of baryons in a typical bubble, which depends on the entropy per baryon S (observed to be of the order of 10^8–10^{11}).

If the field lines were frozen to the comoving frame, the total flux through a comoving fuzzy surface would remain constant and therefore B^2 would scale like a^{-4}, just as density ϵ does in relativistic matter. Thus B_l^2/ϵ is conserved on scales which have not yet unwound. However, as soon as the forces driving the dynamos have stopped the field starts to unwind and reconnect, from small scales upwards, introducing an ordered force density on matter of order B_l^2/l on each scale. On scales which are optically thin, the radiation may be thought of as a uniform "stiff" background, and the two main forces which oppose the straightening of the field lines are (1) the inertia of the matter being dragged by the field, and (2) the viscosity of the radiation dragging on free electrons. Effect (1) gives us the Alfvén velocity on scale l, $v_A=(B_l^2/\epsilon_m)^{1/2}$, where ϵ_m is the mean matter density. Effect (2) gives the "drag-limited" velocity v_d, obtained by balancing the magnetic force and the viscous drag force $F_d \simeq \epsilon_\gamma \sigma_T v_d n_e/c$, where σ_T is the Thomson cross section and n_e is the electron density: $v_d=(B_l^2/\epsilon_\gamma)(l n_e \sigma_T)^{-1}$. At any given time, the velocity on each scale is given by $v_B \cong \min(v_A, v_d)$. The smallest scale which still has any tangles left at time t is that for which $v_B \cong v_H$, where $v_H = l/ct$ is the Hubble velocity.

After the field has straightened out, it leaves the matter on each scale lumpy, producing plasma oscillations sustained by gas and magnetic pressure which eventually damp out by various dissipative mechanisms. On scales exceeding the Jeans mass, however, lumps are stabilized by their own gravity and will collapse after decoupling from the radiation. For gas at the background radiation temperature the Jeans mass is independent of red shift:

$$M_{\mathrm{Jm}} \simeq (nkT/\epsilon_m)^{3/2}(1+\delta\rho/\rho)^{-1/2}M_H \sim S^{1/2}(1+\delta\rho/\rho)^{-1/2}M_\odot, \tag{4}$$

where $M_H \sim \epsilon_m t^3$ is the mass of gas in a Hubble volume. Fluctuations of order unity will appear on the Jeans scale if $(v_B/v_H)_{\mathrm{Jm}} \simeq 1$. If this is the case then the relation

$$v_A/v_H = (v_d/v_A)(\epsilon_\gamma/\epsilon_m)\tau_H, \tag{5}$$

where $\tau_H = n_e \sigma_T ct$, shows that when the universe becomes optically thin at recombination, we have $v_A < v_d$, and hence $v_B \simeq v_A$. Therefore $\delta\rho/\rho = 1$ if $v_A/v_H = 1$. With use of (2)–(4), collapse on the Jeans scale occurs if

$$(v_A/v_H)_{\mathrm{Jm}} \cong f^{1/2}S^{-1/4}\lambda^{3/2}g^{-1/4}T_{\mathrm{GeV}}^{-3/2} \gtrsim 1 \quad (T>1\ \mathrm{GeV}) \tag{6}$$

($\propto T^{-1}$ for $T \lesssim 1$ GeV). [One may verify that for a field this strong, the streaming velocity $v_{eH} \simeq (B_l^2/l n_e n_H m_p \sigma_{eH})^{1/2}$ between ionized and neutral particles is less than v_A even for the recombined plasma with $n_e/n_H \simeq 10^{-4}$, and so the magnetohydrodynamics approximation is still valid.]

This may be compared with the amplitude of density perturbation caused by entropy fluctuations from the same transition,

$$(\delta\rho/\rho)_{\mathrm{Jm},S} \cong f_S(M_{\mathrm{Jm}}/M_b)^{-1/2} \simeq f_S S^{-3/4}\lambda^{3/2}g^{-1/4}T_{\mathrm{GeV}}^{-3/2}, \tag{7}$$

where $f_S \lesssim 1$ is the fractional entropy perturbation of a single bubble. If the initial fraction of energy shared with the magnetic field $f \gtrsim f_S^2 S^{-1}$, then the magnetic effects of the transition dominate over these "isothermal" fluctuations in matter density at late times.

It is more likely that the nonlinear effects of the field are of predominant importance. Regions where reconnection is occurring would be expected to undergo high compression, with a corresponding reduction in the local Jeans mass[9] to as little as $M_\odot$. The condition for this to occur at the epoch when $\epsilon_\gamma \simeq \epsilon_m$ is weaker than (6),

$$(v_A/v_H)_{1M_\odot} \cong f^{1/2}S^{1/6}\lambda^{3/2}g^{-1/4}T_{\mathrm{GeV}}^{-3/2} \gtrsim 1 \quad (T>1\ \mathrm{GeV}), \tag{8}$$

and so star formation could be triggered even if fields were too weak to cause immediate collapse on the background Jeans scale $S^{1/2}M_\odot$.

If the relevant transition occurs at $\simeq 0.2$ GeV (quark confinement?), and if f and λ are not much less than (say) 0.1, then the density perturbations at recombination due to these magnetic effects would lead directly to the immediate collapse of $\sim 10^{1-5}M_\odot$ clouds and the formation of stars or quasars. As a result, the post-recombination universe, instead of being quite simple and homogeneous, would probably resemble the interstellar medium of our galaxy. Since the magnetically generated fluctuations on larger scales go like $\delta\rho/\rho \propto v_A/v_H \propto M^{-5/6}$, the direct effects of the magnetic fields on galaxy formation would be negligible. It is also easy to show that the remnant fields now are several orders of magnitude weaker than observed galactic fields, and that the expected intergalactic fields would be less than about 10^{-13} G, well below currently detectable levels. (Our fields at recombination are at most $\sim 10^{-4}$ G on $\sim 10^5 M_\odot$ scales, whereas Wasserman[2] and others require $\sim 10^{-3}$ G on galactic scales.) On the other hand there are several ways in which pregalactic sources of energy could lead to the formation of galaxies and clusters,[4] so that the field may have important indirect consequences. The observed large-scale matter distribution could have formed in this way even if negligible fluctuations were introduced in the very early universe. The magnetic field in this case allows a microscopic instability to propagate to large enough scale to trigger an astrophysical one, so that the effect of the transitions are not confined to the epoch and scale where they occur, but lead to a long-lived residual complexity. Although I have not demonstrated that such effects necessarily occur, it is clear that one is not necessarily justified in ignoring them, given the ubiquity of magnetic effects in observed high-energy astrophysical environments.[10]

I am very grateful for conversations with A. Achterberg and R. Blandford. This work was supported by a Bantrell Fellowship at Caltech, and by National Science Foundation Grant No. 4-AST-78-21453.

[1]See, for example, A. D. Linde, Rep. Prog. Phys. 42, 389 (1979); T. W. B. Kibble, Phys. Rep. 9, 183 (1980); S. Weinberg, Phys. Scr. 21, 773 (1980); E. Witten, Nucl. Phys. B177, 477 (1981); S. W. Hawking, I. G. Moss, and J. M. Stewart, Phys. Rev. D 26, 2681 (1982); J. Kogut *et al.*, Phys. Rev. Lett. 50, 393 (1983), and references therein.

[2]E. R. Harrison, Astrophys. Lett. 3, 133 (1968), and Mon. Not. Roy. Astron. Soc. 147, 279 (1970); M. J. Rees and M. Rheinhardt, Astron. Astrophys. 19, 189 (1972); I. Wasserman, Astrophys. J. 224, 337 (1978), and references therein.

[3]P. J. E. Peebles, in *The Formation and Dynamics of Galaxies*, edited by J. Shakeshaft (Reidel, Dordrecht, Holland, 1973).

[4]A. G. Doroshkevich, Ya. B. Zeldovich, and I. D. Novikov, Sov. Astron. 11, 233 (1967) [Astron. Zh. 44, 295 (1967)]; J. P. Ostriker and L. L. Cowie, Astrophys. J. Lett. 243, L127 (1981); S. Ikeuchi, Publ. Astron. Soc. Jpn. 33, 211 (1981); C. J. Hogan, Mon. Not. Roy. Astron. Soc. 202, 1101 (1983); C. J. Hogan and N. Kaiser, Astrophys. J., to be published.

[5]L. Biermann, Z. Naturforsch. 5a, 65 (1950); R. H. Kraichnan, Phys. Fluids 8, 1535 (1965); E. N. Parker, *Cosmical Magnetic Fields* (Clarendon, Oxford, 1979); H. K. Moffatt, *Magnetic Field Generation in Electrically Conducting Fluids* (Cambridge Univ. Press, 1978); R. D. Blandford, Astron. J. 88, 245 (1983).

[6]J. A. Stamper *et al.*, Phys. Rev. Lett. 26, 1012 (1971).

[7]R. D. Blandford, J. H. Applegate, and L. Hernquist, to be published.

[8]C. J. Hogan, to be published.

[9]C. J. Hogan, Mon. Not. Roy. Astron. Soc. 185, 889 (1978).

[10]The above calculation treats only the coarsest effects of the field; that is, its direct effect on the motion and distribution of the bulk of the matter. In reality it could introduce qualitatively new phenomena into the big bang; for example, if a small fraction of particles were accelerated by the field to relativistic energies (as in the acceleration of cosmic rays in the galaxy) they might significantly affect the abundances of the rarer light elements. (The production of helium would be essentially unchanged because neither the expansion rate nor the mean square matter density at nucleosynthesis is significantly affected.) However, such phenomena lie outside the scope of this Letter.