Nearly Traceless $1/f$ Noise in Bismuth

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The conductivity fluctuations giving $1/f$ noise in bismuth were found to be extremely nonscalar, with local instantaneous anisotropy in between that of dyads and of traceless tensors. This result requires a symmetry-breaking noise-generating mechanism. Temperature dependence and detailed spectral data show that the $1/f$ kinetics are thermally activated and that the net noise magnitude in thick films is a decreasing function of temperature. A crude model accounting for these results is presented.

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The mechanism by which $1/f$ noise arises in metal films has not been established,[1-4] although strong evidence has been found that the $1/f$ spectrum arises from thermally activated kinetics with a range of activation enthalpies.[1] Some metal films (gold,[4] chrome,[4] and continuous, nearly single-crystal niobium[5]) have previously been found to have highly nonscalar conductivity fluctuations, but these results were not sufficient to rule out with certainty the possibility that the anisotropies were just due to locally anisotropic environments.[4] In this Letter we present evidence that in bismuth the fluctuations must be intrinsically nearly traceless. Together with confirmation of a previous prediction that the magnitude of noise due to defect motion in metals with significant phonon scattering should be a decreasing function of temperature[2] the results point toward a physical model for the noise origins. Bismuth was chosen because, although it is not a typical metal, it has large, highly reproducible and stable, $1/f$ noise. The generalizability of the results is not yet known.

Bi films about 30 nm thick (and one ~ 100 nm thick) were evaporated on glass slides. A cross-shaped pattern, similar to that used in previous studies of other materials,[4,6] was etched in the films with use of photolithography. An isotropy parameter S, defined by

$$S \equiv 2\langle \mathrm{Det}(\delta\underline{\rho})\rangle / \langle \mathrm{Tr}[(\delta\underline{\rho})^2]\rangle, \quad (1)$$

where $\underline{\rho}$ is the planar projection of the resistivity matrix, was measured by techniques described previously.[4,6-9] Also measured on the same or similar films were some statistical properties of the noise,[10] the detailed noise spectra and temperature dependence, and several symmetry properties including the Onsager relations and some resulting from the highly symmetrical pattern. Conventional battery current sources, amplifiers, and digital signal analysis equipment were used.

The statistical properties of the noise were completely Gaussian[10] and the geometrical symmetries of the patterns were obeyed by the noise magnitudes and cross magnitudes, as measured on several paths. We conclude that the noise originates at many independent sites, uniformly distributed throughout the bulk and/or, with less likelihood, the surfaces of the films.

For improved signal-to-noise, temperature-dependent broadband spectra were measured on a conventional long, narrow 140-nm-thick sample nearly identical in shape, resistivity, temperature coefficient, and noise magnitude to Bi samples previously studied by us.[11]

As shown in Fig. 1, the noise spectra in the thick, long, narrow sample were not perfectly $1/f$. The temperature dependence of the frequency of the maximum noise per octave could be fitted with an Arrhenius law $f = f_0 \exp(E^{\pm}/kT)$ where $f_0 \approx 3\times 10^5$ Hz and $E^{\pm} \approx 0.20 \pm 0.05$ eV. This result is consistent with what has been previously reported for Bi films.[1] However, in order to describe

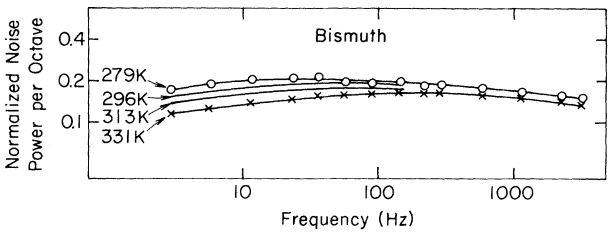

FIG. 1. The temperature-dependent octave-integral spectrum of a 140-nm Bi film. At 279 and 331 K the experimental points are shown, along with smooth curves drawn by eye. At intermediate temperatures only the smooth curves are drawn to avoid clutter. Errors are comparable to the thickness of the symbols. The normalization (accurate to about a factor of 1.5) consists of multiplying the mean square fractional resistance fluctuations per factor of e in frequency by the number of atoms in the relevant part of the sample.

the temperature dependence of the spectra, one must include a decrease in the net magnitude with increasing T as well as the kinetic shift of the spectral position and width. The maximum fractional noise per octave decreased as T^{-x}, $x = 1.1 \pm 0.3$ in this temperature range. In order to calculate the overall temperature dependence of the noise magnitude one must also know how the width (i.e., range of characteristic rates) of the spectrum changes. The two limiting plausible hypotheses are that the width is temperature independent (arising from a range of temperature-independent prefactors, i.e., attempt rates, f_0) or inversely proportional to temperature (arising from a range of activation enthalpies, $E^{\ddagger}$). The data are not sufficient to resolve between these cases. Thus the temperature dependence of the overall magnitude is somewhat uncertain, being about T^{-y}, $y = 1.6 \pm 0.7$ (about 95% confidence) in this range. The existence of roughly such a dependence had previously been predicted[2] for noise in metal films caused by defect motion in the temperature range for which phonon scattering is significant. Given the grain size (~ 50 nm, as measured by scanning electron microscopy) and resistivity (~ 250 $\mu\Omega$ cm, no doubt lower in the center of the film) we expect that phonon scattering was very significant in these films.

The 30-nm samples were found to have a resistivity roughly twice as high as the 140-nm sample. Their noise magnitudes, expressed as fractional variance of the resistance, normalized by their effective volumes, were found to be very roughly five times lower. These results suggest that the $1/f$ noise arises in bulk scattering in Bi and is masked by relatively quiet surface scattering—a result previously found in Bi [12] which is slightly different from most results on metals[1,13,14] and contrasts sharply with results on semiconductors.[6] To test this suggestion we compared the temperature dependence of the noise in a thin film, for which the phonon scattering could not have been dominant, with that of the thick film. The thin film actually showed a very slight increase in normalized noise power with increasing temperature, which seems to confirm that the decrease with temperature in the thick film results from masking of the noisy scattering by quiet phonon scattering. Given the slight residual phonon scattering in the thin films, one would expect a slight decrease in noise with increasing temperature, but it is apparently small compared with other effects such as may arise from temperature dependence of the band structure and carrier wave-vector distribution. The difference in temperature dependence between thick and thin samples is significant and in rough agreement with the defect-motion model.

The most striking result appeared in the parameter S, which was -0.48 ± 0.08 (95% confidence) in the frequency interval 4–16 Hz, for which random error was smallest. No major difference was found between the thick and thin films. Table I shows values of S in different octave intervals computed directly from the digital spectra. Very careful checks were made for nonlinear effects (e.g., due to Joule heating) which can cause errors in determining S; such effects were not significant. Similar values of $S < 0$ have previously been reported in Cr[4], but in samples for which the possibility of significant errors could not be ruled out.

Values of the parameter S are obtained by comparing the cross spectrum of the resistance fluctuations on the two orthogonal paths of a cross-shaped sample with the spectrum of the off-diagonal resistance fluctuations on the same two-port linear system.[4,6,8] Calculation of exact S values from these measured spectra requires the use of a number computed from some field integrals.[8] This computed number, which we believe is accurate to better than 10%, enters very nearly lin-

TABLE I. The computed values of the parameter S for three thin Bi samples and one thick one (the only ones measured) are shown.

	Frequency range (Hz)						
Sample	2–4	4–8	8–16	16–32	32–45	32–64	64–90
1	−0.52	−0.47	−0.43	−0.38	−0.58		
2	−0.20	−0.45	−0.36	−0.43	−0.66		
3	−0.56	−0.54	−0.68	−0.50		−0.46	−0.54
4	−0.63	−0.38	−0.54	−0.35		−0.20	−0.55
Average	−0.48	−0.46	−0.50	−0.42		−0.50	

early into determining the absolute value of S; it is important to realize that the sign of S can be read directly from the data.

Values of $S<0$ mean that when the conductivity along some axis has a positive fluctuation it is probable that there exists another axis for which the fluctuation is negative. Put another way, the correlation coefficient between the two diagonal terms in diagonalized representations of the fluctuations is negative. In fact, for $S<-\frac{1}{3}$, for any two fixed orthogonal axes, the diagonal terms in the fixed-axis representation have a negative correlation coefficient.[8]

Values of $S<0$ cannot be explained by gross structural anisotropies. In the extreme limit, such anisotropies cause any local fluctuation to affect the conductivity only along the single locally allowed axis of current flow, giving $S=0$ regardless of the local mechanism. Carbon granule films are expected to be near this limit and do experimentally give $S=0$.[4] Various theories which postulate independent fluctuations of scattering cross sections with different wave vectors give $S=0$ also.[7,8]

We can, however, present a schematic defect-motion model[15] which in its simplest form predicts $S<0$ with a negative temperature dependence of the noise magnitude and for which plausible modifications would give the $1/f$-like spectrum. In order to illustrate the model we shall discuss a simple two-dimensional version. (See Fig. 2.) Consider a square lattice (this is not essential) with binding sites for some impurity which are not located exactly on any of the diagonals. There are two classes of sites, labeled A and B, which by symmetry must be equivalent in all their scalar properties, including binding energies. The two sites can be interconverted by a rotation—in this case of 90°. The distribution of these impurities between A and B sites is then temperature independent, although the kinetics for fluctuations will typically be thermally activated.

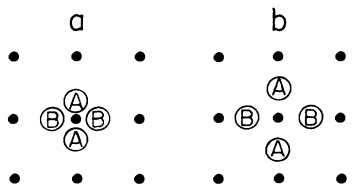

FIG. 2. Schematic pictures showing a two-dimensional version of the point-defect model are shown.

In this two-dimensional version, one would have $S=-1$ by symmetry, i.e., there could be no scalar contribution to the fluctuations. For an equivalent three-dimensional (3D) model, rotation out of the plane to a third class of equivalent sites would be allowed, giving $S=-0.5$. No significance should be attached to the coincidence between this value and that found in Bi, which does not have cubic symmetry. The point is simply that negative S values arise in a natural way, while for realistic 3D models it is hard to obtain purely traceless ($S=-1$) 2D projections.

Simple defect-motion models have an obvious flaw—they predict Lorentzian spectra. The model described in fact predicts a single Lorentzian, even in the 3D version. This problem is common to virtually all simple theories of slow noise processes in bulk crystals, with the general solution being to invoke local strains and other inhomogeneities to account for the observed spread in rates.

For point defects with the symmetry shown in Fig. 2(a), there is no necessary connection between defect translational diffusion and the rotational motions giving anisotropic fluctuations. For those having the symmetry shown in Fig. 2(b), these rates are determined by the same hopping steps. For Bi, with a characteristic noise frequency of ~100 Hz at 300 K and a presumed step size of $\sim 10^{-8}$ cm, a diffusion coefficient of $\sim 10^{-14}$ cm^2/sec would result in the second case. One suspects that, since the noise is not noticeably nonstationary, if translational diffusion were involved it would have to stop at the surface oxide; otherwise the number of defects would thermally equilibrate and probably decrease in time.

The type of model we have presented is appealing in that it describes noise originating in the bulk with approximately the observed symmetry, temperature dependence of kinetics, and temperature dependence of magnitude. The major *ad hoc* assumption is the spreading of the activation energy distribution by local inhomogeneities. Our samples are not so nearly crystalline that we can rule out small motions of or near large defects such as dislocations or grain boundaries. However, previously[11] we found little or no change in the noise magnitude when major changes in the grain structure were produced by annealing. At any rate, models requiring large defects would give a spread in activation energies in a natural way, since near such a defect the material is somewhat amorphous. There would then be no symmetry giving equality of energies between

the states. Instead one would expect a typical flat distribution of energy spacings characteristic of amorphous materials,[16,17] which would lead to an additional factor of T in the noise magnitude. This would not be in good agreement with our data, but there are enough complications in the overall temperature dependence that we cannot rule it out. Similarly, the negative S value would be consistent with such a model, if the motions were something like rotations, but would not arise in as obvious a way as for the point-defect model. To the best of our knowledge, however, no known models other than those involving some defect motion would give negative values for S.

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[1]P. Dutta and P. M. Horn, Rev. Mod. Phys. 53, 497 (1981).

[2]M. B. Weissman, in *Proceedings of the Sixth International Conference on Noise in Physical Systems, Gaithersburg, Maryland, 1981*, edited by P. H. E. Meijer (U.S. GPO, Washington, D.C., 1981), p. 133.

[3]F. N. Hooge, T. G. M. Kleinpenning, and L. K. J. Vandamme, Rep. Prog. Phys. 44, 479 (1981).

[4]R. D. Black, W. M. Snow, and M. B. Weissman, Phys. Rev. B 25, 2955 (1982).

[5]R. D. Black and M. B. Weissman, unpublished results.

[6]R. D. Black, P. J. Restle, and M. B. Weissman, Phys. Rev. B 28, 1935 (1983).

[7]M. B. Weissman, J. Appl. Phys. 51, 5872 (1980).

[8]M. B. Weissman, R. D. Black, and W. M. Snow, J. Appl. Phys. 53, 6276 (1982).

[9]R. D. Black, M. B. Weissman, and P. J. Restle, J. Appl. Phys. 53, 6280 (1982).

[10]P. J. Restle, M. B. Weissman, and R. D. Black, to be published.

[11]R. D. Black, M. B. Weissman, and F. M. Fliegel, Phys. Rev. B 24, 7454 (1981).

[12]F. N. Hooge, J. Kedzia, and L. K. J. Vandamme, J. Appl. Phys. 50, 8087 (1979).

[13]R. F. Voss and J. Clarke, Phys. Rev. B 13, 4790 (1976).

[14]D. M. Fleetwood, J. T. Masden, and N. Giordano, Phys. Rev. Lett. 50, 450 (1983).

[15]As we were preparing this paper, it came to our attention that an identical model has been proposed previously on the basis of a connection between low-frequency internal friction and $1/f$ noise. For a more complete theoretical treatment, we refer readers to Sh. M. Kogan and K. E. Nagaev, Fiz. Tverd. Tela 24, 3381 (1982) [Sov. Phys. Solid State 24, 1921 (1982)].

[16]P. W. Anderson, B. I. Halperin, and C. M. Varma, Philos. Mag. 25, 1 (1972).

[17]W. A. Phillips, J. Low Temp. Phys. 7, 351 (1972).