Density Dependence of the Momentum Distribution for ^{4}He

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Neutron-scattering techniques have been used to determine the momentum distribution $n(\vec{p})$ for ^{4}He for different temperatures and densities. $n(\vec{p})$ is found to be a strong function of density and thus brings into question any direct relationship between the condensate fraction and the pair distribution function $g(r, T)$.

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Since London[1] proposed that a fraction of the atoms in liquid ^{4}He might undergo Bose-Einstein condensation, there has been a considerable effort in trying to confirm this possibility experimentally. A seemingly direct test of the possibility of a zero momentum state for a fraction of the ^{4}He atoms at low temperatures was proposed by Hohenberg and Platzman.[2] The method involves extracting the momentum distribution $n(\vec{p})$ for ^{4}He by scattering neutrons from the liquid with a sufficiently high momentum transfer Q that the impulse approximation is valid. Unfortunately, it is not entirely clear how high a Q is necessary for the impulse approximation to be of sufficient accuracy. This problem has been recently addressed by Weinstein and Negele[3] although their main concern is not in the momentum region applicable to determining Bose-Einstein condensation. Furthermore, if high Q's are used in neutron-scattering experiments, the energy resolution must be greatly increased and an experiment with a Q of 50 $Å^{-1}$ would, for a scattering angle of 120°, need an incoming energy of 2.42 eV and a 0.2% energy resolution to have equal momentum resolution of the present experiment.

A number of neutron-scattering experiments on ^{4}He have been performed with varying results.[4] The differing results for the experiments appear to be due to the fact that the impulse approximation is not valid in the Q region used for the experiments and that proper account was not taken of final-state interactions. A different approach to the problem was taken by Woods and Sears[5] and Sears *et al.*[6] in that high-resolution medium-Q data were analyzed in such a way that the distortions caused by the final-state interactions were minimized. This approach relies on using a series of measurements of the dynamic structure factor $S(Q,\omega)$ made at various Q's and symmetrizing the measurements about the recoil frequencies $Q_0^2/2m$ to minimize final-state interaction effects. The symmetrized $S(Q,\omega)$ distributions are then averaged together to minimize any residual distortions. This technique appears to give reliable measurements of the condensate fraction n_0.

The present experiments were performed at the high flux isotope reactor at Oak Ridge with use of a time-of-flight spectrometer that uses ultrasonically pulsed silicon crystals to pulse the neutron beam. The cross-correlation technique of data accumulation was utilized to obtain a high signal-to-noise ratio. Data were obtained simultaneously at 32 different Q values ranging from 2 to 7 $Å^{-1}$. The data used in obtaining the momentum distributions were restricted to the range 5 to 7 $Å^{-1}$, and averages over various values of Q were tested. The results were insensitive to the number of Q's used as long as the number was about five or larger, and the results shown are for averages over ten momentum values roughly equally spaced in the range 5 to 7 $Å^{-1}$. The momentum resolution was about 0.10 $Å^{-1}$.

The first part of the experiment involved measuring $n(\vec{p})$ at the standard vapor pressure to be sure that the time-of-flight data and triple-axis results[5,6] were consistent. The time-of-flight data were converted to $S(Q,\omega)$ distributions, and symmetrized about the calculated recoil frequencies. $n(\vec{p})$ was then obtained from the constant-angle data by the method of Mook.[7] The condensate fraction was then determined exactly as in Ref. 6 with

$$n(\vec{p}) = n_0\delta(\vec{p}) + (1-n_0)n^*(\vec{p}),$$

where $n^*(\vec{p})$ is assumed to be the momentum distribution of the uncondensed atoms and n_0 the condensate fraction. $n^*(p)$ was obtained as in Ref. 6 by data taken at 2.30 K. The correlation technique of accumulating data is at its best near the peak values of $S(Q,\omega)$ which is the region from which $n(\vec{p})$ at small $\vec{p}$ is obtained. This is also the region of interest from the standpoint of Bose-Einstein condensation so the main focus of the experiment was placed on the small-p region

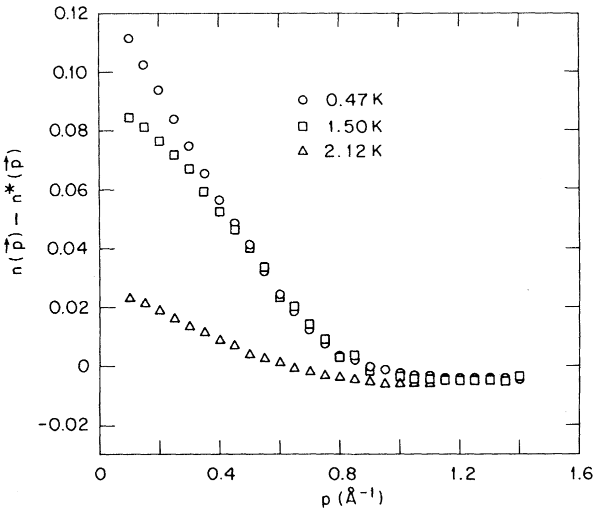

FIG. 1. $n(\vec{p}) - n^*(\vec{p})$ for ^{4}He at standard vapor pressure for different temperatures.

of $n(\vec{p})$. Figure 1 shows the data obtained for three temperatures. The critical quantity for determining n_0 is $n(\vec{p}) - n^*(\vec{p})$ so that the data are shown in this form. We note that below T_λ a dramatic increase is found in this quantity for small $n(\vec{p})$ as expected if a condensate is present. An important addition to the earlier measurements is a result taken at 0.47 K. This result shows that n_0 changes little once the temperature is lowered in the neighborhood of 1 K. This is not unexpected, but it is very important in giving us confidence that zero-temperature calculations for ^{4}He are applicable to the neutron-scattering experiments. Results for p_c, ϵ, β, γ, and n_0 are given in Table I where

$$\epsilon = 4\pi \int_0^{p_c} \{n(\vec{p}) - n^*(\vec{p})\} p^2 dp, \tag{1}$$

$$\beta = 4\pi \int_0^{p_c} n^*(\vec{p}) p^2 dp, \tag{2}$$

γ is the enhancement factor as defined in Ref. 6, and p_c is the momentum where the integrand in (1) goes to zero. n_0 is then given by

$$n_0 = \epsilon/(1 - \beta + \gamma). \tag{3}$$

Our p_c occurs at a lower value than for the data shown in Ref. 6, and our $n(\vec{p})$ at low p is higher. This probably only reflects the fact that the Q's used in the present experiment are somewhat higher and that the resolution is perhaps somewhat better. A low-temperature value of 0.105 is obtained for n_0. This is lower than that obtained in Ref. 6, but well within the range of approximations used in the analysis. In fact it seems clear that the two sets of data show convincingly the strong increase in $n(\vec{p})$ at low p for $T < T_\lambda$ expected for Bose-Einstein condensation. The errors shown on n_0 are from counting statistics only; it is assumed the analysis used in Ref. 6 is appropriate and no uncertainty is allowed for it. The new value of n_0 agrees very well with recent calculations[8-10] which suggest a condensate fraction of about 10%. A measurement was made also at 4.2 K and little change was found for $n(\vec{p})$ for low p between 4.2 and 2.3 K, similar to that reported in Ref. 6.

Measurements were then made at different applied pressures to obtain $n(\vec{p})$ as a function of density. Our measurements were limited to about 15-atm applied pressure as a compromise between having a thin sample cell window for low background and attaining high densities. Constraints on counting time made it impossible to find a $n^*(\vec{p})$ for each pressure. $n^*(\vec{p})$ was established as accurately as possible for 2.3 K at standard vapor pressure since it enters the analysis for each temperature. According to the calculations of Ref. 8, $n^*(\vec{p})$ does seem to change with density. However, n_0 is determined from integrals over an appreciable part of $n^*(\vec{p})$ and these might be expected to be less sensitive to changes in density than $n^*(\vec{p})$ itself. Furthermore, the changes in $n^*(\vec{p})$ are expected to be small at small p compared to the large enhancement caused by Bose-Einstein condensation.

Nevertheless on accurate determination of n_0 as a function of density will have to await a pressure-dependent determination of $n^*(\vec{p})$. Figure 2 shows the results obtained for $n(\vec{p}) - n^*(\vec{p})$ at var-

TABLE I. Condensate fraction and associated parameters as a function of temperature for standard vapor pressure.

Temperature (K)	p_c (Å^{-1})	ϵ	β	γ	n_0
0.47	0.90	0.077	0.275	0.0071	0.105 ± 0.02
1.50	0.89	0.076	0.274	0.091	0.093 ± 0.02
2.12	0.62	0.020	0.134	3	0.005 ± 0.003

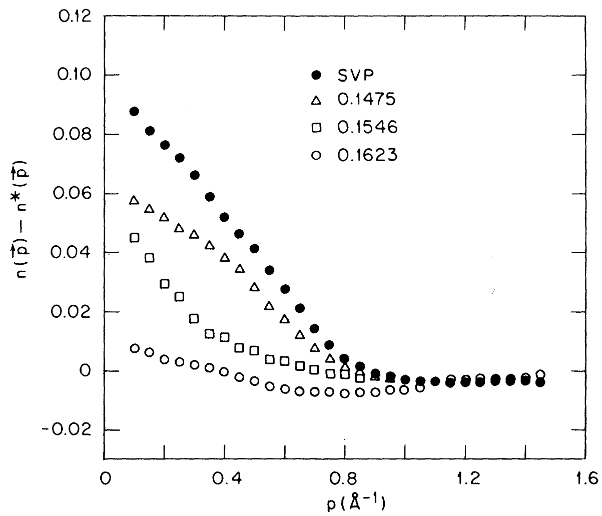

FIG. 2. $n(\vec{p})-n^*(p)$ for ^{4}He at 1.5 K for different densities.

ious densities for $T=1.5$ K and where $n^*(p)$ is the standard-vapor-pressure result. Data presented in this form permit a direct comparison to the temperature-dependent results and the difference plot helps minimize systematic errors such as scattering from the sample container.

Note that $n(\vec{p})$ is strongly affected by even small changes in density. The rapid decrease in the small-$\vec{p}$ part of $n(\vec{p})$ with increasing pressure suggests that n_0 is a strong function of density. The calculations of Lam and Ristig[9] and Whitlock *et al.*[8] support this result although they appear to suggest a somewhat slower decrease in n_0 with density then might be inferred from the data. This may only reflect the result that $n^*(p)$ is sensitive to density and direct comparisons will have to await further measurements.

Figure 2 shows $n(\vec{p})$ varies strongly with density regardless of any assumption about $n^*(\vec{p})$. This result is especially important since the very recent x-ray scattering work of Wirth, Ewen, and Hallock[11] shows unexpectedly that the spatial order in ^{4}He changes little with density. The present experiments in conjunction with the x-ray results suggest that n_0 may not be a sensitive function of the pair correlation function $g(r,T)$. Since Cummings *et al.*[12] have suggested that $g(r,T)$ may be used to determine n_0, it has been argued[13] that relations can be constructed between n_0, $g(r,T)$, and the superfluid fraction. It has been pointed out that the theoretical basis for this type of relationship is weak[14,15] but it has seemed reasonable experimentally. The present results now appear to destroy the empirical success of relating n_0 and $g(r,T)$.

In conclusion the new neutron-scattering results on $n(\vec{p})$ give excellent confirmation of earlier results for n_0 as a function of temperature and extend those results down a factor of 2 in temperature giving strong evidence that ^{4}He at 1 K is well approximated by zero-temperature theories. The results of $n(\vec{p})$ vs density show the first experimental proof that $n(\vec{p})$ is strongly dependent on density. The density-dependent results taken in conjunction with the x-ray work on $g(r,T)$ call for a reexamination of a simple connection between n_0 and $g(r,T)$.

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