Experimental Evidence of Intermittencies Associated with a Subharmonic Bifurcation

M. Dubois, M. A. Rubio,[a] and P. Berge

Service de Physique du Solide et de Résonance Magnetique, Centre d'Etudes Nucléaires de Saclay, F-91191 Gif-sur-Yvette, France

(Received 17 June 1983)

The first experimental observation is reported of type-III Pomeau-Manneville intermittencies in a hydrodynamical system: the Rayleigh-Bénard instability studied in confined geometry. From optical measurements, return maps have been constructed and distribution of the laminar period lengths has been measured. These results are in complete agreement with the theoretical model.

PACS numbers: 47.25.Qv, 05.40.+j

The problem of chaos, involving a small number of degrees of freedom, has received much attention.[1] It is now well established that at least three different routes may lead to this deterministic chaos[2] and among them is the so-called Pomeau-Manneville scenario.[3] This particular route is characterized by intermittent turbulent bursts breaking an otherwise regular signal.[4] It has been suggested that these intermittencies qualitatively explain some experimental findings,[5] but up to now, except in the case of type-I intermittencies observed in a model system,[6] the available experimental data have not allowed a complete quantitative comparison with the theoretical model to be made.

We report here the first observation, in a hydrodynamical system,[7] of the so-called type-III intermittencies in the Pomeau-Manneville classification; these intermittencies arise in the process of destabilization of a limit cycle, and appear simultaneously with a period-doubling bifurcation. The turbulent bursts which appear are associated with an increase in the amplitude of the subharmonic mode.

Furthermore, the experimental data have been quantitatively compared with the theoretical predictions: The complete agreement clearly establishes the relevance of the theoretical model to account for the experimental situation.

The basic experimental situation is Rayleigh-Bénard convection in confined geometry; the test fluid is silicon oil with Prandtl number $N_{\mathrm{Pr}} \simeq 38$. The rectangular cell ($L_x = 2d$, $L_y = d, 2d$ where d, the depth of the cell, is 1.25 cm) is inserted in a measuring system, previously described by Dubois and Bergé.[8] The state of the convection pattern is visually monitored by looking at interferometric pictures, but the quantitative measurements of the time-dependent properties are performed on optical images, similar to those provided by the Schlieren method. A parallel light beam is sent through the cell and an image is then formed by a lens. An adjustable diaphragm is placed at the focus and a photodiode which can be positioned at any point of the image receives a light intensity directly related to the variation of the local temperature gradient around a chosen mean value $\nabla_z T$ or $\nabla_x T$.

When the Rayleigh number is increased, we observe, as in other cases,[9] a sequence of different structures, each of them with its specific oscillators and its route to turbulence. Among these routes, we have observed the following sequence. For $333 < N_{\mathrm{Ra}}/N_{\mathrm{Ra},c} < 377$ (where N_{Ra} denotes Rayleigh number), a particular structure—probably formed with three rolls along L_x—is stationary. At $N_{\mathrm{Ra}}/N_{\mathrm{Ra},c} \simeq 377$, a monoperiodic regime takes place with a frequency f_0 near 1.7×10^{-2} Hz ($d^2/D_T = 1950$ sec) given by the pulsation of a cold "plume"; then, at $N_{\mathrm{Ra}}/N_{\mathrm{Ra},c} \simeq 416.7$, the subharmonic frequency $f_0/2$ appears in the convective behavior, associated with stochastic intermittencies. A typical time-dependent behavior is shown in Fig. 1. We very clearly observe the growth of the subharmonic amplitude, together with the decrease of the "fundamental" amplitude. When the subharmonic amplitude reaches a high value, the increasing being very rapid, the signal loses its regularity and a turbulent burst appears. Immediately after this, there is a reappearance of the regular behavior, corresponding to a return to the preceding periodic behavior, with a given amplitude of the subharmonic frequency. This initial amplitude, which can have a completely arbitrary value, determines the length of the regular period until the next turbulent burst occurs: The lower the subharmonic amplitude at the beginning of the regular (laminar) period, the longer the duration of this regime until the next turbulent burst occurs; so these durations are stochastic, leading to a mean turbulent state. Obviously, the corresponding Fourier spectrum pre-

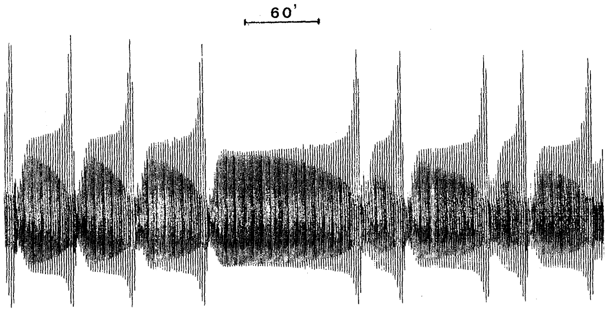

FIG. 1. Time dependence of light intensity, roughly proportional to the horizontal temperature gradient near the cold plume. $N_{\mathrm{Ra}}/N_{\mathrm{Ra},c} \approx 420$.

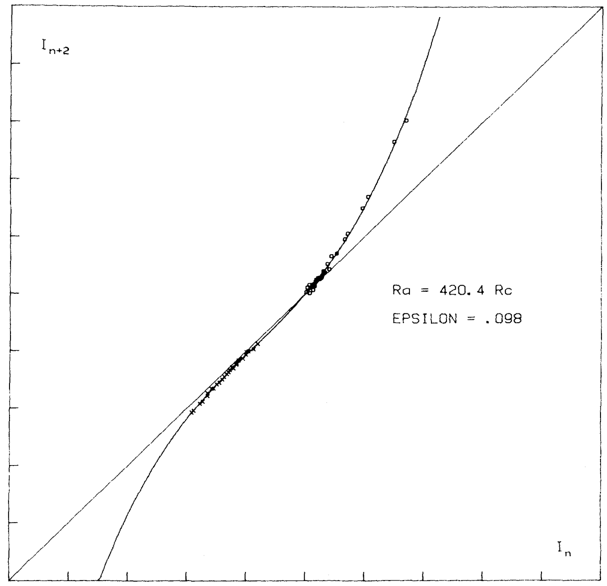

FIG. 2. Return map $I_{n+2}=f(I_n)$, constructed from the data shown in Fig. 1 superposing two laminar periods. The amplitudes of the light modulation in the turbulent bursts have not been drawn.

sents a broadband noise near zero frequency and a broadening at the bottom of the peaks f_0, $f_0/2$, and their harmonics. This intermittent behavior is reversible when the Rayleigh number is varied (no hysteresis), and then the approach to the chaotic state is continuous.[10]

The onset of intermittencies corresponds to the appearance of a new unstable direction in the phase space, which, at least, is three dimensional. To point out the dynamical properties, it is useful to look at Poincaré return maps, which are best adapted to the actual experimental situation and also constitute the framework of the theoretical model.[3] In a general form, these intermittencies can be described by the following relation:

$$I_{n+1} = -(1+\epsilon)I_n + AI_n^2 + BI_n^3 + \text{higher-order terms}. \tag{1}$$

A and B are constants; ϵ is the parameter of the bifurcation; the I_n values are the successive maximum values of the relevant physical signal. In order to discuss the stability of the subharmonic state, we have to apply twice the recurrent transformation, giving the following relation by keeping the most significant orders:

$$I_{n+2} = (1+2\epsilon)I_n + aI_n^2 + bI_n^3 \tag{2}$$

(a and b are constants). From the experimental data (time dependence as shown in Fig. 1), we get the I_n values as the successive maxima of the light intensity modulations seen by a photodiode. Then the return maps $I_{n+2}=f(I_n)$ are constructed. One example is given in Fig. 2: The crosses represent the (decreasing) amplitudes of the "fundamental" mode (odd n), and the squares, the (increasing) amplitudes of the "subharmonic" mode (even n), as the time evolves apart from the ghost of the fixed point (monoperiodic regime). The full curve represents the best fit by the theoretical relation (2) and has been obtained by a least-squares adjustment procedure. It yields $\epsilon = 0.098$. ϵ is expected to be proportional to the experimental control parameter $\epsilon_I = (N_{\mathrm{Ra}} - N_{\mathrm{Ra},I})/N_{\mathrm{Ra},I}$ ($N_{\mathrm{Ra},I}$ is the Rayleigh number which corresponds to the threshold of the intermittent behavior).

One point has to be emphasized: The experimental return maps do not always show this specific behavior which is seen in Fig. 2, depending on the position of the measurement point with respect to the structure. Indeed the convective motion is influenced differently by one or the other of the oscillators along the different stream lines and measurements at different points reflect the behavior along different directions in the phase space.[11]

Another signature of the intermittent behavior is the distribution of the lengths t of the laminar periods. If we know the path followed by the representative point in the phase space, it is obvious that the time duration will depend on the distance from the unstable monoperiodic point where the reentry is made after the turbulent burst. In fact, this time duration t is proportional to the number of iterations n which lead the representative point from the reentry point ($I = I_n$) to the end of the channel. We have calculated $n(I_n)$ with $a = 0$

(this corresponds to our experimental situation): For each iteration step dn, the corresponding intensity variation is $2\epsilon I - bI^3$. Then, integrating from the reentry point and assuming that the probability of reinjection $P(I_n)$ is uniform (independent of the reentry point, and of I_0),[12] we get

$$P(t) \sim P(n) = cte\, \frac{dI_0}{dn} \sim \frac{(e^{-4\epsilon t})^{1/2}}{(1-4e^{4\epsilon t})^{3/2}} \qquad (3)$$

for high values of t, or in its integral form

$$N(t_0) \sim \int_{t_0}^{\infty} P(t) \sim \{\exp(-4\epsilon t_0)/[1-\exp(-4\epsilon t_0)]\}^{0.5}. \qquad (4)$$

In Fig. 3, where the y axis represents the number of laminar periods, with a length greater than T_0, we can see the comparison of the experimental results with the expected behavior according to the relation (4). The agreement is very good, in the range of N_{Ra} values corresponding to the domain of existence of these intermittencies ($\epsilon_I < 0.2$). For higher N_{Ra} values, the actual structure becomes unstable.

For the type-I intermittencies the maximum of the curve representing the probability $P(\tau)$ versus the duration τ of the laminar period is located at comparatively long times[3,13] and is followed by a rapid decrease corresponding to a clear bounding of the durations. However, for the type-III intermittencies the maximum probability is predicted to be located at short times followed by a slow decrease of the curve. The experimental results presented here agree rather well with this last prediction. This fact, together with the previous remark (Ref. 11), definitively identifies the observed phenomenon with type-III intermittencies.

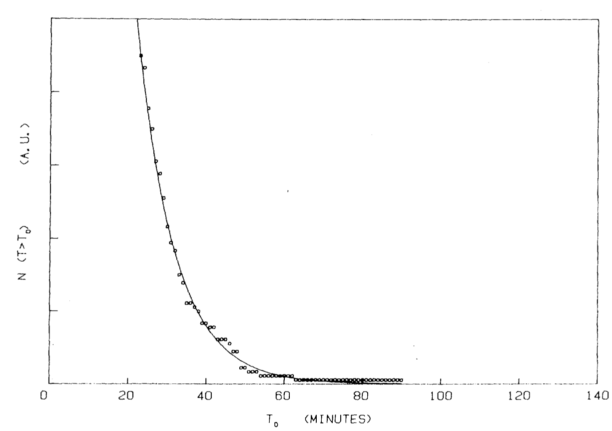

FIG. 3. Number of laminar periods with length greater than T_0. $N_{\mathrm{Ra}} \simeq 420$. The best fit, with the relation (4), gives $\epsilon = 0.095 \pm 0.003$, in agreement with the value found from the return map (Fig. 2).

In conclusion, we would say that, although intermittencies have been already observed in convection experiments[14,15] and also in other systems,[16,17] the particularity of the experimental situation described in this paper is that here all the properties which are relevant in the theoretical model have been experimentally verified, and show excellent agreement between the physical situation and the theory. The fact that in certain previous experiments the final chaotic state, although representing the ultimate step of a well-known evolution, is not always understood (for example the actual number of degrees of freedom is not known) is an additional indication of the interest of the present example, where all the behaviors follow the theoretical model.

We thank V. Croquette, P. Manneville, and Y. Pomeau for helpful discussions. Our thanks also go to B. Ozenda and C. Poitou for very efficient technical assistance. This work was supported in part by the Stiftung Volkswagenwerk.

[a]Permanent address: Departamento de Física Fundamental, Universidad Nacional de Educación á Distancia, Aptdo. Correos, 50487 Madrid, Spain.

[1]See, for example, J. B. McLaughin and P. C. Martin, Phys. Rev. A 12, 186 (1975); M. L. Rabinovitch, Usp. Fiz. Nauk 21, 123 (1978) [Sov. Phys. Usp. 21, 443 (1978)]; E. Ott, Rev. Mod. Phys. 53, 655 (1981).

[2]J. P. Eckmann, Rev. Mod. Phys. 53, 643 (1981).

[3]Y. Pomeau and P. Manneville, Commun. Math. Phys. 74, 189 (1980).

[4]P. Manneville, in *Symmetries and Broken Symmetries*, edited by N. Boccara (IDSET, Paris, 1981), p. 517; or J. E. Hirsch, B. H. Huberman, and D. J. Scalapino, Phys. Rev. A 26, 2117 (1982).

[5]P. Bergé, M. Dubois, P. Manneville, and Y. Pomeau, J. Phys. (Paris), Lett. 41, L341 (1980); Y. Pomeau, J. C. Roux, A. Rossi, S. Bachelart, and C. Vidal, J. Phys. (Paris), Lett. 42, L271 (1981); J. C. Roux, in "Order in Chaos," Proceedings of a Conference at Los Alamos (to be published); V. Croquette, Pour la Science 62, 62 (1982).

[6]C. Jeffries and J. Perez, Phys. Rev. A 26, 4 (1982).

[7]The type-III intermittencies have been observed in a forced compass by V. Croquette (private communication).

[8]M. Dubois and P. Bergé, J. Phys. 42, 167 (1981).

[9]P. Bergé, M. Dubois, and V. Croquette, in *Convective Transport and Instability Phenomena*, edited by J. Zierep and H. Oertel (Euromech, Karlsruhe, 1981).

[10] C. Tresser, P. Coullet, and A. Arneodo, J. Phys. (Paris), Lett. 41, L243 (1980).

[11] For example, in certain points of the image of the structure, while the fundamental amplitude is always decreasing along the laminar period, the amplitude of the subharmonic mode may remain practically constant (no growth) and finally decreases very near the turbulent burst. But, whatever the measurement point, the f_0 and $f_0/2$ amplitudes do not vary in a homothetic way. This is the opposite behavior to that observed in type-I intermittencies where, if the subharmonic mode is present, the two return maps, corresponding to each mode, remain homothetic.

[12] P. Manneville, J. Phys. (Paris) 41, 1235 (1980).

[13] M. Dubois, in *Symmetries and Broken Symmetries*, edited by N. Boccara (IDSET, Paris, 1981), p. 91.

[14] J. P. Gollub and S. V. Benson, J. Fluid Mech. 100, 4119 (1980).

[15] J. Maurer and A. Libchaber, J. Phys. (Paris), Lett. 41, L515 (1980).

[16] T. Yamada and H. Fujisaka, Suppl. Prog. Theor. Phys. 64, 269 (1978).

[17] H. J. Scholz, T. Yamada, H. Brand, and R. Graham, Phys. Lett. 82A, 321 (1981).

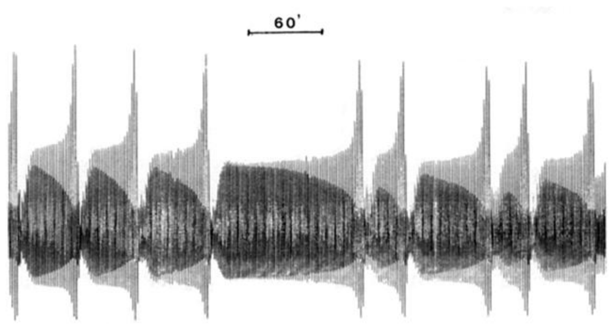

FIG. 1. Time dependence of light intensity, roughly proportional to the horizontal temperature gradient near the cold plume. $N_{\mathrm{Ra}}/N_{\mathrm{Ra},c} \approx 420$.