Correlation Length Exponent for the O(n) Model in Two Dimensions for $n=0$

Nienhuis[1] recently presented an argument, subject to plausible assumptions, in favor of a simple formula for the correlation length exponent ν of the O(n) model for $d=2$. For the case $n=0$, corresponding to the self-avoiding walk (SAW) model of a dilute linear polymer, his argument predicts $\nu=\frac{3}{4}$.

This result is surprising for several reasons, among which are the following:

(a) It disagrees with a variety of recent estimates based on normally reliable approximation procedures.[2-5] For example, a four-loop momentum-space renormalization group (RG) analysis gives[2] 0.77, large-cell Monte Carlo position-space RG gives[3] 0.756 ± 0.004, phenomenological RG gives[4] 0.7503 ± 0.0002, and extrapolation of the first sixteen values of ρ_N, the mean square end-to-end distance of a SAW, gives 0.747 ± 0.001 (for the most reliable lattice, the close-packed triangular, the number is even lower: 0.746 ± 0.001).[5] In fact, the only recent calculation that is even consistent with the Nienhuis result is the Monte Carlo estimate $\nu=0.753 \pm 0.004$.[6]

(b) The Nienhuis argument that $\nu=\frac{3}{4}$ agrees perfectly with the "Flory formula"[7] $\nu_F(d)=3/(d+2)$, for $1 \leq d < 4$. This formula is thought to be very rough, and is strongly violated near $d=4$ where it predicts that $2\nu_F(d)=1-(4-d)/6$, while momentum-space RG calculations shown that $2\nu(d)=1-(4-d)/8$. For $d=3$, there is theoretical and experimental evidence that ν is about 2% smaller than ν_F.[7,8] That the Flory formula even comes close to representing the true value of ν has been shown to arise from the fact that its derivation makes two approximations which contribute with opposite signs.[7] Thus it is important to determine if the Nienhuis formula is correct, since if it is then perhaps one will be motivated to seek insight concerning why these two errors *exactly* cancel each other for the special case $d=2$.

Our purpose here is to test the Nienhuis result $\nu=\frac{3}{4}$ by using what has traditionally been the most reliable method of exponent determination, extrapolation from exact enumerations. To this end, we focus on the triangular lattice. Grassberger's result was based on enumerations of SAW's of up to sixteen steps. We enumerate exactly the $c_{17}=103\,673\,967\,882$ and $c_{18}=438\,296\,739\,594$ walks of seventeen and eighteen steps, respectively. This is a nontrivial calculation, requiring 80 h on an IBM 3081 computer; the next order would require roughly 332 h. We find the exact results $\rho_{17}c_{17}=5\,385\,834\,447\,738$ and $\rho_{18}c_{18}=24\,767\,425\,671\,540$.

We find that the additional two terms make clear that there is distinct curvature in the successive estimates of ν that Grassberger calculates [Eq. (3.2) of Ref. 5]. If we allow for the fact that $\rho_N=AN^{2\nu}[1+B/N^{\Delta}+C/N+\ldots]$, then we find that the leading nonanalytic correction, B/N^{Δ}, is characterized by $\Delta-1<0$. Hence the limiting slope of Fig. 1 of Ref. 5 must be $-\infty$, and the data must display a minimum.[8] We used many extrapolation methods, and all our analysis consistently predicts $\nu=\frac{3}{4}$ to within 0.3%.

In summary, we have made a nontrivial extension of the series for the mean square end-to-end distance of the SAW problem for the triangular lattice. We have analyzed the extended series, and found agreement with the value $\nu=\frac{3}{4}$ recently presented by Nienhuis.[9] Thus the apparent discrepancy between the Nienhuis argument and numerical estimates for ν is resolved.

We wish to thank E. Brézin, S. Havlin, and A. Margolina for helpful discussions, S. Redner for assistance in programming and the National Science Foundation, the U. S. Army Research Office, and the U. S. Office of Naval Research for financial support.

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Received 2 June 1983
PACS numbers: 05.50.+q, 64.60.Cn, 64.60.Fr, 75.40.Fa

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[9] Nienhuis also presents a *conjecture* $\gamma=\frac{43}{32}$ for the "susceptibility" exponent; our new c_N are consistent with his conjecture.

