Shell-Model Description of the Low-Energy Structure of Strongly Deformed Nuclei

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An effective fermion interaction of maximum particle rank four, comprised of operators which form the integrity basis of an SU(3) → R(3) algebra, has been found sufficient to reproduce almost exactly, within a single leading irreducible representation of pseudo SU(3), the ground- and gamma-band rotational structure as well as the concomitant interband and intraband $B(E2)$ strengths of states in the strongly deformed nuclei. Results are given for ^{164}Er.

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The description of the dynamics of nuclei which exhibit rotational-like spectra $[E_I \propto I(I+1)]$ has a long history. A significant early contribution is found in the pioneering work of Bohr and Mottelson.[1] This led to the development of the highly successful deformed potential or Nilsson model.[2]

Attempts to provide a microscopic description of such nuclei in terms of an underlying shell-model picture have been frustrated, in part, by the enormous basis dimensionalities encountered. Elliott[3] developed an SU(3) symmetry truncation for light ds-shell nuclei. Its success derives from the dominance of the harmonic oscillator potential, an assumption that breaks down for heavier nuclei because the spin-orbit splitting overpowers and destroys the real oscillator shell structure.

Raju, Draayer, and Hecht[4] suggested the possible validity of a pseudo SU(3) scheme for heavy nuclei. It is motivated in large part by certain regular features of the Nilsson model and should be most appropriate when the quadrupole deformation is large. Recently, building on that effort, we[5] proposed a shell-model truncation scheme for heavy deformed nuclei. Basis states are built by strong-coupling neutron and proton configurations that individually possess the maximal pseudo SU(3) symmetry.

In this as in other theories in which SU(3) is a dominant symmetry, it is expected that states spanning the leading irreducible representation (irrep) of SU(3) [i.e., the (λ, μ) which has the largest value of $2\lambda+\mu$] will dominate the low-energy excitation spectrum. Physically, in the pseudo SU(3) scheme, the states of the leading irrep are those many-particle configurations which have the largest intrinsic quadrupole deformation. To form states of good angular momentum, SU(3) is reduced with respect to R(3). This reduction results in a $(\lambda, \mu)KL$ classification of states, where L is the orbital angular momentum and K is an L-multiplicity label. For given (λ, μ) the states lie in bands of L values based on each K. If m (n) denotes the maximum (minimum) of λ and μ, these bands are given by $L=K, K+1, \ldots, K+m$ if $K\neq 0$, and by $L=m, m-2, \ldots, 1$ or 0 if $K=0$ where K can take the values $K=n, n-2, \ldots, 1$ or 0.

The purpose of this Letter is to show that operators of the SU(3) → R(3) integrity basis suffice to reproduce, within a single leading irrep of pseudo SU(3), the ground- and gamma-band structure of strongly deformed nuclei.[6] This includes subtleties of the observed interband and intraband $B(E2)$ strengths and the splitting of the 2^+ states. To achieve similar results in a phenomenological rotational model calculation would, at a minimum, require including an L^4 term in the interaction and band mixing.

The interaction Hamiltonian is rotationally invariant and must therefore be an R(3) scalar. Judd *et al.*[7] have shown that there are five independent R(3) scalars [built of the generators of SU(3): the angular momentum L and the quadrupole operators Q] that form the so-called SU(3) → R(3) "integrity basis." All R(3) scalars are expressible as a polynomial function of those five operators. The basis set includes the second- and third-order Casimir invariants of SU(3) (C_2 and C_3) which are independent of L and K, $X_2 = L\cdot L$ which is independent of K but produces an $L(L+1)$ splitting of L states, and two operators which have both an L and a K dependence, $X_3=(LQL)$ and $X_4=(LQQL)$. Only the X operators split states within an irrep of SU(3). Accordingly, we have chosen to write our residual interaction Hamiltonian in the form

$$H(4) = aX_2 + bX_3 + cX_4 + d(X_2)^2,$$

$$H(3) = \kappa_2 Q\cdot Q + \kappa_3[Q\times Q]\cdot Q + \kappa_4[L\times Q]_1\cdot[Q\times L]_1. \quad (1)$$

The first (four-parameter) form is the most general interaction of maximum particle rank four that the SU(3) → R(3) algebra supports. The second (three-parameter) form is given because the two-body interaction is expected to dominate and have a $Q\cdot Q$ nature and, furthermore, we have found that $(L\cdot L)^2$ plays only a minor role if X_3 and X_4 have the forms $[Q\times Q]\cdot Q$ and $[L\times Q]_1\cdot[Q\times L]_1$. Adding $(L\cdot L)^2$ to the latter would lead, because of completeness, to identical results for both. As is well known, $Q\cdot Q=4C_2-3L\cdot L$, and it is clear that this term alone cannot reproduce experiment in a single SU(3) irrep since it does not produce K splitting of the L degeneracies. Similarly, the product of three quadrupole operators is related to the two third-order invariants by $[Q\times Q]\cdot Q=\sqrt{\tfrac{2}{7}}(4C_3+\tfrac{9}{2}X_3)$. Recalling that X_3 is proportional to (LQL), we see that this type of interaction effectively samples the quadrupole deformation of rotated shell-model configurations. It acting alone can produce the desired K splitting but then inverts the L spectrum.

For the sake of brevity we shall only present results for ^{164}Er. Its structure is typical of strongly deformed nuclei and it has been the focus of much recent attention. Results for other rare-earth nuclei as well as some for the actinides will be the subject of a longer paper which, among other things, will show the stability of the interaction parameters with changing mass numbers. A shell-model picture for ^{164}Er is that of eighteen valence protons and fourteen valence neutrons outside the $Z=50$ and $N=82$ closed shells, respectively. If the interaction is such as to maximize the quadrupole deformation, then one can estimate occupancies from a Nilsson level scheme. One finds that the most probable dominant configuration has ten protons and eight neutrons occupying normal-parity orbitals of their respective major shells, while eight protons and six neutrons occupy the concomitant abnormal-parity intruder levels, the $h_{11/2}$ and $i_{13/2}$, respectively. In this study, we assume that the latter play no role, for within a single unique parity configuration the short-range potential is expected to dominate and favor for low-energy eigenstates a seniority-zero coupled configuration.[8] For the nucleons distributed among the normal-parity orbitals, the leading irreps coming from the [22222] and [2222] partitions of the proton and neutron pseudo shells are the (10, 4) and the (18, 4), respectively. The presence of a strong $Q_p\cdot Q_n$ part in the long-range quadrupole-quadrupole interaction then implies, according to the results of Ref. 5, that the strong-coupled (28, 8) irrep dominates the structure of the lowest eigenstates.

In Table I, we present the results for the excitation energies obtained by diagonalizing the H of (1) in the basis spanned by the states

$$|(28,8)KL,\,S=0,\,J=L\rangle \times|(h_{11/2})^8,\,v=0,\,J=0\rangle\,|(i_{13/2})^6,\,v=0,\,J=0\rangle. \quad (2)$$

The parameter values (in kiloelectronvolts), determined by a nonlinear least-squares regression analysis, are $\kappa_2=-9.37$, $\kappa_3=-0.261$, and $\kappa_4=-0.053$ for $H(3)$. If a $\kappa_4'(X_2)^2$ term is added to $H(3)$, the new parameters are $\kappa_2=-10.9$, κ_3

TABLE I. Experimental (Ref. 10) and theoretical (three- and four parameter) excitation energies (in kiloelectronvolts) for the ground-state band (I_g) and the γ vibrational band (I_γ) of ^{164}Er.

I_g	Expt	Theor(4)	Theor(3)	I_γ	Expt	Theor(4)	Theor(3)
0	0	0	0	2	860	858	860
2	91	87	83	3	946	943	941
4	300	289	276	4	1058	1055	1050
6	614	600	577	5	1198	1194	1184
8	1025	1012	981	6	1359	1358	1350
10	1518	1514	1497	7	1545	1547	1534
12	2083	2092	2088	8	1745	1758	1765
14	2703	2730	2782	9	1977	1994	1992
16	3411	3407	3567	10	2184	2244	2299
18	4121	4102	4443	11	2479	2523	2559
20	4868	4790	5411	12	2733	2803	2956
22	5651	5445	6472	13	3027	3117	3234
				14	3267	3415	3739

$=-0.270$, $\kappa_4=-0.049$, and $\kappa_4'=-0.008$ for $H(4)$. The agreement with the experimental energies is excellent for both $H(3)$ and $H(4)$ up to 2–3 MeV. At that point coupling to seniority-two states becomes important. We reiterate that it is impossible to fit the data with just second- and third-order terms. Although in contrast with $Q\cdot Q$, $[Q\times Q]\cdot Q$ can split K values, it inverts the L spectrum.[9]

Members of the ground-state band (see Table I), are dominated by $K=0$ components (95%–100%), with the $K=2$ components carrying the remaining strength. The $4^+(K=4)$, $6^+(K=6)$, and $8^+(K=8)$ eigenstate bandhead energies lie at 3.38, 7.55, and 13.38 MeV, respectively. The energy bandhead ratio, $E(K=4)/E(K=2)$, is thus equal to 3.9 which is significantly larger than the vibrational value of 2.0. This result indicates[10] that the K splitting in our SU(3) model corresponds to a large anharmonicity in the intrinsic γ vibrational motion. [The relation between the SU(3) formalism and the collective potential energy surface formalism will be the subject of another investigation.]

A more sensitive test of the induced splitting is the $B(E2)$ transition strengths. The $B(E2)$'s were calculated with use of the pseudo SU(3) quadrupole operator only. Its coefficient was normalized to the experimental $B(E2; 2^+\to 0^+)$. The predictions (which are virtually identical with use of either the third- or fourth-order wave functions) are shown in Fig. 1 along with experimental values.[11,12] One notes that overall agreement is good. The crossover $B(E2; 2_\gamma\to 0_g)=0.050\ e^2\ b^2$ which is consistent with the experimental[12] value of 0.036 $\pm 0.01\ e^2\ b^2$. The calculated static quadrupole moment of the $2_g{}^+$ state is $Q=-2.06\ e$ b.

Finally, we remark that the excited 0^+ bands seen in strongly deformed nuclei have one important feature; namely, they do not have enhanced collective $B(E2)$ transitions to either the ground or the gamma band. In our scheme these bands would arise from either nonleading SU(3) irreps of the dominant shell-model partition or from irreps from other partitions. In either case the $B(E2)$'s will not be large even in a more realistic (less severely truncated) model space calculation. Even more interesting is the question of the nature of the interaction between the $K=0$, $K=2$, and abnormal-parity configurations in the backbending region. This will be reported on in the future.

In conclusion, we have found that the leading irrep of strong-coupled pseudo SU(3) wave functions forms an adequate basis for describing low-lying collective phenomena in heavy deformed nuclei provided the effective interaction is allowed to include third- and fourth-order terms of the $SU(3)\to R(3)$ integrity basis. This does not necessarily imply the importance or even the existence of three- and four-body interactions in nuclei; it is simply a statement of the fact that the effective interaction appropriate to the SU(3) truncated model space requires such terms. Specifically, we have shown that the third- and fourth-order scalars can induce splitting within a single SU(3) irrep that is consistent with the gross characteristics of the experimentally observed ground and gamma-vibrational bands seen for well deformed nuclei. The success of the model suggests the need for several further investigations, a principal one being that of deriving the particular form of our interaction believed to be appropriate in a larger, less severely truncated, model space.

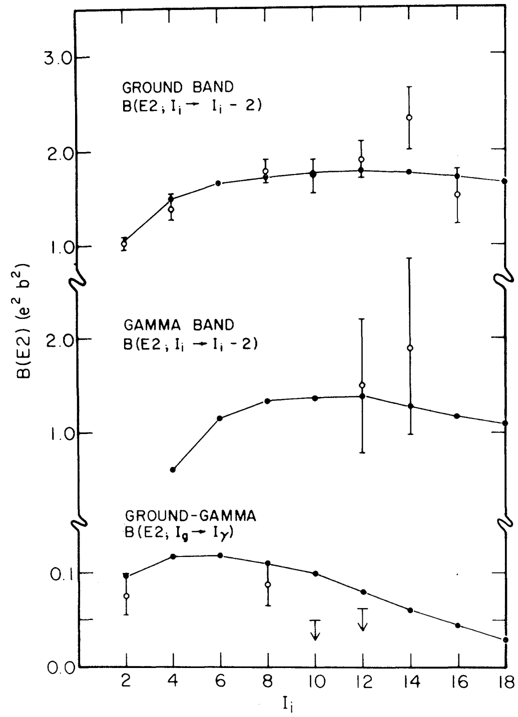

FIG. 1. Experimental (Refs. 11 and 12, circles with error bars) and theoretical (dots connected by lines) $B(E2)$ values for ^{164}Er.

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[1]A. Bohr and B. R. Mottelson, K. Dan. Vidensk.

Selsk., Mat. Fys. Medd. 27, 16 (1953).

[2]S. G. Nilsson, K. Dan. Vidensk. Selsk., Mat. Fys. Medd. 29, 16 (1955).

[3]J. P. Elliot, Proc. Roy. Soc. London, Sec. A 245, 128, 562 (1958).

[4]R. D. Ratnu Raju, J. P. Draayer, and K. T. Hecht, Nucl. Phys. A202, 433 (1973).

[5]J. P. Draayer, K. J. Weeks, and K. T. Hecht, Nucl. Phys. A381, 1 (1982).

[6]P. P. Raichev and R. P. Rusey, Yad. Fiz. 27, 1501 (1978) [Sov. J. Nucl. Phys. 27, 792 (1978)].

[7]B. R. Judd, W. Miller, Jr., J. Patera, and P. Winternitz, J. Math. Phys. (Cambridge, Mass.) 15, 1787 (1974).

[8]The breaking of seniority in these high-j orbitals plays a crucial role in the description of the backbending phenomena for high-spin states. The effects of that begin to appear in ^{164}Er at around 2 MeV excitation energy.

[9]This behavior is related to the description of the intrinsic nuclear shape. If the quadrupole operator is written in terms of β and γ intrinsic deformation variables, a third-order invariant in Q leads to a potential surface term of the form $\beta^3 \cos 3\gamma$. This term leads to an infinitely deep potential well for large β. It is necessary to have a fourth-order invariant ($\sim\beta^4$) to restore the stability of the intrinsic potential. See also G. Rosensteel and D. Rowe, Ann Phys. (N.Y.) 126, 343 (1980).

[10]T. S. Dumitrescu and I. Hamamoto, Nucl. Phys. A383, 205 (1982).

[11]S. W. Yates *et al.*, Phys. Rev. C 21, 2366 (1980), and references therein.

[12]A. Buyrn, Nucl. Data Sheets 11, 385 (1974).