Measurement of Neutron Quantum Interference in Noninertial Frames

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Neutron interferometry has been performed on an accelerated interferometer under conditions where gravitational effects on the interfering beams very nearly compensate each other. The observed neutron phase shift is found to be in excellent agreement with that calculated from Schrödinger's equation transformed to the accelerated frame where it is found essential to include the spherical wave aspect. The present experiment thus verifies the validity of the classical transformation laws for noninertial frames also in the quantum limit.

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The influence of gravity on the phase in a neutron interferometer was measured by Colella, Werner, and Overhauser in a series of beautiful experiments which have become known as the COW experiment.[1,2] In these investigations a single-crystal interferometer of Laue type[3,4] is used which is rotated about the incident beam so that the interfering beams travel at different height (and thus potential) in the gravitational field. From the observed interference pattern a gravity-induced phase shift is deduced which increases with increasing potential difference between the two interfering beams. Apart from corrections which were attributed to bending of the interferometer crystal, the major part of the observed phase shift could be calculated from Schrödinger's equation by introducing the varying gravitational potential $-m_g\vec{g}\cdot\vec{r}$ as an ordinary potential[5-8] into the equation. The conclusion drawn from the COW experiment was that the classical principle of equivalence has been experimentally proven to be valid also in the quantum limit.[8] This means that the laws of quantum physics are the same in a frame with gravitational potential $-m_g\vec{g}\cdot\vec{r}$ as in a corresponding frame lacking this potential but having acceleration $-\vec{g}$ instead. It was noted by COW themselves[8] that, strictly speaking, there is a missing link in the argument and that a complete test of the equivalence principle in the quantum limit would involve repeating the experiment in an accelerated frame of reference traveling in gravitation-free space. However, COW surmised such an experiment unnecessary "if we believe that the Schrödinger equation holds in an accelerated frame."[8] We considered this belief to be worth checking by experiment in a very direct manner.

In our experiment the interferometer is accelerated (while oscillating in the horizontal plane) and the neutrons are free from any force (potential) in that plane, while in the COW experiment the interferometer is at rest and the neutrons are subject to the gravitational potential. Thus our experiment yields complementary information.

With the D18 neutron interferometer at the Institute Laue-Langevin in Grenoble we noted the sensitivity of the interference pattern to vibrations.[9,10] Following these observations we designed a special traverse support for the interferometer crystal in order to expose it to controlled forced oscillations normal to the (220) Bragg planes that served for division and recombination of the interfering beams. The principal experimental setup is illustrated in Fig. 1. The traverse features leaf-spring guidance for smooth and practically frictionless movement. The interferometer performs sinusoidal oscillation when it is driven by a function generator via a pair of standard loudspeaker magnets the coils of which are coupled to the sliding part of the traverse. The leaf springs have a structure of special design by which spurious rotation is kept below the detection limit of 0.7 μrad. This is an essential point since rotation can cause an additional phase shift of its own.[8,11-14]

In a typical measurement, the intensity I_0 of the outgoing beam is measured in a stroboscopic manner at the inversion points of the oscillation, i.e., when the momentary acceleration $a_\pm$ corresponds to $a_+ = X_0\omega^2$ and $a_- = -X_0\omega^2$, respectively, where X_0 is the amplitude and ω the frequency of the oscillation. In order to obtain a complete

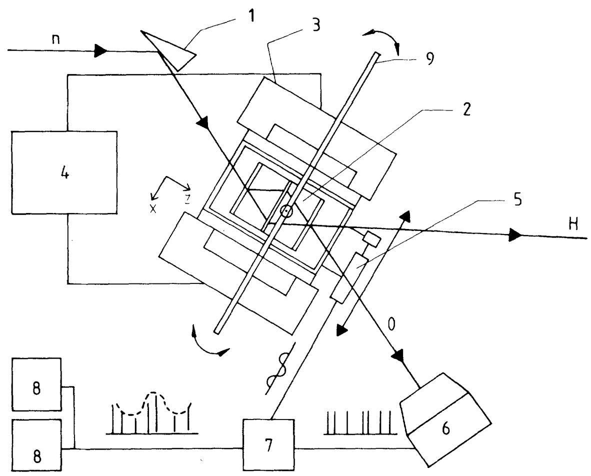

FIG. 1. Experimental setup. n, incoming neutron beam; 1, fore crystal; 2, interferometer on traverse; 3, loudspeaker magnets; 4, function generator; 5, position transducer; 6, neutron detector measuring I_0; 7, position-to-pulse-height converter; 8, pair of single-channel analyzers; and 9, Al phase-shifter plate.

train of interference fringes both for a_+ and a_- the measurement of I_0 is repeated at different angular positions of a stationary Al phase-shifter plate as indicated in Fig. 1. A typical pair of interferograms is shown in Fig. 2. A total of 45 pairs were measured at a constant wavelength $\lambda = 0.18305$ nm. Amplitudes of 10 to 45 μm and frequencies of 3 to 19 Hz were used. Acceleration-induced phase shifts covered the range ± 3 rad.

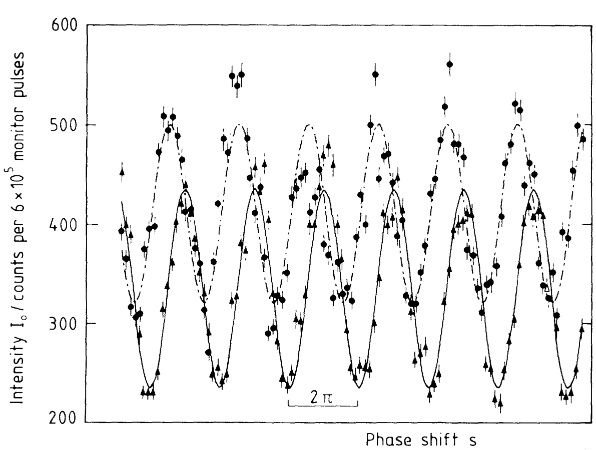

FIG. 2. Pair of interferograms taken with amplitude $X_0 = 10\,\mu$m and oscillation frequency $\nu = \omega/2\pi = 16$ Hz corresponding to an acceleration of $a_\pm = \pm 0.1$ m s^{-2} (a_+, triangles; a_-, circles). The curves have been fitted to the data points. The abscissa gives the phase shift introduced by the Al phase-shifter plate.

The measurements were evaluated by numerical fit of the function $f(s) = A + B\cos(s + C)$ to the data points. s is the phase shift generated by rotation of the Al phase-shifter plate. A, B, and C are fitting parameters. Let C_+ and C_- be the C parameters obtained for a pair of interferograms with accelerations a_+ and a_-, respectively, then $\beta \equiv \frac{1}{2}(C_+ - C_-)$ is the average phase shift caused by the acceleration of magnitude $a \equiv \frac{1}{2}(a_+ - a_-) = X_0\omega^2$. Fringes maintained reasonable contrast up to values of a of about 0.34 m s^{-2}. As is seen from the plot shown in Fig. 3, the phase shift β is proportional to the acceleration a over the covered range. The full line is the result of linearly averaging the measured points. Its equation is

$$\beta = 0.015(45)\ \text{rad} + [8.99(38)\ \text{rad m}^{-1}\ \text{s}^2]a. \quad (1)$$

In order to compare this result with theory we consider the single-particle Schrödinger equation for a crystal at rest in an inertial frame,

$$[-(\hbar^2/2m_i)\nabla^2 + V(\vec{r})]\Psi = E\Psi. \quad (2)$$

Here m_i is the inertial mass of the neutron and $V(\vec{r})$ the crystal potential responsible for diffraction. If the crystal is at rest in a noninertial frame with acceleration $\vec{a}$ then[7]

$$[-(\hbar^2/2m_i)\nabla^2 + V(\vec{r}) + m_i\vec{a}\cdot\vec{r}]\Psi = E\Psi \quad (3)$$

is Schrödinger's equation transformed according to the laws of the macroscopic principle of equivalence of noninertial frames.

On the other hand, if we write the acceleration of gravity as $\vec{g}$ and the gravitational mass as m_g,

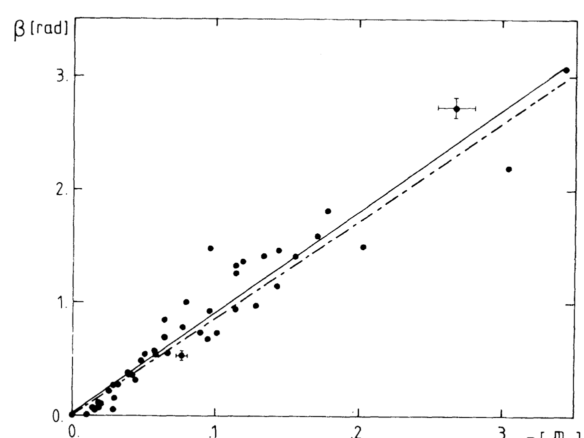

FIG. 3. Measured phase shifts β as function of acceleration a. Full curve is obtained by linearly averaging the data points; broken curve is phase shift calculated from dynamical diffraction.

and introduce the gravitational potential $-m_g\vec{g}\cdot\vec{r}$ into (2), we obtain in accordance with Werner[15]

$$[-(\hbar^2/2m_i)\nabla^2+V(\vec{r})-m_g\vec{g}\cdot\vec{r}]\Psi=E\Psi. \qquad (4)$$

In (3) $-\vec{a}$ is the acceleration seen from the accelerated frame, so that $-\vec{a}$ corresponds to $\vec{g}$ in (4). Consequently (3) and (4) are strictly equivalent if and only if $m_i=m_g$.

We have calculated the phase shift due to acceleration starting from (3) which yields corrected fundamental equations and dispersion surfaces of dynamical diffraction as was pointed out by Werner.[15] Within the crystals we have taken into account the tiepoint migration of dynamically diffracted wave fields under the action of the acceleration $\vec{a}$, an effect which is very similar to the tiepoint migration occurring in a slightly deformed crystal lattice. Directly connected with the tiepoint migration is a gradual change of the wave vectors of the plane-wave components of the wave fields.

In the evaluation of the COW experiment[5-8] this effect has not been taken into account so far, although it is quite important since it not only contributes to the phase shift, as we show below, but also explains[16] the fading of contrast observed with increasing acceleration.[6,8] Furthermore, we have calculated the phase shift in the more realistic case of spherical waves by using the theory for the zero absorption Laue-case interferometer as developed by Bauspiess, Bonse, and Graeff.[17]

We describe the mentioned change of wave vectors as a function of the usual normalized angle y which is proportional to the deviation $\Delta\Theta$ from the exact Bragg angle Θ_B:

$$y=-\Delta\Theta\sin 2\Theta_B/|\chi_1|, \qquad (5)$$

where χ_1 is the Fourier component of order 1 of $V(\vec{r})$. In an accelerated frame y changes according to[16]

$$dy=\tan\Theta_B(2am_i^2/h^2k^2|\chi_1|)dz, \qquad (6)$$

when the waves propagate by dz in the direction z which is normal to the crystal wafers (Fig. 1).

The change in y is the same within the crystals and on the path sections between them; and integration over sections inside and outside the crystals leads in the first place to a phase shift β_0 which was also calculated by Greenberger and Overhauser[7] and alone taken into account,

$$\beta_0=(\tan\Theta_B/\cos\Theta_B)(4\pi am_i^2/h^2k)Z_w(Z_w+t), \qquad (7)$$

where t is the wafer thickness and Z_w the wafer distance. In the second place, however, integration in the crystals results in an additional phase shift β_d due to the y-dependent interference of the wave fields belonging to different branches of the dispersion surface. Taking into account both β_d and β_0, we find for the interferometer which we used [$t=0.435\,54(8)$ cm, $Z_w=2.72936(9)$ cm]

$$\beta\equiv\beta_0+\beta_d=(8.6545\text{ rad m}^{-1}\text{ s}^2)a. \qquad (8)$$

The broken curve in Fig. 3 corresponds to Eq. (8).

As is seen, (8) is very well confirmed by the measurements. On the other hand, neglect of the dynamical interbranch phase effects β_d yields

$$\beta=\beta_0=(7.841\text{ rad m}^{-1}\text{ s}^2)a, \qquad (9)$$

which obviously differs significantly from the measurements.

The ratio β_d/β_0 increases as t/Z_w becomes larger. For our interferometer t/Z_w equals 0.16. The interferometer used in the COW experiment had[8] $t/Z_w=0.07$ so that the influence of dynamical diffraction effects is smaller but still not negligible. In fact when we calculate the phase shift including the dynamical phase effect we find excellent agreement with the measured values of COW. The remaining small difference between theory and experiment might be due to bending of the COW interferometer.[6,8] In our experiment bending could not occur because the acceleration was acting parallel to the crystal wafers. It thus confirms the necessity to consider dynamical effects. Furthermore, our treatment is capable of explaining[16] the contrast fading observed in the COW experiment.

In conclusion, by interferometry with an accelerated interferometer we have experimentally verified the validity of the classical transformation laws for noninertial frames in the quantum limit. So far the experimental accuracy obtained is about 4%. By improving the oscillation traverse and the stroboscopic observation technique we expect to improve the accuracy to better than 1%. The result is within 3.7% of the theoretical prediction following from a theory taking into account the spherical wave aspect as well as dynamical phase effects within the interferometer crystal.

The measurement of the Sagnac effect by Werner, Staudenmann, and Colella[13] is related to our experiment in that it also is neutron interferometry in an accelerated frame. However, the acceleration depends on the direction of the neutron velocity in the rotating frame and is different for the two directions in which the beams propagate. In contrast to this, our experiment is performed in a system of constant acceleration and is thus the direct counterpart to the COW experiment.

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