## Crossover of Fractal Dimension in Diffusion-Limited Aggregates

In a series of recent articles Witten and Sanders[1] and Meakin[2] have carried out detailed simulations of diffusion-limited aggregation. Most simulations were done on a lattice, with the diffusing particle randomly changing its direction at *every* site. When the particle reaches a site adjacent to the aggregate, it "sticks" and becomes a member of the aggregate. These simulations yield highly ramified structures, with fractal dimension $D \simeq 1.70$ (for Euclidean dimension $d=2$).

A more general model of diffusion-limited aggregation should take into account *two* distinct length scales. One is the "sticking length" $a$ of the order of the size of the diffusing particles; a particle sticks when its center is at a distance $a$ from that of a member of the aggregate. Another length of importance is the mean free path $l_0$ of the diffusing particles. All previous simulations assumed $a = l_0$ (=1 lattice spacing; simulation of a continuum model[3] was also done with the same assumption). However, if one allows $a < l_0$ (which may be relevant for some physical applications) structures that exhibit crossover of the fractal dimension should be obtained. Since on scales $l$, $a < l < l_0$, aggregation can be described by ballistic motion of the incoming particles, we considered aggregation (in $d=2$), generated by particles that move on straight lines, with uniformly distributed orientation and impact parameter (with respect to a "seed" placed at the origin). A particle sticks when its trajectory first crosses a bond which has an occupied site at (at least) one of its ends.[4] Clusters of 80 000 sites are generated in less than one minute on the IBM 3061. A growth sequence of one such cluster is shown in Fig. 1. We plotted $\ln N(R)$ vs $\ln R$, where $N(R)$ is the number of occupied sites $i$ at distances $R_i \leq R$ from the origin. $D$, defined by $N(R) \sim R^D$, was calculated by discarding a few small and a number of large $R$ values; we find $D \simeq 1.93 \pm 0.02$. An actual simulation of diffusion-limited aggregation, with $a \ll l_0$, should therefore give rise to aggregates with high fractal dimension on local scales $a < l < l_0$, and lower $D$ at scales $l > l_0$. This crossover effect is different from that reported recently by Witten and Meakin,[5] who studied aggregation in the presence of randomly distributed seed sites, leading to structures with low local $D < d$, and $D = d$ at large scales.

It should be noted that since the aggregate generated on small scales may be influenced by screening due to large branches, our simulation (which assumes uniform distribution of the direction of incidence) may serve only as an estimate of the local $D$.

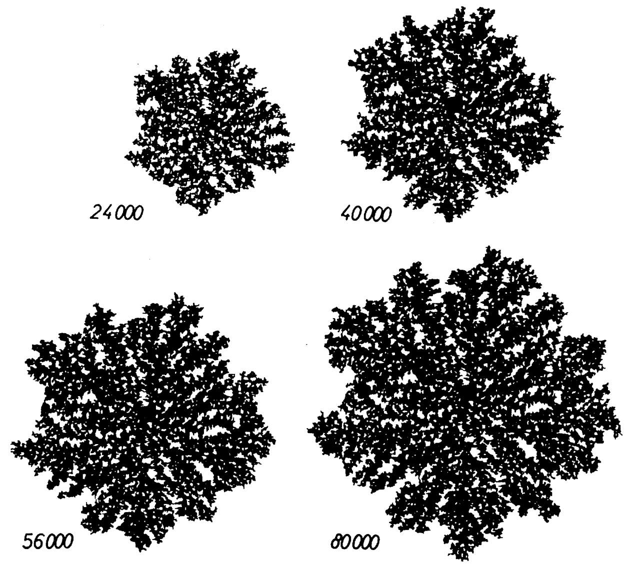


FIG. 1. Growth sequence of a cluster; number of sites on figures.

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[2]P. Meakin, Phys. Rev. A 27, 1495, 2616 (1983).

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[4]This model was considered by Vold and by Sutherland, and discussed in a recent preprint by Meakin, which we received after completion of this work.

[5]T. A. Witten, Jr., and P. Meakin, National Science Foundation–Institute for Theoretical Physics Report No. NSF-ITP-83-82 (unpublished).