Magnetic Vortices in Rotating Superfluid ^{3}He-B

P. J. Hakonen, M. Krusius, M. M. Salomaa, and J. T. Simola

Low Temperature Laboratory, Helsinki University of Technology, SF-02150 Espoo 15, Finland

and

Yu. M. Bunkov

Institute for Physical Problems, U.S.S.R. Academy of Sciences, 117334 Moscow, U.S.S.R.

and

V. P. Mineev and G. E. Volovik

L. D. Landau Institute for Theoretical Physics, U.S.S.R. Academy of Sciences, 117334 Moscow, U.S.S.R.

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Rotating superfluid ^{3}He-B is found to possess a new contribution to the NMR frequency shift, which changes sign on reversal of either the angular velocity of rotation or the magnetic field. For $p = 29.3$ bars this gyromagnetic effect shows a discontinuity in magnitude at the first-order phase-transition temperature $T/T_c = 0.6$, at which a change in the vortex-core structure takes place. These observations support the conclusion that the vortex core possesses a spontaneous intrinsic magnetization.

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A wealth of new fundamental phenomena pertaining to vorticity in anisotropic superfluids has been discovered in recent NMR measurements on rotating superfluid ^{3}He.[1] Here we report first measurements on gyromagnetism in superfluid ^{3}He-B. The physical effect is quite unique and has no counterpart in superfluid ^{4}He. In contrast to usual gyromagnetic features, the effect in ^{3}He-B is inherently related to the peculiar symmetry break in the p-wave paired condensate. Gyromagnetism is observed in the measurement via the NMR frequency shift, which depends on the relative directions of the angular velocity of rotation $\vec{\Omega}$ and the applied magnetic field $\vec{H}$. The gyromagnetic contribution to the frequency shift is interpreted as arising from the spontaneous magnetic moment of vortices. This would imply that at least a fraction of the vortex core consists of liquid ^{3}He in a nonunitary pairing state, significantly different from the isotropic Balian-Werthamer state.

The equilibrium B phase is described by the order-parameter matrix $A_{i\alpha} = \Delta(T) R_{i\alpha}(\hat{n}, \theta) e^{i\varphi}$, where $\Delta(T)$ is the energy gap and φ is an arbitrary phase factor. This state is obtained from the simplest $^3P_0$ state, which corresponds to the order parameter $A_{i\alpha} = \Delta(T)\delta_{i\alpha}e^{i\varphi}$, by a relative rotation of spin (α) and orbital (i) coordinate spaces through an angle θ about an axis $\hat{n}$, described by the matrix $R_{i\alpha}(\hat{n}, \theta)$. Therefore, superfluid ^{3}He-B manifests a subtle broken relative spin-orbit symmetry. The angle θ minimizing the nuclear dipole interaction is $\theta = \arccos(-\frac{1}{4})$.

The experimental quantity of interest here is the NMR frequency shift $\Delta\nu$ which in the B phase measures the angle β between the anisotropy axis $\hat{n}$ and the external field. The frequency shift in the NMR spectrum thus reflects the $\hat{n}$ texture and allows one to deduce the symmetries and relative magnitudes of the different textural free-energy contributions.[2] These include the magnetic field term $F_H = -a(\hat{n}\cdot\vec{H})^2$, the bending free energy F_B, and the surface term F_S, which provides the boundary conditions. In the high-field limit, appropriate to the present measurements performed at 28.4 mT, $F_S = -d(\hat{s}_i R_{i\alpha} H_\alpha)^2$, where $\hat{s}$ is the surface normal.

During rotation the liquid is threaded by a lattice of vortices whose equilibrium density equals $n_V = 2\Omega/(h/2m_3)$. At the angular velocities $\Omega \sim 1$ rad/s employed, the intervortex distance is less than the textural bending length ξ_H. Hence the additional free-energy term due to rotation may be described by an averaged orientation of $\hat{n}$ (see Gongadze, Gurgenishvili, and Kharadze[3]):

$$F_V = \tfrac{2}{5} a\lambda(\hat{\Omega}_i R_{i\alpha} H_\alpha)^2, \quad (1)$$

where the vortex parameter λ is proportional to n_V and thus to Ω.

The present measurements are performed on a cylindrical sample of ^{3}He with radius $R = 2.5$ mm and with the axis aligned with $\hat{\Omega}$.[4] In Figs. 1 and 2 the frequency shifts of the measured NMR absorption peaks are presented as functions of reduced temperature. Typical NMR signals illustrating the appropriate absorption peaks are also

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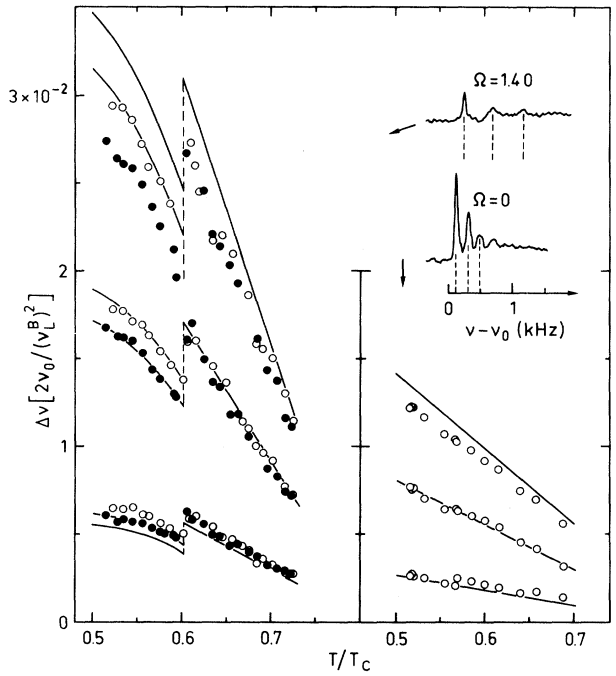

FIG. 1. The lowest three spin-wave resonance frequencies measured at 29.3 bars in the stationary state (right-hand side) and during rotation at $\Omega = 1.40$ rad/s (left-hand side): open circles, $\vec{H}$ parallel with $\vec{\Omega}$; closed circles, $\vec{H}$ antiparallel with $\vec{\Omega}$; solid lines are the calculated eigenfrequencies. The inset illustrates the measured NMR absorption spectrum at $T/T_c = 0.51$.

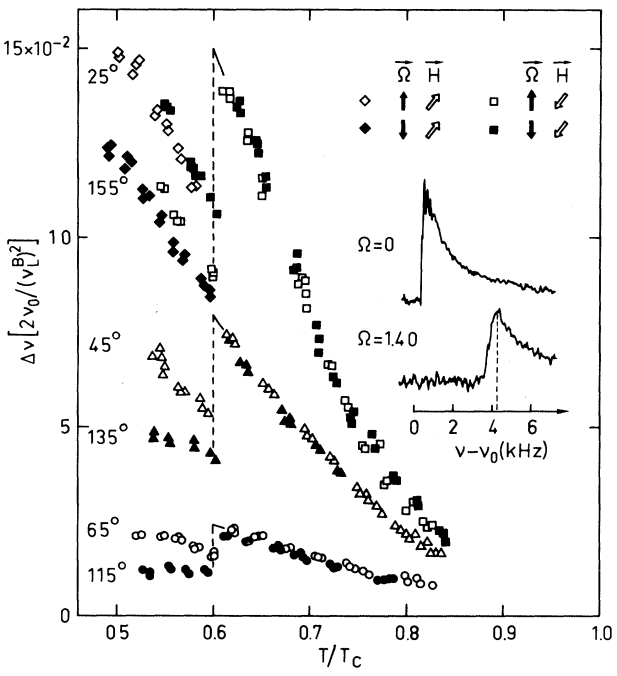

FIG. 2. The frequency shift of the NMR absorption peak in a tilted magnetic field, measured at $\Omega = 1.40$ rad/s. The angle μ between $\vec{\Omega}$ and $\vec{H}$ is quoted. The open and filled symbols represent forward and reverse rotation, repsectively. For $\mu = 25°$ and 155°, the directions of $\vec{\Omega}$ and $\vec{H}$ are shown by the arrows. The NMR signals in the inset were measured at $T/T_c = 0.53$ with $\mu = 25°$.

shown. The shift $\Delta\nu$ from the Larmor frequency ν_0 is given in terms of the Leggett frequency $\nu_L{}^B$. The pressure in these measurements is 29.3 bars and T_c is 2.72 mK.

Two outstanding features are common to Figs. 1 and 2: (1) In the rotating state there occurs a jump at $T = 0.6T_c$ in the magnitude of the frequency shift, which implies a discontinuity in the vortex free energy F_V. This phenomenon represents a first-order phase transition in the structure of the vortex core since it displays the properties expected of a transition associated with each individual vortex, but not of the vortex lattice as a whole.[1] The transition temperature is pressure dependent and runs almost parallel with the A-B phase boundary. (2) The NMR frequency shift depends on the directions of rotation and magnetic field: The same net effect results by reversing $\vec{H}$ or $\vec{\Omega}$. The symmetry of this gyromagnetic frequency shift is described by a textural free energy term, which is odd in $\vec{\Omega}$ and $\vec{H}$,

$$F_{gm} = \tfrac{4}{5} a \kappa (\hat{\Omega}_i R_{i\alpha} H_\alpha), \qquad (2)$$

proposed by Volovik and Mineev.[5] The gyromagnetic frequency shift also displays a discontinuity at the vortex-core transition, as is apparent from Figs. 1 and 2. We conclude that both F_V and F_{gm} depend on the detailed structure of the vortex core.

In Fig. 1 the NMR frequency shifts correspond to a series of absorption maxima separated by roughly equal spacing. These peaks are resonances due to the excitation of spin waves localized on the axially symmetric "flare-out" texture.[1,2]

The flare-out texture is found by numerical minimization of the textural free energy with appropriate boundary conditions for the $\hat{n}$ vector. Then the spin-wave spectrum is solved and compared to the experimental result.[6] The calculated frequency of the second eigenmode in the stationary state is first adjusted by varying the magnetic bending length ξ_H. The only other parameter involved is the dipolar coherence length ξ_D which sets the length scale in the spin-wave equation. The results for the stationary state are shown in

Fig. 1 on the right-hand side as solid lines. In the temperature interval of our data the ξ_H obtained by the fitting procedure is well approximated by $\xi_H = 0.67[\epsilon/(1-0.91\epsilon)]^{1/2}$ mm where $\epsilon = 1 - T/T_c$. The deviations of the measured and calculated spin-wave spectra are attributed to the fact that a fixed estimated value $\xi_D = 10$ μm was used.

The values of λ and κ which were found by fitting the spin-wave spectrum for the rotating case are shown in Fig. 3 as solid lines. Both λ and κ are proportional to Ω.

These parameters can also be derived from measurements of the frequency shifts in a magnetic field tilted with respect to $\hat{\Omega}$.[4] The data in Fig. 2 represent the overall shift of the main absorption peak (cf., the inset) which arises from a reorientation of the $\hat{n}$ texture in the bulk liquid during rotation. The normalized NMR shift equals $\sin^2\beta$, which in the bulk is found by minimizing the energy terms $F = F_H + F_V + F_{gm}$, neglecting finite-size effects. This leads to the following condition on the $\hat{n}$ orientation[5]:

$$\lambda\left[u\cos 2\mu \pm \frac{u^2 - \frac{1}{2}}{(1-u^2)^{1/2}}\sin 2\mu\right] + \frac{\kappa}{H}\left[\cos\mu \pm \frac{u}{(1-u^2)^{1/2}}\sin\mu\right] = 1, \quad (3)$$

where μ is the tilting angle of the magnetic field $\vec{H}$ with respect to $\vec{\Omega}$ and $u = 1 - \frac{5}{4}\sin^2\beta$. By use of this expression the data of Fig. 2 have been converted to values of λ and κ; the results are shown in Fig. 3.

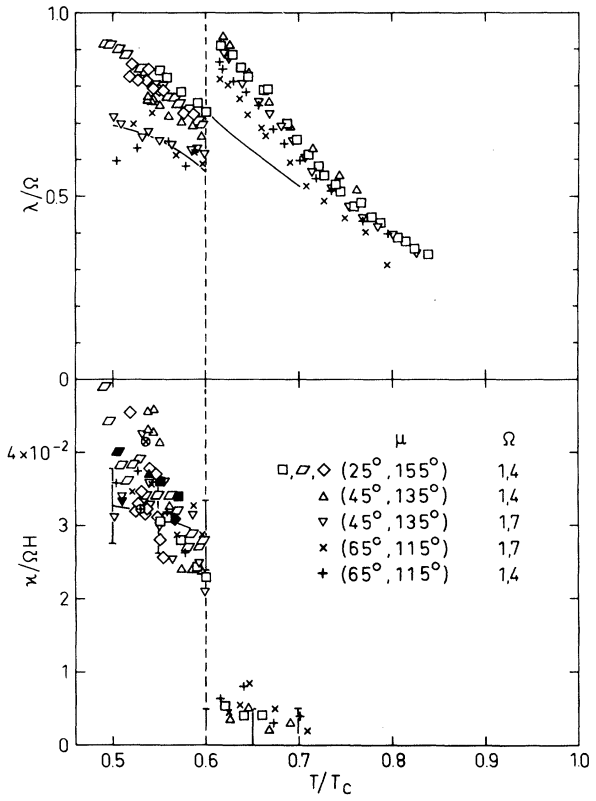

FIG. 3. The normalized vortex and gyromagnetic parameters λ/Ω [Eq. (1)] and $\kappa/\Omega H$ [Eq. (2)] in s/rad. The solid lines correspond to the calculated spin-wave eigenfrequencies shown in Fig. 1, with error bars indicated for κ. For $T/T_c > 0.6$, κ is so small that it cannot be resolved from the spin-wave analysis. The open data points represent tilted field measurements of Fig. 2 after use of Eq. (3) to convert the frequency shifts to values of λ and κ. The different symbols refer to separate experiments with the parameters as quoted; filled symbols indicate measurements as a function of Ω at constant T.

Referring to Fig. 3, where the data obtained at different field orientations lie approximately on a single curve for λ/Ω, we conclude that neglecting the boundary effect is not crucial, especially for T close to T_c, where $\xi_H \ll R$.

Here we suggest an explanation for these gyromagnetic phenomena: In the Ginzburg-Landau approach the gyromagnetic term [Eq. (2)] may be ascribed to the properties of both the bulk liquid and the vortices, i.e., $\kappa = \kappa^{\text{bulk}} + \kappa^{\text{core}}$.

The vortex contribution to F_{gm} is a consequence of an intrinsic magnetic moment which is concentrated inside the vortex core, where the order parameter is nonunitary. The possible nonunitary nature of vortices in a $^3P_2$ paired superfluid state, which may exist inside neutron stars, has recently been discussed by Sauls, Stein, and Serene.[7] Simple symmetry arguments which use the relative spin-orbital symmetry breaking show that nonunitarity also takes place inside the cores of vortices in the B phase, producing a magnetic-moment density $\sim(\Delta^2/\epsilon_F)(\chi_n/\gamma\hbar)$, analogously to neutron stars.[7] Here χ_n is the magnetic susceptibility of normal ^{3}He, ϵ_F is the ^{3}He Fermi energy, and γ is the gyromagnetic ratio of a ^{3}He nucleus. Multiplying this by the cross-sectional area of the core $S \sim \pi\xi^2$, where $\xi \sim \hbar v_F/\Delta$ denotes the coherence length, one obtains for the magnitude of the magnetic moment per unit length $|M_V| \sim \chi_n \hbar/\gamma m_3$. If the vortex is axially symmetric, $\vec{M}_V$ is directed along $R_{ic}\hat{z}_i$ where $\hat{z}$ is the axis of the vortex, i.e., in this case $\hat{z} = \hat{\Omega}$. Thus the gyromagnetic energy due to the vortices with density n_V is $F_{gm}^{\text{core}} = -n_V \vec{M}_V \cdot \vec{H}$; accordingly, $\kappa^{\text{core}} \sim \chi_n \Omega/\gamma a$, which is of the same order of magnitude as

the experimental data for κ.

The bulk-liquid contribution to the gyromagnetic energy[5] is due to the orbital momentum of Cooper pairs.[8] This momentum results from the rigidity of the quantum state for the Cooper pairs, which in the B phase is the eigenstate of the generalized total momentum $\hat{\vec{J}}_i = \hat{\vec{L}}_i + R_{i\alpha}\hat{\vec{S}}_\alpha$ with zero eigenvalue. Here $\hat{\vec{L}}$ and $\hat{\vec{S}}$ are the angular and spin momentum operators. In an applied magnetic field a spin density of Cooper pairs $\vec{S}^p = (\chi^p/\gamma)\vec{H}$ (χ^p is the pair spin susceptibility) is induced which leads to the pair orbital momentum density $L_i{}^p = -(\chi^p/\gamma)R_{i\alpha}H_\alpha$.[8] This term ensures that $\vec{J}$ remains zero and yields under rotation the gyromagnetic energy $F_{gm}{}^L = -\vec{L}^p \cdot \vec{\Omega}$ with $\kappa^L = 5\chi^p\Omega/4a\gamma$. However, one may show[9] that this effect reduces to a surface effect rather than a bulk one. This feature reflects a fundamental property of the pair orbital momentum in superfluid ^{3}He. As a result of the large overlap of the Cooper pairs, whose size $\propto \xi$ is much larger than the interatomic spacing b, i.e., $b/\xi \sim T_c/\epsilon_F$, their motion with the orbital angular momentum $\vec{L}^p$ essentially transforms to the center-of-mass motion of the pairs on the surface of the container; this has no orientational effect in the bulk liquid. Only a tiny fraction of the orbital momentum $\vec{L} \cong (b/\xi)^2\vec{L}^p$ is associated with the intrinsic rotation of the Cooper pairs and is responsible for the bulk-energy term $-\vec{\Omega}\cdot\vec{L}$.[10] Thus $\kappa^{\text{bulk}} \sim (T_c/\epsilon_F)^2\chi^p\Omega/\gamma a$ and may be neglected in comparison with κ^{core}.[11]

We have presented the first observation of gyromagnetism in an anisotropic superfluid, which strongly supports the conclusion that a spontaneous magnetic moment[12] is concentrated in the cores of quantized vortex lines in ^{3}He-B.

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[10]A parallel situation prevails in the A phase (Ref. 9): Although all the Cooper pairs posses the same orbital angular momentum $\hbar\vec{l}$, where $\vec{l}$ is the axis of quantization, the major part of this momentum density $\vec{L} = (\rho_s/2\hbar)\vec{l}$ transforms to a center-of-mass motion of the pairs at the surface of the container and only a fraction $\vec{L}_0 \sim (T_c/\epsilon_F)^2\vec{L}$, which is called the intrinsic orbital momentum in the A phase, corresponds to the relative rotation of Cooper pairs.

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