Characteristic Lengths in the Wavy Vortex State of Taylor-Couette Flow

Kwangjai Park, Gerald L. Crawford, and Russell J. Donnelly
Department of Physics, University of Oregon, Eugene, Oregon 97403
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The wavy vortex state of Taylor flow has been examined with use of a novel linear optical scanning apparatus which allows the wavy vortex flow to be studied in considerable detail: Measurements of the wavy mode amplitude and pair-size distribution show that characteristic lengths scale with the size of the apparatus.

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There has been increasing interest recently in studying the wavy vortex state in Couette flow.[1-4] This effect is interesting because it is a more complicated instability than Taylor vortex flow, evolves from it at slightly higher Reynolds number, yet is close enough to the Taylor vortex state that one believes progress can be made in understanding the behavior of the flow with relatively simple mathematical models.[5] Coles first characterized this flow by means of the number of vortex pairs N in the flow and the number of waves m around the annulus.[1] As the Reynolds number R increases beyond the onset for the wavy mode R_w it is found that the time-averaged pair spacing becomes nonuniform through the flow[3] and that N can change spontaneously by destruction or creation of new pairs. A mechanism for this change was proposed by Ahlers, Cannell, and Lerma[3] and a different mechanism was recently proposed by Park and Crawford.[6]

The pair-size distribution reported by Ahlers, Cannell, and Lerma[3] shows a variation of pair size over a rather long length scale. This observation has stimulated Brand and Cross[5] to advance dynamical equations for wavy vortex flow with solutions containing a penetration depth or a healing length of the order of 10 times the gap size d. We have assembled a 256-channel optical scanning device which allows us to look at the wavy flow with unprecedented precision. We report here new measurements which show no evidence for such a healing length; indeed it appears that the operative characteristic length is the total length L of the apparatus.

Our apparatus consists of cylinders of radii $R_i = 2.235$ cm and $R_2 = 2.54$ cm with aspect ratio Γ $(= L/d)$ made variable by adjusting the position of a nonrotating plug at the top of the flow. The fluid consists of glycerol and water solutions with kinematic viscosity $\nu = 4.99$ cS at 20 °C, with Kalliroscope tracer added at a few percent by volume, carefully adjusted to emphasize the location of the inflowing pair boundaries. The outflowing boundaries have a less stable optical signature and are out of phase with the inflowing boundaries. Rotation rates Ω are set by a programmable frequency synthesizer, the temperature monitored by a Hewlett-Packard model 2804A quartz thermometer, and reflected light from the flow focused on a Fairchild CCD 111A linear optical array. The output of the array is digitized and stored on a flexible disk. Because of speed limitations of our interface bus, we take a 4-ms exposure of one axial slice of the flow pattern and allow the pattern to rotate 342° before making the next exposure. We take forty pictures of somewhat more than one vortex pair in evenly spaced time intervals while the pattern revolves 38 times. From these forty pictures, by automated analysis, we obtain data such as are shown in Fig. 1. These data can be fitted by sinusoidal functions, and from these fits we can accurately deduce the pair spacing and the apparent wavy mode amplitudes. We move the array downwards and repeat the process until we have a high-resolution picture of the entire flow.

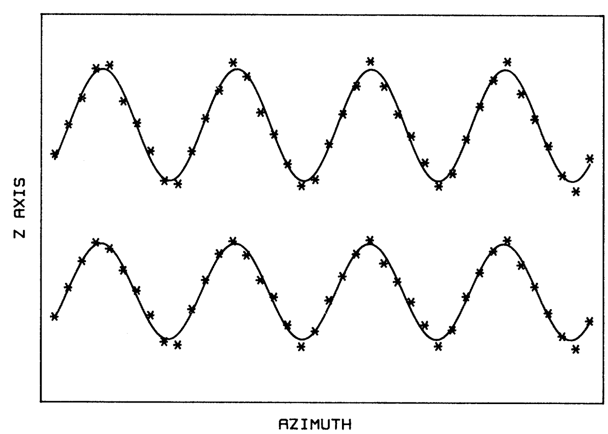

FIG. 1. Locations of two adjacent-inward flowing pair boundaries as a function of azimuthal position. Forty pairs of data points and their least-squares fits are shown.

This procedure gives us an axial resolution of 0.005 cm and a dynamic range equivalent to twelve-bit accuracy (as contrasted to four or six bits for video scanners).

We show in Fig. 2 the pair-size distribution for a flow at $R = \Omega R_1 d/\nu = 159$. (For our geometry, Taylor vortex flow begins at $R_c = 123$, and wavy flow at $R_w = 140$.) We see that the pair-size distribution has a minimum about 5 pairs of cells from each end. These data were obtained by use of a ramping rate $a^* = dR/dt^* = 0.1$, where $t^* = t/(Ld/\nu)$, $N = 33$, and $m = 2$.[7] If a ramping rate of $a^* = 15$ (150 times the quasistatic rate) is used to traverse across R_w, a wavy mode with $m = 3$ rather than 2 is produced in our apparatus. If a wavy mode state with $m = 2$ is first established just above R_w under the quasistatic criterion, and is then ramped to a higher Reynolds number state with a rapid acceleration, the $m = 2$ mode is retained, and a nonequilibrium pair-size distribution is obtained. This distribution relaxes over a period of time to a distribution identical to that obtained under the quasistatic procedure. The relaxation time in this case is highly history dependent. Figure 3 compares two such distributions at $R = 159$ with $N = 33$, where $a^* = 0.1$ in the quasistatic case and $a^* = 15$ in the nonequilibrium case. The nonequilibrium distribution has much more scatter than the equilibrium and is also quite flat in the central portion. This flatness could be misinterpreted as suggesting the existence of a penetration length, when in fact it is only an artifact of a nonequilibrium situation.

The question of the existence of a penetration length cannot be settled by observations at a single aspect ratio. We have, therefore, examined three aspect ratios, $\Gamma = 30.89$, 53.90, and 70.41. In order to extract characteristic lengths from the data we have decomposed the pair-size distribution as shown in Fig. 2 into the sum of a Gaussian function in the main part of the cylinder,

$$a_1 = A \exp(-z^2/\sigma^2), \quad -L/2 < z < L/2,$$

and a decaying exponential at each end of the cylinder,

$$a_2 = B \exp[-(L/2 - |z|)/\alpha] + C,$$

where A, B, C, α, and σ are parameters of the fit. We attach no theoretical significance to our choices of the forms of a_1 and a_2.

Our results show that a_1 is by far the dominant term in the pair-size variation. Results for several of our measurements are summarized in Table I. Here, $R = \Omega R_1 d/\nu$, and χ^2/n is the reduced chi-square goodness of fit, with n the number of degrees of freedom.[8] The data of Table I show that σ increases with L, and that σ/L is a constant within experimental error.

The determination of α is severely limited—it can rest on as few as two points. The evidence, poor as it is, tends to support the idea that α/L is also a constant, but that result cannot be established statistically beyond question.

The conclusion we wish to draw from this experiment is that if the Reynolds number is ramped sufficiently slowly to establish what Benjamin[9] has called the primary flow ($a^* = 0.1$ in this case), the effects of the ends of the apparatus extend

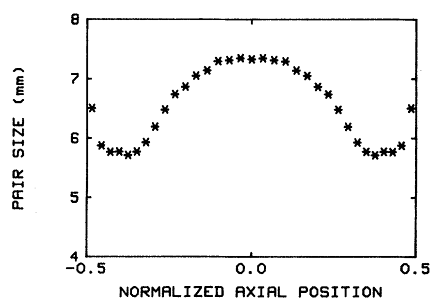

FIG. 2. Distribution of pair size in the cylinder for $R = 159$, $N = 33$. The data in this figure have been folded from $z = 0$. We have ascertained that under normal circumstances the distribution is symmetric about the center. Because of the large amounts of data required to obtain each distribution, most runs cover only two-thirds of the cylinder length.

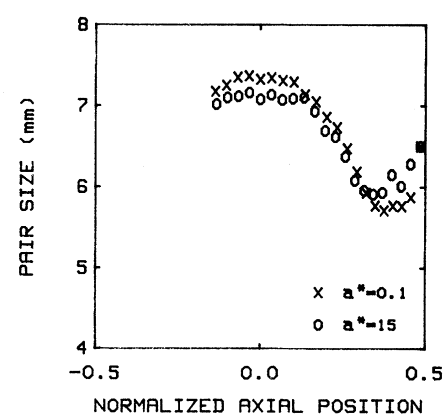

FIG. 3. Comparison of pair-size distributions for "slow" ($a^* = 0.1$) and "fast" ($a^* = 15$) ramping ($m = 2$ in both cases). Note that rapid ramping results in more scatter in the data and a roughly uniform pair-size distribution in the central region.

TABLE I. Experimental data for three aspect ratios on the characteristic length of the Gaussian pair-size distribution a_1.

L (cm)	Γ	N	R	σ (cm)	σ/L	χ^2/n
9.42	30.89	14	159 ± 0.5	2.32 ± 1.32	0.246 ± 0.140	0.25
16.44	53.90	23	149 ± 0.5	4.14 ± 3.93	0.250 ± 0.239	0.35
21.48	70.41	33	149 ± 0.5	6.40 ± 2.57	0.298 ± 0.120	0.20
21.48	70.41	34	149 ± 0.5	5.40 ± 1.76	0.251 ± 0.082	0.36
21.48	70.41	35	149 ± 0.5	5.27 ± 2.66	0.245 ± 0.124	0.15

throughout the flow: There is no penetration or healing length at each end of the flow with uniform pair spacing inside. Previously reported experiments on nonwavy Taylor vortex flow point to the same conclusion: (i) Experimental observations for Taylor vortices show that for the range of lengths in which a state characterized by N pairs is stable, the equilibrium pair size is uniform throughout the cylinder (with the exception of one half-pair at each end).[10] (ii) If the plug position is changed inside this range, the effect is felt throughout the length, and a different uniform pair size is established when equilibrium is reached.[10] (iii) The amount of time needed to reach this equilibrium involves the entire length L of the apparatus.[11]

The situation which emerges from recent experiments is clear. When the primary flow is set up by ramping the Reynolds number quasistatically, the effects of the ends are felt throughout the apparatus in both Taylor vortex and wavy vortex flow. In wavy vortex flow we find no evidence for a "healing length" of order $10d$ unless we ramp too rapidly or are too close to a transition. The necessity for quasistatic ramping requires an inconceniently long experiment for an apparatus of even quite modest aspect ratio. If one ramps too quickly, the flow will be in the midst of a transient relaxation (of perhaps quite long duration), and the results will not, in general, be reproducible from one laboratory to another. There is a growing suspicion that results will be sensitive to the radius ratio R_1/R_2 as well. These rather unwelcome conclusions likely account for the apparent conflicts in observations by different investigators over the years, and may call for more realistic theories than have been developed to date.

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