Magnetic Surface Waves in Plasmas

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The existence of a new type of surface wave in plasmas is demonstrated. These waves are intimately connected with the self-generation of magnetic fields in the laser-plasma interaction. The waves resemble waveguide modes in that a number of discrete modes can exist. The modes are localized to within a collisionless skin depth of the surface and, in the collisionless fluid limit, there is no restriction on the distance the waves can propagate.

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Spontaneously generated magnetic fields in laser-produced plasmas have been observed for many years.[1-3] These observations, along with their obvious impact on the inertial-confinement fusion program, have been the motivation for the many papers that have appeared on the subject in the last decade.[4-8] Transport of energy along surfaces,[8,9] anomalously fast plasma blowoff,[8] and insulation of the laser-heated electrons from the target interior[7,8] (known in the laser fusion community as flux limitation) have all been attributed to properties of self-generated magnetic fields. All these phenomena require sharp discontinuities in plasma properties (e.g., density, temperature, and atomic charge) for their existence. Therefore, the understanding of the normal surface modes in a plasma is crucial to the understanding of these phenomena. In this Letter, I demonstrate the existence of an entirely new set of plasma surface modes. It will be shown that (1) the self-generated magnetic field plays an essential role in the propagation of these waves; (2) a number of discrete modes exist, as in a waveguide; (3) in the collisionless fluid limit, there is no restriction on the distance the waves can propagate; (4) the waves are localized around the surface on scale lengths of the order of a collisionless skin depth; and (5) the phase and group velocities are very dependent on the density and temperature profiles at the surface. While this work has been motivated by programmatic aspects of the inertial-confinement fusion program, it is felt that the results are quite general and applicable to any plasma that contains sharp density and/or temperature gradients.

We choose a density profile similar to the one illustrated in Fig. 1(a). In regions A and C we require the density gradient scale lengths to be large compared with the scale length of the density jump in region B. We will permit the density to vary in the x direction only. The ions are assumed to be cold and fixed. Quasi charge neutrality is also assumed. Collisions have been neglected. The temperature profile is permitted to be arbitrary and no heat flux is permitted. We choose the magnetic field to lie in the z direction and to vary only as a function of x, y, and time. We will look for waves localized in x around region B and propagating in the y direction. The equations for the electron hydrodynamics are

$$\nabla \cdot n\vec{v} = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\vec{\psi} - \vec{\Omega}) - \nabla \times \vec{v} \times (\vec{\psi} - \vec{\Omega}) = -\nabla \times \frac{1}{mn} \nabla p, \tag{2}$$

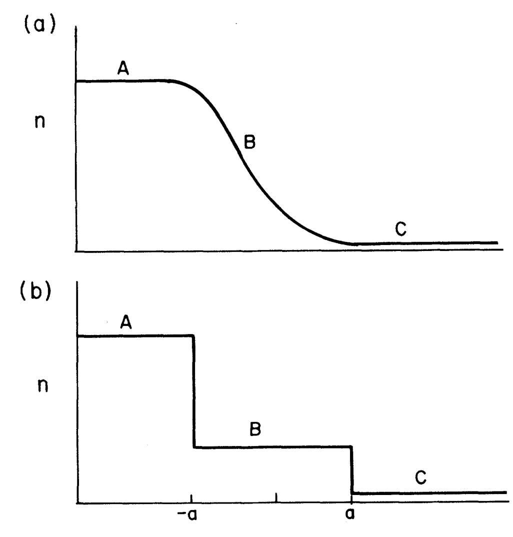

FIG. 1. (a) Density profile. The surface, region B, separates region A from region C. (b) Density profile for analytic solution. In region B the profile is nearly flat.

and

$$\frac{\partial(\epsilon/n)}{\partial t}+\vec{v}\cdot\nabla\left(\frac{\epsilon}{n}\right)=-p\vec{v}\cdot\nabla\left(\frac{1}{n}\right), \qquad (3)$$

where $\epsilon=3p/2=3nT/2$. Here, m is the electron mass, n is the density, v is the velocity, p is the pressure, T is the temperature, ϵ is the internal energy density, $\vec{\psi}=\nabla\times\vec{v}$ is the electron vorticity, and $\vec{\Omega}=e\vec{B}/(mc)$ is the magnetic vorticity. Equation (2) was obtained by taking the curl of Euler's equation. In almost all the previous work on self-generated magnetic fields,[4,5,9] electron inertial effects, represented here by the electron vorticity, have been neglected. These effects must be included to obtain the waves of this Letter. The inclusion of the energy equation, Eq. (3), is also necessary. The flow of internal energy provides a solenoidal pressure force which acts as the source of the magnetic field in Eq. (2). Ampere's law is

$$\lambda_s^2\nabla\times\vec{\Omega}=-\vec{v}, \qquad (4)$$

where $\lambda_s=c/\omega_p$ is the collisionless skin depth and ω_p is the local plasma frequency. Equations (1)-(4), the curl of Eq. (4), and the assumption of a given fixed density profile provide a closed set of equations for the unknowns $\vec{v}$, T, ψ, and Ω. If we linearize these equations about the temperature profile $T_0(x)$ and assume a y and time dependence of $\exp(-i\omega t+iky)$, we obtain

$$\frac{d}{dx}\left(\lambda_s^2\frac{d\Omega}{dx}\right)-(1+k^2\lambda_s^2)\Omega+\xi v_\varphi^{-2}\Omega=0, \qquad (5)$$

where ξ is defined by $\xi=2\lambda_s^2T_0/3L_n^2m$, v_φ is the phase velocity, and $L_n=[d\ln(n)/dx]^{-1}$ is the density gradient scale length. The boundary conditions for the surface waves are that Ω vanishes as $|x|$ goes to infinity. Equation (5) will be recognized as the Sturm-Liouville equation[10] of which the Schrödinger equation is a limiting case. The problem of finding surface modes reduces to the problem of finding the eigenmode of Eq. (5) for each eigenvalue, v_φ^{-2}. In other words, each eigenmode will have a different dispersion relation which depends on the eigenvalue v_φ^{-2}.

Equation (5) can be solved analytically if we assume the density profile of Fig. 1(b). In regions A and C the density gradient scale length is taken to be infinite, forcing ξ to zero. In region B we require the density gradient scale length to be much longer than λ_s and a, but not so large that $1-\xi v_\varphi^{-2}+k^2\lambda_s^2$ is greater than or equal to zero. In region B, ξ is taken to be constant. The problem is now equivalent to the problem of solving Schrödinger's equation in an asymmetric square potential well. The bound solutions correspond to surface waves. The number of surface modes will depend on the depth and width of the "potential well." In regions A and C one finds evanescent solutions decaying as $\exp(q_ax)$ and $\exp(-q_cx)$, respectively. Here, q_a and q_c are defined by $q_a=(\lambda_a^{-2}+k^2)^{1/2}$ and $q_c=(\lambda_c^{-2}+k^2)^{1/2}$. The subscripts on the skin depths, λ, indicate the region to which they pertain. In region B, one finds sinusoidal oscillations with wave number $q_b=(\xi_bv_\varphi^{-2}/\lambda_b^2-\lambda_b^{-2}-k^2)^{1/2}$, where ξ_b is the value of ξ in region B. One sees that, for small k, the waves are localized in x to within a few skin depths of the surface. This is in sharp contrast to most earlier work.[4] In order to connect the solutions, one requires Ω and $\lambda_s^2\Omega$ to be continuous across the boundaries of the regions. With this condition, one obtains the dispersion relation

$$\cos(2q_ba)\lambda_b^2q_b(\lambda_a^2q_a+\lambda_c^2q_c)+\sin(2q_ba)(\lambda_a^2q_a\lambda_c^2q_c-\lambda_b^4q_b^2)=0. \qquad (6)$$

One can see by inspection that Eq. (6) only has a solution when the magnitude of $\cos(2q_ba)$ is small. This occurs whenever the condition $q_ba\simeq(2j+1)\pi/4$ is satisfied. Here, j is an integer that varies from zero to infinity. We see that j is the index for discrete surface modes corresponding to the discrete bound states in the Schrödinger problem. If we take q_ba to be exactly equal to $(2j+1)\pi/4$, then Eq. (6) can be simply solved for the phase velocity of each mode, yielding

$$v_\varphi^2=\frac{2}{3}\,\frac{T_0a^2}{mL_b^2}\left((2j+1)^2\frac{\pi^2}{16}+\frac{a^2}{\lambda_b^2}+k^2a^2\right)^{-1}, \qquad (7)$$

where

$$k^2=\frac{-1}{2}\left(\frac{1}{\lambda_a^2}+\frac{1}{\lambda_c^2}\right)+\frac{1}{2}\left[\left(\frac{1}{\lambda_a^2}+\frac{1}{\lambda_c^2}\right)^2-4\left(\frac{1}{\lambda_a^2\lambda_c^2}-\frac{\lambda_b^8}{\lambda_a^4\lambda_c^4a^4}(2j+1)^2\frac{\pi^2}{16}\right)\right]^{1/2}. \qquad (8)$$

The temperature at the surface, the thickness of the surface, and the density gradient scale length at the surface all play a significant role in determining the propagation speed. We see that when ka is small, the waves are nondispersive. We also note that, for small k, higher modes travel at slower speeds. It is important to note that, unlike the waves studied in Ref. 5, all the points on the wave front

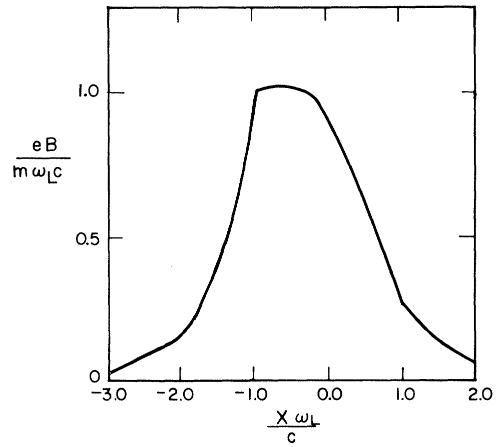

FIG. 2. Magnetic field profile. The solid line is for a density profile of $n_a = 1.75n_0$, $n_b = 0.2n_0$, and $n_c = 0.5n_0$, where n_0 is an arbitrary density. The thickness, a, is taken to be $1.0c/\omega_L$, where $\omega_L^2 = 4\pi n_0 e^2/m$.

travel with the same phase velocity, Eq. (7). Thus, the distance that the waves can travle is not limited by the curvature of the wave front, as in that case. With the eigenvalues, Eqs. (7) and (8), one can obtain the eigenmodes. Furthermore, the phase velocities given by Eq. (7) of this Letter and Eq. (25) of Ref. 5 are significantly different. There is only agreement, for small j, in the limit that a/λ_b is very large and ka is very small. Figure 2 is the display of the fundamental mode for a specific density profile. Profiles of this nature have been observed both in particle simulations[8] and experimentally.[3]

In conclusion, the existence of an entirely new type of surface wave in plasmas has been demonstrated. This wave shares many properties with waveguide modes. It is expected that this new wave will play a crucial role in the understanding of surface energy transport,[8] magnetic insulation,[11] and energy decoupling in the laser-plasma interaction. Additionally, the understanding of how this surface wave couples to radiation will provide a valuable diagnostic of the surface[12] as well as possibly leading to a coherent ultraviolet or soft x-ray light beam.[13]

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