Nuclei: A Superfluid Condensate of α Particles? A Study within the Interacting-Boson Model

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The authors have studied the question of whether pairs of neutrons and pairs of protons of the usual superfluid phases form a bound state to give rise to a superfluid condensate of "α particles." They indeed find indications for this to be the case from a BCS-like study for bosons using the proton-neutron interacting-boson model as well as from an even-odd effect in the number of pairs using experimental binding energies.

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The question of α-particle correlations in nuclei and whether nuclei are a superfluid condensate of α particles rather than an ordinary fermion-pair condensate has long[1-3] been raised (we here prefer to use the term α particle in a broad sense to indicate any kind of bound state between two protons and two neutrons rather than "quartet" or "quadrupel"). For instance Danos and Gillet and Arima and Gillet,[1] Eichler and Yamamura,[2] and Kamimura, Marumori, and Takada[3] have investigated this problem in quite some detail more than a decade ago. More recently this old question seems to have received renewed attention,[4,5] partly because of the success of the proton-neutron interacting-boson model (IBM2).[6] But also Dussel, Lotta, and Perazzo[5] pointed out not long ago that condensation of α particles might indeed be plausible from a systematic investigation of experimental α-particle Q values.

As a matter of fact the question whether pairs of fermions condense as such or as bipairs is an important question not only in atomic nuclei, but has its relevance in several other domains of physics as well. Indeed our investigation has been inspired by a recent work of Nozières and Saint-James[7] where the condensation of excitons versus biexcitons in a semiconductor has been studied in an interacting-boson model with use of a BCS formalism for bosons.

We here essentially follow their ideas, applying them to IBM2. This can, of course, only be a quite crude model for the question of α-particle condensation since the underlying fermion structure of the bosons certainly plays an important role. A completely fermionic treatment of the problem seems, however, quite difficult though some early attempts in this direction exist.[2,3,8] For instance, the exactly solvable model by Eichler and Yamamura[2] exhibits many features which we will also find in our study. Our results can, of course, only be very qualitative but on this level we find some intriguing agreement with experiment; this concerns, for instance, the quite pronounced even-odd effect in the number of pairs and its dependence on the neutron excess which we found from the study of experimental binding energies and which can be considered as the analog of the even-odd effect of ordinary nuclear pairing.

Our starting point is the phenomenological Hamiltonian of the neutron-proton interacting-boson model in its simplest version since we here want to make only a qualitative study (we use standard notation[6]):

$$H=\sum_{\rho}\epsilon_s s_\rho{}^\dagger s_\rho+\epsilon_d\, d_\rho{}^\dagger\, d_\rho-\kappa Q_\pi Q_\nu \tag{1}$$

with

$$Q_{\rho\mu}=d_{\rho\mu}{}^\dagger s_\rho+s_\rho{}^\dagger\tilde{d}_{\rho\mu}+\chi[d_\rho{}^\dagger d_\rho]_{2\mu},$$

where $\tilde{d}_\mu=(-)^\mu d_{-\mu}$ and ρ stands for proton (π) or neutron (ν). The operators s_ρ and d_ρ are pure boson operators approximating proton or neutron pairs coupled to angular momentum 0 and 2, respectively, thus retaining the most important degrees of freedom of nuclear dynamics. The parameters entering (1) have been adjusted from experiment and it is known that one obtains a quite good description of the systematics of the low-lying part of the spectra of even-even nuclei.[6]

The Hamiltonian (1) exhibits a quite strong interaction between neutron and proton pairs and it is thus predisposed for the study of α correlations. The question whether (1) gives rise to a condensation of α particles can be investigated most conveniently by generalizing the BCS formalism for fermions (formation of fermion pairs) to bosons (formation of boson pairs) (for a study of this generalization see, e.g., Nozières and Saint-James,[7] Schuck and Ethofer,[9] and Rig and Schuck[10]). We thus introduce the following Bogoliubov transformation among the boson operators s_ρ and d_ρ:

$$\sigma_\pi^\dagger = u_s s_\pi^\dagger - v_s s_\nu + x_\pi, \quad \delta_{\pi\mu}^\dagger = u_d d_{\pi\mu}^\dagger - v_d \tilde{d}_{\nu\mu}, \quad \sigma_\nu^\dagger = u_s s_\nu^\dagger - v_s s_\pi + x_\nu, \quad \delta_{\nu\mu}^\dagger = u_d d_{\nu\mu}^\dagger - v_d \tilde{d}_{\pi\mu}, \tag{2}$$

with $u_i^2 - v_i^2 = 1$; $i = s, d$. In contrast to the fermion case there can appear constant terms such as x_π and x_ν, the significance of which becomes clear if we write down the vacuum which corresponds to the quasiboson operators (2)[10]:

$$|\Phi\rangle \propto \exp\{z_\pi s_\pi^\dagger + z_\nu s_\nu^\dagger + c_s s_\pi^\dagger s_\nu^\dagger + c_d [d_\pi^\dagger d_\nu^\dagger]_0\} |0\rangle . \tag{3}$$

Here $|0\rangle$ stands for the total vacuum and the new parameters are related to the ones in Eqs. (2) as follows:

$$z_\rho = -x_\rho / u_s; \quad c_s = v_s / u_s; \quad c_d = 5^{1/2} v_d / u_d . \tag{4}$$

The first two terms in the exponent of (3) correspond to the condensate of single bosons (of the usual BCS type) which is taken care of by the constants x_π and x_ν in (2) and the last two terms in the exponent correspond to the boson-pair condensate.

We now look for the minimum of the energy with respect to the parameters of the transformation (2) under the constraints that the total number of particles as well as the neutron excess has a definite value:

$$\delta\langle \varphi | H - \lambda n - \mu(n_\nu - n_\pi) | \varphi \rangle = 0 . \tag{5}$$

The calculus is standard and leads to the following set of coupled equations:

$$2u_i v_i (\epsilon_i - \lambda) + \Delta_i (u_i^2 + v_i^2) = 0, \quad i = s, d, \tag{6a}$$

with

$$\Delta_s = 5\kappa u_d v_d, \quad \Delta_d = \kappa (u_s v_s + \chi^2 u_d v_d + y_\pi y_\nu), \tag{6b}$$

and

$$(\epsilon_s - \lambda) y_\nu + \Delta_s y_\pi = \mu y_\nu,$$
$$y_\nu = -u_s x_\pi - v_s x_\nu, \tag{6c}$$
$$\Delta_s y_\nu + (\epsilon_s - \lambda) y_\pi = -\mu y_\pi,$$
$$y_\pi = -u_s x_\nu - v_s x_\pi . \tag{6d}$$

Equations (6c) and (6d) have, besides the trivial solution $y_\nu = y_\pi = 0$, a nontrivial one which for the case where we have neutron excess is given by

$$y_\nu = -u_s x; \quad x = x_\nu = (n_\nu - n_\pi)^{1/2},$$
$$y_\pi = -v_s x; \quad x_\pi = 0 . \tag{7}$$

In this case we have only neutrons in the single-boson condensate of (3) and all protons are bound in α particles. From (6a) and (6b) we obtain the final expressions for our problem:

$$\left. \begin{matrix} u_i^2 \\ v_i^2 \end{matrix} \right\} = \frac{1}{2} \frac{\epsilon_i - \lambda}{E_i \pm 1}, \quad i = s, d; \tag{8}$$

$$E_i = [(\epsilon_i - \lambda)^2 - \Delta_i^2]^{1/2}; \quad u_i v_i = -\tfrac{1}{2} \Delta i / E_i, \tag{9}$$

$$\Delta_s = -\tfrac{1}{2}\kappa\, 5\Delta_d / E_d , \tag{10a}$$

$$\Delta_d = -\tfrac{1}{2}\kappa [(1 + x^2)\Delta_s / E_s + \chi^2 \Delta_d / E_d]. \tag{10b}$$

The parameter λ is determined as usual from the particle number condition and $\mu = E_s$. We see that Eqs. (8)–(10) resemble ordinary BCS equations for fermions with only a number of signs reversed because of the boson structure of our problem. For instance the quasiparticle energies contain now a difference of two positive numbers under the square root where for fermions we have a sum. In our numerical study however, it turns, out that the particle number condition always prevents the quasiparticle energies from becoming imaginary.

One of the arguments for ordinary proton or neutron pairing in nuclei which is usually put forward is the even-odd effect. This means that it usually costs more energy to take out a proton or a neutron from an even-even nucleus than from an odd one because of the extra binding of the pairs. This gives rise to the very well known sawtooth behavior for the Q values of neutrons (or protons) as a function of neutron (or proton) number. In analogy to this we should find in the case of α-particle superfluidity an even-odd effect as a function of the number of pairs. This implies that it should be more difficult to take a neutron or proton pair out of a nucleus which (i.e., in practice whose open-shell configuration) consists only of α particles than from a nucleus which consists of α particles plus one odd pair. In order to investigate this we took as our inert core the doubly magic nucleus ${}^{132}_{82}\mathrm{Sn}_{50}$ and filled in eight α particles which leads to ${}^{164}_{98}\mathrm{Dy}_{66}$. We then added an odd neutron or proton pair leading to $^{166}_{100}\mathrm{Dy}_{66}$ and $^{166}_{98}\mathrm{Er}_{68}$, respectively. From their averaged (in order to smooth out eventual isotopic effects) binding energies[11] we subtracted that of ^{164}Dy which, e.g., gives us the point at $n=17$ of the full line in Fig. 1. We then subtracted from the binding energy of $^{168}_{100}\mathrm{Er}_{68}$ (nine α's) the averaged binding energies of $^{166}_{98}\mathrm{Er}_{68}$ (minus one neutron pair) and $^{166}_{100}\mathrm{Dy}_{66}$ (minus one proton pair) which yields the point at $n=18$ on the full line of Fig. 1. This procedure was continued up to the addition of eighteen α particles to the $^{132}_{82}\mathrm{Sn}_{50}$ core leading to the pronounced sawtooth behavior as a function of the number of pairs as shown in Fig. 1, corroborating thus the idea of α-particle superfluidity.

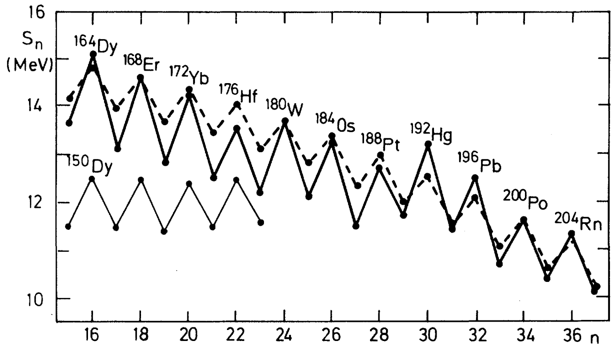

FIG. 1. Even-odd staggering as a function of boson number. The topmost full line corresponds to the chain beginning with ^{164}Dy (as described in the text) and the lower full line to the chain of ^{150}Dy as obtained from the mass tables. The broken line corresponds to the present calculation.

Before discussing the significance of the other curves in Fig. 1 let us return to our theory and investigate how from there such an even-odd effect finds its natural explanation. We calculate the ground-state energies for even and odd boson numbers using ^{132}Sn as an inert core:

$$E_n{}^0=\langle\Phi|H|\Phi\rangle=2\epsilon_s v_s{}^2+10\epsilon_d v_d{}^2-5\kappa[2u_s v_s u_d v_d+\chi^2(u_d v_d)^2], \tag{11a}$$

$$E_{n+1}{}^0=\langle\Phi|\sigma_\rho H\sigma_\rho{}^\dagger|\Phi\rangle=E_n{}^0+\epsilon_s(u_s{}^2+v_s{}^2)+10\kappa(u_s v_s)(u_d v_d), \tag{11b}$$

and determine the parameter v_d in both cases by minimizing the energy. (v_s is eliminated by the constraint on the particle number.) In order to take account of the A dependence of ϵ_s and ϵ_d (which is certainly present over such a long mass range as shown in Fig. 1) we made the following *Ansatz*:

$$\epsilon_{s,d}=\epsilon_{s,d}{}^0[132/(132+gn)]^{1/3},$$

where $\epsilon_s{}^0$ and g are adjustable parameters found to be 13.7 MeV and 0.15, respectively, in the present analysis. The difference $\epsilon=\epsilon_d{}^0-\epsilon_s{}^0$ has been taken to be 0.2 MeV, a value compatible with the IBM2 parameters. For κ we took 0.5 MeV and χ was put equal to zero because we found—within reasonable ranges—very little dependence of our results on this parameter and $\chi=0$ can be considered as its mean value. With these parameters we then calculated from (8)-(11) the broken curve of Fig. 1 which shows semiquantitative agreement with the corresponding experimental data.

One certainly cannot expect more from our theory and the present form of the IBM2 Hamiltonian where the internal structure of the bosons is entirely neglected; one also might think that for a more realistic investigation a more complete IBM2 Hamiltonian including for instance the Majorana force and other bosons should be taken.[8] One then could even take the mass differences considered here for the determination of certain parameters in the IBM2. Although the effect of deformation may be relatively small for energy differences considered here, one definitely should allow for a deformed trial function (3). Nevertheless our model study seems to indicate that protons and neutrons in doubly open nuclei, instead of forming two superfluids of proton and neutron pairs, rather agglomerate to form α particles which condense to a superfluid phase.

Before drawing definite conclusions we should, however, keep several points in mind. The effects we are after are genuine four-fermion correlations which therefore should not already be contained in a fermion wave function of the mean-field type. It is, however, clear that the asymmetry energy contained in the usual Hartree-Fock-Bogoliubov theory gives rise, at least partially, to the effects that we are here discussing in our interacting-boson model. How much of the effect is due to genuine four-fermion correlations can in the end probably only be decided from a true four-fermion calculation.

That the bosonic description might not be entirely devoid of exhibiting true four-body correlations could, however, be concluded from the following observation: Suppose that the nucleus $Z=N=50$ existed; our full sawtooth line of Fig. 1 already discussed above would consist then of $n/$

2 α particles and sixteen unpaired neutron bosons. Had we unpaired fermions instead of unpaired bosons, the fermion-pair superconductivity would be strongly suppressed because of the blocking effect, as is very well known.[10] On the contrary in our case of *boson-pair condensation* the superconductivity effect is *enhanced* in the presence of unpaired bosons as is, e.g., easily seen from Eq. (10b) where the interaction strength κ becomes multiplied by a factor of $1+x^2$ where x^2 is the number of unpaired bosons. This effect is also verified experimentally and demonstrated in Fig. 1 where we show besides the sawtooth line for sixteen neutron bosons plus $n/2$ α's also another sawtooth line where we successively diminished the number of unpaired bosons. We see that reducing the number of unpaired bosons from sixteen to nine reduces the amplitude of the even-odd staggering by about a factor of 2, a quite nice demonstration of the above-mentioned enhancement effect indeed.

In conclusion we can say that our BCS-like theory for bosons in conjunction with the IBM2 Hamiltonian suggests that open-shell nuceli (open in protons and neutrons) may form a superfluid condensate of α particles rather than separate superfluid phases of proton and neutron pairs. This is of course to be understood in the context that finite systems never undergo a real phase transition, but this problematic point is the same for the condensation of α particles as it is for the one of neutron or proton pairs; the well-known answers to the latter case[10] apply therefore equally to the case of α particles.

Of course further studies on our problem are in order. Firstly, one should perhaps take a more general Hamiltonian (inclusion of, e.g., the g boson is easy). Secondly, preliminary studies show that in the filling in of α particles the system has a tendency to deform; it would therefore be very interesting to study the phase transition from sphericity to deformation within our model. Thirdly, one should try to reformulate our theory using real fermions and not bosons since the underlying bifermion structure of the bosons probably plays an important role for this problem. Investigations on these points are in progress.

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