Relationship between Faraday Rotation and Hall Effect in Amorphous Rare-Earth–Transition-Metal Alloys

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The Faraday rotation and spontaneous Hall effect were investigated in amorphous thin films with compositions $(GdFe)_{1-x}M_x$ $(M = Bi, Sn, Au)$. Bismuth and tin are found to increase the Faraday rotation, φ_F, the Hall resistivity, ρ_H, and the sample resistivity, ρ, while Au decreases φ_F, ρ_H, and ρ as compared to values observed for GdFe. Specifically, φ_F is proportional to the Hall angle (ρ_H/ρ). A model first proposed by Voigt is developed to account for this proportionality.

In this paper we report data showing that the Faraday rotation of polarized light increases with an increase in the spontaneous Hall effect in amorphous rare-earth–transition-metal alloys. It is also shown that changes in magnetization alone cannot be responsible for this relationship.

As early as 1893 Kundt[1] emphasized the proportionality of Faraday rotation and Hall effect with magnetization. Judy[2] has recently reviewed a model proposed by Voigt[3] which relates Faraday rotation to Hall currents. It is this model that we use to explain our experimental results. Darwin[4] and also Gurevich and Yassievich[5] have presented theoretical discussions relating the dielectric tensor to the conductivity tensor but no data were given. Krinchik[6] examined data on Fe, Ni, and Co and concluded that magneto-optical phenomena and Hall effect are influenced by an internal effective magnetic field.

It is also apparent from separate theoretical discussion of both Faraday rotation and spontaneous Hall effect in ferromagnetic metals that a relationship should exist. Argyres[7] treats the Faraday and Kerr effects on the basis of the band theory of metals using the spin-orbit interaction. This interaction provides a mechanism which links electric currents, having polarized magnetic moments, with optical properties of the material.

A similar approach concerning the Hall effect in ferromagnetic metals is taken by Karplus and Luttinger[8] and also Berger.[9] The spin-orbit interaction of the polarized conduction electrons gives rise to a Hall current perpendicular to the magnetization. Berger has called this a "side jump" mechanism but Lyo and Holstein[10] have pointed out that the theories of Karplus and Luttinger and

of Berger are equivalent.

The samples we have studied are listed in Table I; details of sample preparation are given elsewhere.[11,12] They are ferrimagnetic alloys of the form $(Gd_x Fe_{1-x})_{1-y}M$ $(M = Bi, Sn, or Au)$. Hansen and Urner-Wille[11] and Urner-Wille, Hansen, and Witter[12] have found that the saturation Faraday rotation, ψ_F, in Gd-Fe is more than 10^5 deg/cm and that ψ_F increases when Sn and Bi are added. The addition of Au decreases φ_F.

A wavelength of 633 nm was used for the optical measurements obtained with a He-Ne laser. In this paper, the wavelength dependence is not described. However, it was found[13] in Gd-Fe-Bi that the value of Faraday rotation showed only a shallow maximum in the 600 to 800 nm range and then decreased slowly for shorter wavelengths. The rotation measurements were performed on films 80 to 100 nm thick with use of an optical hysteresigraph.[11] Magnetic fields up to 15 kOe were used. This value of field is sufficient for saturating these ferrimagnetic samples.

With use of the van der Pauw[14] technique the Hall resistivity and sample resistivity are obtained. The Hall effect in amorphous magnetically ordered materials such as Gd-Fe is almost entirely due to the asymmetric scattering of the conduction electrons by the magnetic ions. It is known as the extraordinary Hall effect or the "spontaneous" Hall effect.

The values of Hall resistivity ρ_H and the Faraday φ_F rotation are the basic measurements. Dividing these by the saturation magnetization $4\pi M_s$ we obtain the spontaneous Hall coefficient, $R_s = \rho_H/4\pi M_s$, or the Kundt's constant, $K_s = \varphi_F/4\pi M_s$. Another important parameter is the ratio

TABLE I. Results given for 4.2 K; μ_B/Fe, Bohr magnetons per Fe based on Gd having $7\mu_B$; ρ, sample resistivity; ρ_H, Hall resistivity; φ_F, specific Faraday rotation; and $4\pi M_{total}$, total magnetization of both Fe and Gd as described in the text. Compositions marked with asterisks showed negative ρ_H and φ_F.

Sample Composition Atomic fraction	data point	μ_B/Fe	ρ $\mu\Omega$cm	ρ_H $\mu\Omega$cm	$\phi_F \times 10^5$ deg/cm	$4\pi M_{Total}$ kG
$Gd_{0.26}Fe_{0.74}$*		2.18	274	11.5	2.7	23.1
$Gd_{0.18}Fe_{0.82}$	×	2.20	232	7.6	2.1	21.6
$Gd_{0.23}Fe_{0.77}$		2.18	256	11.4	2.8	17.9
$(Gd_{0.26}Fe_{0.73})_{0.86}Au_{0.14}$		2.55	232	8.0	2.6	23.1
$(Gd_{0.28}Fe_{0.72})_{0.85}Au_{0.15}$*	O	2.63	260	11.0	2.1	21.7
$(Gd_{0.26}Fe_{0.74})_{0.7}Au_{0.30}$		2.55	140	4.0	1.8	17.7
$(Gd_{0.26}Fe_{0.72})_{0.65}Au_{0.35}$		2.93	205	2.1	1.5	17.7
$(Gd_{0.25}Fe_{0.75})_{0.92}Bi_{0.08}$*		2.21	388	19.1	3.7	17.5
$(Gd_{0.26}Fe_{0.74})_{0.86}Bi_{0.14}$	Δ	2.73	417	16.5	3.9	19.4
$(Gd_{0.26}Fe_{0.74})_{0.82}Bi_{0.18}$		2.90	366	17.7	3.8	18.4
$(Gd_{0.26}Fe_{0.74})_{0.91}Sn_{0.09}$		2.52	339	15.7	4.1	21.6
$(Gd_{0.27}Fe_{0.73})_{0.9}Sn_{0.10}$*		2.52	324	16.2	4.3	21.1
$(Gd_{0.24}Fe_{0.76})_{0.87}Sn_{0.13}$	□	2.30	261	16.0	4.6	19.0
$(Gd_{0.27}Fe_{0.73})_{0.82}Sn_{0.18}$*		2.33	287	14.6	4.4	17.9

ρ_H/ρ. This gives the Hall angle θ_H satisfying $\tan\theta_H = E_y/E_x = \rho_H/\rho$, where E_y is the Hall field and E_x is the applied electric field which gives rise to the current in the specimen.

In ferrimagnetic order, as found in Gd-Fe, and Gd-Fe with Bi, Sn, or Au additions, two magnetic sublattices, one for Gd and one for Fe, are oriented antiparallel. For this the saturation magnetization $4\pi M_s$ is the difference of the two sublattice magnetizations. In order to obtain the values of the individual sublattice magnetizations, we assume that each Gd atom has seven unpaired spins with all atomic moments pointed parallel. For pure Gd this gives a $4\pi M$ of 24 700 G. Using the chemical composition as determined from electron microprobe analysis, we calculate the fractional volume of Gd to obtain $4\pi M_{Gd}$. The value of $4\pi M_{Fe}$ is then given by $4\pi M_{Gd}$ plus or minus the measured value of $4\pi M_s$ depending on which sublattice is dominant. From $4\pi M_{Fe}$ we calculate μ_B/Fe as given in Table I using crystalline densities because amorphous density values are not available. It must be emphasized that when the Fe sublattice is dominant the measured values of ρ_H and φ_F are positive and when the Gd sublattice is dominant ρ_H and φ_F are negative. As noted in Table I, those compositions marked with an asterisk have measured negative values

of ρ_H and φ_F.

The signs of ρ_H and φ_F are defined in terms of the direction of magnetization. Pure Fe[15,16] has positive ρ_H and φ_F and pure Gd[15,17] has negative values. There have been a number of investigations concerning the change of sign of ρ_H[18,19] and φ_F[20] and the associated contribution of the two sublattice magnetizations. McGuire, Gambino, and O'Handley[15] have taken the approach that in ferrimagnetic Gd-Co and Gd-Fe the Gd and Co or Fe all contribute to ρ_H. Since Gd and Co or Fe have opposite signs for ρ_H and point in opposite directions the Hall scattering is additive. Therefore, we use $4\pi M_{tot} = |4\pi M_{Fe}| + |4\pi M_{Gd}|$ to obtain a realistic value for the Hall coefficient, R_s. Values of R_s can be calculated from Table I. R_s ranges from 3.5×10^{-10} Ω cm/G for Gd-Fe to 9.6×10^{-10} when 0.18 Bi is added.

The same value of $4\pi M_{tot}$ is taken to determine the Kundt's constant, $K = \varphi_F/4\pi M_{tot}$. Generally the temperature variation of φ_F represents the sublattice magnetizations given as

$$\varphi_F(T) = A \, 4\pi M_{Fe}(T) - B \, 4\pi M_{Gd}(T),$$

where A and B are constants. The magneto-optical coefficients determine the temperature dependence and can have negative or positive sign. As obtained from the data in Table I, K has val-

ues of 9.72 deg/cm G for Gd-Fe to 20.6 for 0.18 Bi. Both fundamental parameters R_s and K (which have units per gauss) vary strongly with composition. As will be discussed, it is the change of R_s and K with resistivity that is most important. It is simplest to discuss this variation in terms of φ_F and ρ_H.

The measurements of ρ_H and φ_F listed in Table I increase with each other; however, they do not increase in any easily identifiable way with increasing Fe moment. It is noted that the Au as well as the Bi and Sn cause an increase in Fe moment. Both Bi and Sn cause ρ_H and φ_F to increase but Au in Gd-Fe causes a decrease in ρ_H and φ_F.

The dependence of R_s on resistance in magnetic alloys has been discussed in detail by Berger.[9] For concentrated amorphous magnetic alloys such as we have in this work, ρ_H or R_s depends on ρ^2. This dependence can be described using the Berger "side jump" model. It is noted that Asomoza et al.[21] working with silver–rare-earth amorphous and crystalline alloys give support to the side jump model. The increase of ρ_H with resistivity is thus accounted for by theory. Since φ_F increases with ρ_H, then obviously φ_F increases in some way with resisitivity. Our proposed model is that the Faraday rotation increases primarily because the Hall angle increases and only indirectly because resistivity increases.

In Fig. 1, we plot φ_F vs ρ_H/ρ to illustrate in its simplest form this dependency of Faraday effect on Hall effect. There is a reasonable linear relationship.

Our discussion of Faraday rotation in its relationship to Hall effect follows the Voigt model[3] based on the description given by Judy.[2] When a light wave enters a conducting medium, the optical electric field (E_0) causes current carriers to move coherently with the incident electric field. If these current carriers experience a Hall effect then there is also a Hall electric field (E_y) synchronous with the optical field E_0. This leads to a new field E_0' rotated at an angle θ' as given by $\tan\theta' = E_y/E_0$.

When the optical field is reversed at each half wavelength, then E_y is also reversed and the rotation θ' remains in the same direction. Thus, for each wavelength of light there is a Faraday rotation of $2\theta'$.

The polarized light traveling in the amorphous alloy undergoes a change of wavelength (λ) given by λ/n, where n is the index of refraction. The specific Faraday rotation φ_F should equal $2\theta'$
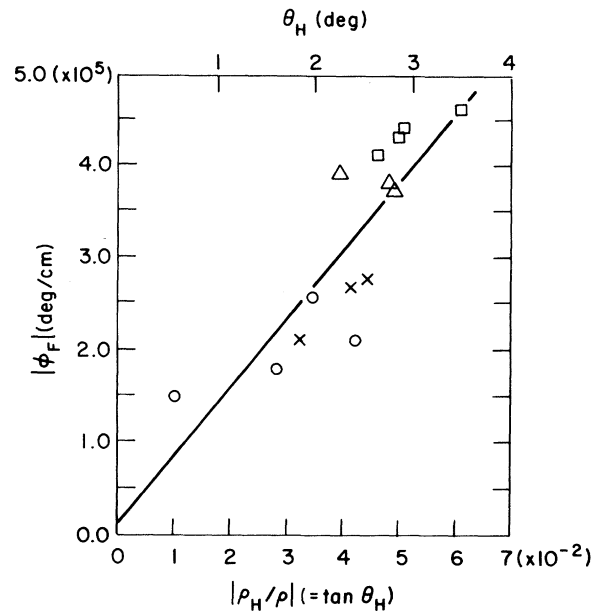

FIG. 1. Specific Faraday rotation φ_F vs ρ_H/ρ at 4.2 K. The Hall angle in degrees is given at top of graph. Data points are defined in Table I.

times the number of wavelengths per centimeter, giving $\varphi_F = 2n\theta'/\lambda$ or $\varphi_F = 2n\theta_H/\lambda$ if we assume that θ', the optical Hall angle, is equal to θ_H the dc Hall angle. This relationship is subject to experimental test if n is known for these alloys. For example, with use of the data from Fig. 1 of $\theta_H = 4°$ when $\varphi_F = 5.3\times10^{-5}$ deg/cm, a value of $n = 4.2$ is necessary. The value of n has not been measured in these alloys but $n = 3.4$ at 630 nm is given[22] for crystalline Fe. Therefore, $n = 4.3$ is not unreasonable.

The simple model presented here where we have attributed the Faraday rotation in amorphous alloys to an interaction of polarized light with the Hall-effect conduction electrons can lead to further considerations in the band theory of metals. For example, if our simple model is primarily an intraband effect of the conduction electrons then the Faraday rotation could also be an intraband effect. Both Erskine's[23] analysis of magneto-optical absorption for ferromagnetic metals and Karplus and Luttinger's[8] for Hall effect give the most importance to interband transitions. This means that more information about the frequency dependence of the Faraday rotation and Hall effect is needed. At higher frequencies, interband contributions become more important and conduction-electron polarization may change. It is then possible that the spontaneous Hall effect may

change sign as a function of frequency. For example, iron and cobalt maintain the same sign of ρ_H and φ_F but nickel does not. Nozieres and Lewiner[24] have presented a detailed theory of side jump and skew Hall scattering for a two-band semiconductor containing spin impurities. Changes of sign for the Hall effect are accounted for in this two-band model which may also have application to metals.

The discussion of band theory in amorphous metals presents problems because of atomic disorder. However, short-range order and observed coordination numbers in amorphous metals makes them similar to fcc crystals. Recent work by Moruzzi et al.[25] follows this approach and develops the concept of electronic structure for disordered systems such as studied here.

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